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# Biaxial Beam–Column Equation for Wood Members

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## Abstract

The National Forest Products Association (NFPA), upon the recommendation of its Technical Advisory Committee and its Special Advisory Committee, has adopted the author's design equation for compression members under combined bending and compression for the next edition of the National Design Specification (NDS). Included in the formulation are new slenderness reduction factors (now called the "column stability factor $C_P$" and the "beam stability factor $C_L$"). This paper compares the new and old equations and indicates their effect on design.

## Introduction

In 1986, the author proposed a new design equation for wood beam-columns (Zahn 1986), which is more accurate and more general that what has been used until now. That proposal has been adopted by the NFPA. It was originally written in the notation of the current NDS but is presented here as it will appear in the new edition.

## Stability Factors $C_P$ and $C_L$

The column stability factor $C_P$ is

$$C_P = \frac{1 + (F_{cE}/F_c^*)}{1.6} - \sqrt{\frac{[1 + (F_{cE}/F_c^*)]^2}{2.56} - \frac{F_{cE}/F_c^*}{0.8}} \tag{1}$$

in which

$$F_{cE} = \frac{0.3E'}{(l_e/d)^2} \tag{2}$$

$$F_c^* = F_c C_D C_M C_t \tag{3}$$

$E'$ = allowable modlus of elasticity. $l_e$ = effective length of compression member, $d$ = least dimension of rectangular compression member, $F_c$ = tabulated compression design value parallel to grain. $C_D$ = load duration factor, $C_M$ = moisture content factor, and $C_t$ = temperature factor.

For flatwise bending, the beam stability factor $C_L$ is unity. For edgewise bending,

$$C_L = \frac{1 + (F_{bE}/F_b^*)}{1.91} - \sqrt{\frac{[1 + (F_{bE}/F_b^*)]^2}{3.66} - \frac{F_{bE}/F_b^*}{0.957}} \tag{4}$$

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in which

$$F_{bE} = \frac{0.438E'}{R_B^2} \tag{5}$$

$$F_b^* = F_b C_D C_M C_t C_f C_c \tag{6}$$

$R_B \equiv \sqrt{l_e d/b^2}$ = slenderness ratio of bending member, $F_b$ = tabulated bending design value, $C_f$ = form factor, and $C_c$ = curvature factor.

These equations are simply the Ylinen column formula with its adjustable parameter $c$ set equal to 0.8 for columns (based on recent data) and 0.957 for beams (based on calibration to past practice, namely the FPL 4th-power parabola). The 4th-power parabola (in the notation of the new NDS) was

$$C_P = 1 - \frac{1}{3}\left(\frac{l_e/d}{K}\right)^4 \tag{7}$$

in which $K = 0.67\sqrt{E/F'_c}$ and $F'_c$ = allowable compression design value parallel to grain. For $l_e/d \leq 11$, you were to use $C_P = 1$, which created a slight discontinuity at $l_e/d = 11$. For $l_e/d > K$, you were to use the Euler equation, namely

$$C_P = \frac{0.30E/F'_c}{(l_e/d)^2} \tag{8}$$

For columns, the new equation is more conservative than past practice. Figure 1 shows a comparison of the new and old equations along with column test data from several sources (Buchanan 1984, Johns and Buchanan 1982, Malhotra 1972, Neubauer 1972, and Forest Products Laboratory Unpublished). In Fig. 1, the discontinuity at $l_e/d = 11$ is ignored, and the factors of safety have been removed from the equations for case of comparison with test data.

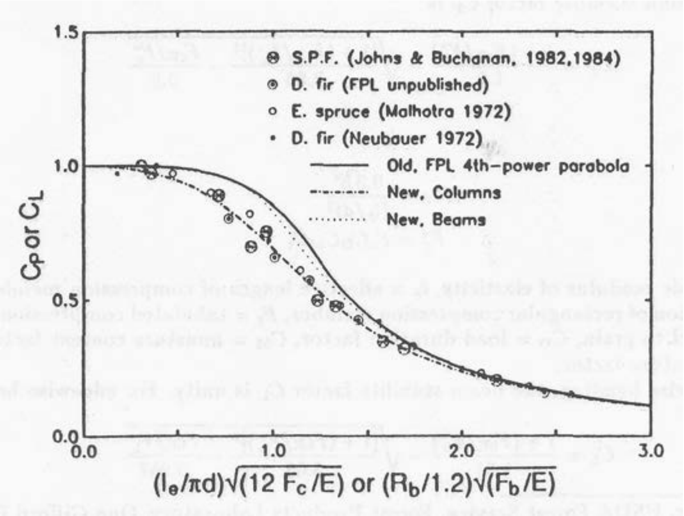


**Figure 1.** New equations for columns and beams compared with old FPL 4th-power parabola and data from several sources. Marker size is congruent with sample size, which ranged from 1 to 100.

## Biaxial Beam–Column Equation

The new interaction equation for combined flexure and axial loading has been extended to include biaxial bending. The code requires that members be so proportioned that

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{b1}}{F'_{b1}[1-(f_c/F_{cE1})]} + \frac{f_{b2}}{F'_{b2}[1-(f_c/F_{cE2})]} \le 1.0 \tag{9}$$

in which

$$f_c < F_{cE1} \equiv \frac{0.3E'}{(l_{e1}/d_1)^2} \tag{10}$$

and

$$f_c < F_{cE2} \equiv \frac{0.3E'}{(l_{e2}/d_2)^2} \tag{11}$$

and $F'_c$ includes the column stability factor calculated both edgewise and flatwise and taking the smaller value. Each allowable stress $F$ in the denominator takes a load duration factor applicable to the load that induced the corresponding applied stress $f$ in the numerator. Bending stress induced by eccentricity of the axial load is increased by the factor $[1 + 0.234(f_c/F_{cE1})]$ or $[1 + 0.23.1(f_c/F_{cE2})]$.

The old interaction equation was

$$\frac{f_c}{F'_c} + \frac{f_b}{F'_b - Jf_c} \le 1.0 \tag{12}$$

in which

$$J \equiv \frac{(l_e/d) - 11}{K - 11}, \qquad 0 \le J \le 1 \tag{13}$$

Bending stress induced by eccentricity of the axial load was increased by the factor 1 +1.5$J$/6. The denominator of the second term was introduced by Newlin. He originally subtracted $f_c$ from in $F'_b$ an attempt to account for the so-called ***P–Δ*** effect (bending caused by the axial load). The $J$ factor was added later in an attempt to gradually phase in this effect with increasing $l_e/d$. A more exact analysis of the ***P–Δ*** effect by Southwell employs a moment modification factor, which is effective at all values of $l_e/d$. Newlin's analysis was more conservative.

The new equation for combined flexure and axial loading is more liberal than that of past practice for two reasons: (1) the axial load term is squared (based on recent data) and (2) the ***P–Δ*** effect is now accounted for by Southwell moment modification factors rather than the Newlin factor presently in use.

The net effect on design depends on the length of the member and on whether the load is primarily axial compression or bending. For members that are of intermediate length and primarily loaded in compression, the required size will increase. This will decrease the allowable spans for some truss designs. On the other hand, for members that are very short or for which the ***P–Δ*** effect predominates, the required member size will decrease. This will increase the allowable spans of some truss designs. Overall, no allowable span will change by more than ±l or 2 ft. (±0.5 m). This change is is addition to changes brought about by new design values obtained from the Canadian and U.S. In-Grade Testing Program.

## Glulam and Other Reconstituted Wood Products

Appendix G of the current NDS allows products whose modulus of elasticity has a coefficient of variation of $\leq$50.11 to be designed with a smaller factor of safety on the elastic buckling load, namely

$$F'_c = \frac{0.418E}{(l_e/d)^2} \tag{14}$$

In similar fashion, Appendix O of the current NDS allows beams whose modulus of elasticity has a coefficient of variation of $\leq 0.11$ to be designed using

$$F'_b = \frac{0.609E}{R_B^2}. \tag{15}$$

These provisions will be retained in the new NDS. As of this writing, discussions are underway to permit materials that are *more homogeneous* than solid sawn lumber to be designed using the Yline column equation with $c$ = 0.9 rather than 0.8. The justification appeals partly to a calibration to past practice (FPL 4th-power parabola) and partly to the fact that more homogeneous materials should experience less interaction between crushing and buckling, which has the effect of increasing $c$. If that proposal is adopted, the more liberal column equation for laminated and other reconstituted wood products would be

$$C_P = \frac{1 + (F_{cE}/F_c^*)}{1.8} - \sqrt{\frac{[1 + (F_{cE}/F_c^*)]^2}{3.24} - \frac{F_{cE}/F_c^*}{0.9}} \tag{16}$$

A final comment: I believe that paragraph 10 Appendix G of the current NDS was inadvertently retained. This provision allows the option of using an obsolete design practice that was supplanted by the adoption of the FPL 4th-power parabola. Paragraph 10 states that "It has long been common practice to simplify wood column design by using . . ." the smaller of $F'_c$ and $F_{cE}$. However, $F'_c$ and $F_{cE}$ are always upper bounds (with the usual factor of safety) on the true failure criterion. Now that a more accurate column equation has been adopted in the new NDS, use of this provision could result in an increase of up to 45% in the calculated column capacity; that is, a reduction of the factor of safety to a value <1. As of this writing, the TAC committee has not yet had time to act on my suggestion that this provision be eliminated.

## Appendix I. — References

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