

New Column Design Formula

John J. Zahn

Abstract

The old column formulas of the 1986 National Design Specification for Wood Construction are compared with the new 1991 column formulas. The new design method is considered to be more conservative; a change that is necessitated by recent column test data. It will also be better able to accommodate future changes in the wood resource base and the introduction of new wood products because it has an adjustable parameter that models degrees of homogeneity and straightness.

Introduction

In 1986, Zahn recommended the adoption of new design criteria for wood-beam columns; in particular, the adoption of Ylinen's column formula (Zahn, 1986; Ylinen, 1954). The recommendation was well received, but implementation in the 1986 National Design Specification for Wood Construction (NDS) (NFPA, 1986) was delayed

pending the incorporation of certain other reforms, especially the new design values for in-grade lumber. This paper will compare the new column formulas with the old formulas, cite the reasons for making this major change, and provide an example of how it will affect column design.

Old Column Formulas

In the 1986 NDS, the maximum design value for compression, F'_c , is marked with a prime to denote that it has been "reduced for slenderness." That reduction for slenderness is determined by the following formulas:

Short Columns. For $l_e/d \leq 11$:

$$F'_c = F_c \quad [1]$$

where l_e is the effective length appropriate to the support

conditions, d is the cross-section dimension measured in the direction that buckling displacement can occur, and F_c is the tabulated design stress for compression parallel-to-grain multiplied by all applicable modification factors, such as those for load duration, moisture content, etc. Equation 1 simply shows that, for short columns, there is no reduction.

Intermediate Columns. For $11 < l_e/d < K$, where:

$$K = 0.671\sqrt{E/F_c} \quad [2]$$

the reduction is:

$$F'_c = F_c \{1 - (1/3) [(l_e/d)/K]^4\} \quad [3]$$

where E is the modulus of elasticity.

Long Columns. For $K \leq l_e/d \leq 50$,

$$F'_c = 0.30E/(l_e/d)^2 \quad [4]$$

This is Euler's historic formula for the elastic buckling of a column.

Needed Reforms

Safety. The 1986 NDS Equation 3 for the intermediate range of slenderness is the so-called FPL fourth-power parabola (FPL, 1987). It was based on data for large timbers (Newlin and Gahagan, 1930) and clear Sitka spruce (Newlin and Trayer, 1941). It is not conservative for modem, in-grade lumber (Buchanan, 1984; Johns and Buchanan, 1982; Malhotra, 1972; Neubauer, 1972; Zahn, unpublished data). Figure 1 shows in-grade data for columns, and the 1986 (dashed line) and 1991 formulas. In this figure, the factors of safety are set equal to unity for direct comparison with column strength data. The abscissa is $l_e/(\pi d) \sqrt{12F_c/E}$ so that data for several species may be combined on a single figure. Note that the column stability factor of the 1986 NDS is ~25% greater than in the 1991 NDS (NFPA, 1991) at $l_e/(\pi d) \sqrt{12F_c/E} = 1$, a value of slenderness that lies in the range of most frequent application. It should be noted that calculated column capacity in the 1991 NDS will also differ from the 1986 NDS due to the adoption of new design values for dimension lumber.

Ease of use. The slenderness range is further separated into three ranges: short, intermediate, and long. Each has its own design criterion. This complicates attempts to program the design criteria on electronic calculators. A single formula would involve fewer decisions, and would avoid the discontinuity at the boundary between short and intermediate ranges.

New Column Formula

In the 1991 NDS, a prime is used to denote that the tabulated design value, F_c , has been multiplied by all the applicable modification factors, such as load duration, moisture content, temperature, etc. Table 2.3.1 in the 1991 NDS specifies the modification factors that apply to each tabulated design value. The resulting quantity is called the "allowable design value." The effect of slenderness is

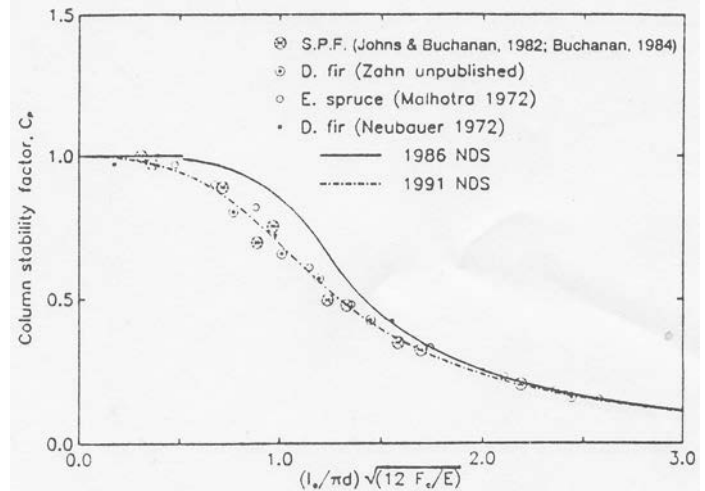


Figure 1. 1991 and 1986 NDS column formulas compared with data from several sources. Marker size is congruent with sample size, which ranged from 1 to 100.

accounted for by one of the modification factors, namely the "column stability factor," C_p . Once again, l_e/d is limited to a maximum value of 50.

The column stability factor is calculated as follows:

$$C_p = \frac{1 + (F_{ce}/F_c^*)}{1.6} - \sqrt{\frac{[1 + (F_{ce}/F_c^*)]^2}{2.56} - \frac{F_{ce}/F_c^*}{0.8}} \quad [5]$$

where F_c^* is the tabulated compression design value multiplied by all applicable modification factors except $C_F < 1.0$, and C_p ; $F_{ce} = K_{ce}E'/(l_e/d)^2$; $K_{ce} = 0.3$ for visually graded lumber; and $K_{ce} = 0.418$ for products with a coefficient of variation of $E \leq 0.11$.

Here, C_F is the size effect factor. In the 1986 NDS, this applied only to bending. In the 1991 NDS, it applies to tension and compression as well.

The column stability factor is defined so that it is essentially unity for small l_e/d and essentially F_{ce}/F_c^* for very large l_e/d , thereby eliminating the need for separate formulas for "short" and "long" ranges of l_e/d . The numbers 0.8, 1.6, and 2.56 in various denominators in Equation 5 are c , $2c$, and $4c^2$, respectively, where c is an adjustable parameter. With this parameter, the formula will agree with test data in the middle range of l_e/d . In Figure 1, the value $c = 0.8$ provides a good fit to data for in-grade lumber. Theoretically, c should be closer to unity for materials that are more homogeneous or more straight. Therefore, there is a separate column stability factor for glued-laminated (glulam) columns, which has exactly the same form as Equation 5, except that c has been set equal to 0.9. In principle, as new wood-based materials are introduced, they could be tested and assigned an appropriate c value without changing the basic form of the column design equation.

Example

To calculate the axial compressive load capacity of a nominal, Douglas fir, 7-ft-long (2.1 m) 2x4 (38x89 mm)¹,

¹For metric conversion, the nominal size is given as the equivalent standard dressed size in millimeters.

we assume that the member is laterally supported against buckling in the weak direction. Tabulated design values are $F_c = 1000$ psi (6.894 MPa) and $E = 1,700,000$ psi (11.72 GPa). The slenderness ratio is $l/d = 84/3.5 = 24$.

1986 National Design Specification

Under the 1986 NDS, we calculate:

$$K = 0.671 \sqrt{1,700,000/1000} = 27.7$$

This is greater than l/d and, therefore, the column is in the intermediate range. Using Equation 3, we obtain:

$$F_c^* = 1000 [1 - (1/3) (24 / 27.7)^4] = 811 \text{ psi}$$

and calculate the capacity, P , to be:

$$P = 811(1.5)(3.5) = 4260 \text{ lb}$$

1991 National Design Specification

Under the 1991 NDS, we first calculate the Euler Buckling stress:

$$F_{cE} = [(0.3)(1,700,000)] / 24^2 = 885 \text{ psi}$$

and use it and F_c^* to calculate the column stability factor. Here we shall assume that there are no other applicable modification factors, so $F_c^* = 1000$ psi.

$$C_P = \frac{1 + (885/1000)}{1.6} - \sqrt{\frac{[1 + (885/1000)]^2}{2.56} - \frac{885/1000}{0.8}} = .647$$

Therefore, the column capacity is:

$$P = 1000(.647)(1.5)(3.5) = 3400 \text{ lb}$$

This constitutes about a 20% reduction from that which the 1986 NDS allowed. This reduction is typical and was necessary in order to accurately reflect what we know about the column strength of in-grade lumber.

Summary

Very long and very short columns have the same design capacity under the new design criterion as under the old criteria. In the intermediate range of slenderness, however, the new column design equation is considerably more conservative than what has been used in the past. This will bring the margin of safety for intermediate-length columns into agreement with the margin of safety for short and long columns. This reform was made necessary by new information about the strength of wood columns. Because long columns fail suddenly and without warning, it has always been the intent to employ a slightly greater margin of safety in their design than is used for short columns. This new formula accomplishes this better than any tangent parabola or other special intermediate formula ever could, because it bases all column design on a combination of the long- and short-column capacities.

In addition, the form of the equation is now such that member homogeneity and straightness are modeled by a single adjustable parameter. As the wood resource base changes and reconstituted products become more competitive with solid wood products, it is likely that new column materials will be introduced. Future code reforms to accommodate new products or changes in the solid wood resource base can be easily accomplished by adjusting the numerical value of this parameter.

References

- Buchanan, A.H. 1984. Strength model and design methods for bending and axial load interaction in timber members. Ph.D. thesis, University of British Columbia, Vancouver, British Columbia, Canada.
- Forest Products Laboratory (FPL). 1987. Wood handbook: Wood as an engineering material. Agriculture Handbook No. 72. USDA Forest Service, Washington, DC. 466 p.
- Johns, K.C., and A.H. Buchanan. 1982. Strength of timber members in combined bending and axial loading. p. 343-368. In Proceedings of the International Union of Forestry Research Organizations. Wood Engineering Group S5.02.
- Malhotra, S.K. 1972. A rational approach to the design of solid timber columns. Applications of Solid Mechanics. Study No. 7. University of Waterloo, Ontario, Canada.
- National Forest Products Association (NFPA). 1986. National design specification for wood construction. NFPA, Washington, DC.
- National Forest Products Association (NFPA). 1991. National design specification for wood construction. NFPA, Washington, DC. (in press).
- Neubauer, L.W. 1972. Full-size stud tests confirm superior strength of square-end columns. *Transactions of the American Society of Agricultural Engineers*. 15(2):346-349.
- Newlin, J.A., and J.M. Gahagan. 1930. Test of large timber columns and presentation of the Forest Products Laboratory column formula. Technical Bulletin No. 167. U.S. Department of Agriculture, Forest Service, Washington, DC.
- Newlin, J.A., and G.W. Trayer. 1941. Stresses in wood members subjected to combined column and beam action. Forest Products Laboratory Report No. 1311, U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, Madison, WI.
- Ylinen, A. 1954. A method of determining the buckling stress and the required cross-sectional area for centrally loaded straight columns in elastic and inelastic range. p. 529-550. In Publications of the International Association for Bridge and Structural Engineering, Vol. 16. International Association for Bridge and Structural Engineering, Zurich, Switzerland.
- Zahn, J.J. 1986. Design of wood members under combined load. *Journal of Structural Engineering*. 112(9):2109-2126.
- J.J. Zahn, Research Engineer, USDA Forest Service, Forest Products Laboratory, Madison, WI.