

REACTION RATE MODEL FOR THE FATIGUE STRENGTH OF WOOD

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ABSTRACT

In this paper, we consider the fatigue strength of wood structural members. That is, we develop a mathematical model for time-dependent strength under sinusoidal load. This work extends the model for time-dependent strength under constant load and ramp load derived previously by two of the authors. It is based on the statistical theory of the absolute reaction rate in a version favorably reviewed in the literature. Under the isothermal condition, the model predicts that the time at fracture is independent of stress frequency. The need to evaluate experimentally some of the model parameters that may depend on stress frequency indirectly through temperature changes is discussed.

Keywords: Absolute reaction rate, bending strength, creep rupture, Douglas-fir, duration of load, fatigue, frequency, sinusoidal load, strength, temperature, wood.

NOMENCLATURE

E	Activation energy (J/mol)	β	Stress coefficient in process of rupturing (1/MPa)
f	Fraction of unbroken bonds	γ	Stress coefficient in process of reforming (1/MPa)
I₀(x)	Modified Bessel function of the 1st kind, of order zero	δ	Volume of moving element (m ³)
K	Rate function (1/sec)	σ	Applied stress (MPa)
k	Boltzmann's constant (J/K)	ψ	Average stress per bond (MPa)
R	Gas constant (J/mol.K)	Ω	Circular frequency; stress frequency (1/sec)
T	Absolute temperature (K)	ω	Frequency of motion (1/sec)
t	Lifetime(s)		
α	Dimensionless parameter		

Subscripts

b	Bond rupture
c	Constant load; mean stress
o	Initial condition; stress amplitude
r	Bond reformation
s	Sinusoidal load
u	Standard ultimate strength test

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INTRODUCTION

Wood structural members exhibit time-dependent rupture behavior. The design of these members requires an appropriate consideration of the effect of load duration. The estimation of load-duration effects is usually obtained from mathematical models whose parameters were determined from long-term test data. The tests may employ either constant load or ramp load, in which the applied load increases linearly with time. For the same failure load, the lifetime under ramp load always exceeds the lifetime under constant load. The required test times can extend from a fraction of an hour to 10 yr or more. Because there are many other loading conditions of practical interest, we investigated how the results from these tests can be applied to the design of wood structural members under other loadings.

In the past, fatigue in wood was recognized in aircraft design; in recent years, fatigue has been a factor in the design of wind turbine generators (Tsai and Ansell 1990). However, mathematical modeling of fatigue and fatigue test data for wood are limited in the literature. In this study, we develop a mathematical model for the time-dependent strength of wood structural members under sinusoidal load. Our model is an extension of the model for constant load and ramp load (Liu and Schaffer 1991) that is based on the theory of reaction rate for the fracture of solids originally proposed by Tobolsky and Eyring (1943). The physical parameters are the same. When we equate the sum of the applied mean stress and a function of the sinusoidal stress amplitude with the stress caused by constant load, we find that the models for the two loading conditions are identical in form.

Tobolsky and Eyring (1943) proposed that the breaking of polymeric threads under load is due to the slipping of bonds at a rate that is dependent on both stress and temperature. Their rupture rate equation was used to show that constants obtained from creep tests of certain polymerics can successfully predict lifetimes, not only for creep tests but also for con-

stant strain-rate tests, over wide ranges of stress-time history and temperature (Graham et al. 1969).

A reaction rate model of the rupture of polymeric filaments subjected to constant load, ramp load, and sinusoidal load was derived by Coleman (1956). Making use of the observed phenomenon that the elongation at break is relatively independent of the rate of load, Coleman derived a general superposition principle for the calculation of the time to break a filament by creep failure under an arbitrary loading history. In Coleman's model, the variable of distortion appears in place of the number of unbroken bonds as applied by Tobolsky and Eyring (1943).

Hsiao and Ting (1966) and Hsiao et al. (1968) chose to employ a more general model than that by Tobolsky and Eyring (1943). Hsiao, Ting, and coworkers used a different set of constants for frequency of motion and stress coefficient for bond fracture and bond reformation in the reaction rate theory. They suggested that, under small loads, the reformation processes may be responsible for the frequently observed nonlinearity of the relationship between the applied load and the logarithm of time-to-fracture. Their expression for the rate of bond fracture, after identification of constants, is identical to that of Tobolsky and Eyring (1943). Hsiao (1966) applied the model to estimate the fracture time of solids under constant load. In studying the effect of temperature on the strength of wood, Schaffer (1973) used the same model (Hsiao et al. 1968) for constant load and derived a new model for ramp load.

Coleman (1956) developed a model based on distortion as the basic mechanism of bond rupture and applied it to sinusoidal load. Henderson et al. (1970) reviewed the literature of the application of the theory of reaction rate to the fracture of solids. They found that the evidence better supports slipping than distortion as the basic mechanism of bond rupture leading to fracture of certain polymer systems. Therefore, we applied the model that uses slipping as the basic mechanism of bond rupture to the same load considered by Coleman,

namely sinusoidal. It will be shown here that the resulting models are functionally identical.

The models by Hsiao (1966) for constant load and Schaffer (1973) for ramp load were extended by Liu and Schaffer (1991) to analyze the nonlinear behaviors between the applied stress and the logarithm of time-to-fracture as observed in test data of wood and other solids. The present study further extends that work to a superposition of constant load and sinusoidal load.

BASIC EQUATIONS

The time-dependent nature of solids can be formulated using the statistical theory of the absolute reaction rate for a given orientation of the constituent elements or bonds with respect to the direction of applied stress. Let f be the fraction of unbroken bonds and a function of orientation and time. The rate of change off is given as follows (Hsiao et al. 1968):

$$\frac{df}{dt} = K_r(1 - f) - K_b f \quad (1)$$

in which

$$K_r = \omega_r \exp \left[-\frac{E}{RT} - \gamma \psi(t) \right] \quad (2)$$

is the rate of reformation of broken bonds;

$$K_b = \omega_b \exp \left[-\frac{E}{RT} + \beta \psi(t) \right] \quad (3)$$

is the rate of rupturing of unbroken bonds; ω_r and ω_b are respectively the frequencies of the jump motion of the bonds with respect to forming and breaking processes; E is the activation energy; R the universal gas constant; T the absolute temperature; γ and β are stress coefficients that modify the energy barrier as a consequence of the applied stress in the direction of each element, and $\psi(t)$ is a stress function that may be interpreted as the average stress on the unbroken bonds. For a completely oriented system, if the applied stress is σ in the direction of the bonds, then

$$\psi(t)f(t) = \sigma \quad (4)$$

and f is independent of the system orientation.

For high values of stress, $K_r \ll K_b$. To a first approximation, K_r in Eq. (1) can often be ignored (Hsiao 1966; Henderson et al. 1970; Schaffer 1973; Hansen and Baker-Jarvis 1990; Kozin and Bogdanoff 1990). The omission of K_r can also be justified by the fact that until failure is imminent, f is nearly one.

From Eq. (1) with $K_r = 0$ and Eqs. (3) and (4), we have

$$\frac{df}{dt} = -\omega_b f \exp \left[-\frac{E}{RT} + \frac{\beta \sigma}{f} \right]. \quad (5)$$

The solution of Eq. (5) follows.

METHOD OF SOLUTION

Consider a sinusoidal stress,

$$\sigma = \sigma_c + \sigma_o \sin \Omega t \quad (6)$$

in which σ_c is the constant mean stress, σ_o is the amplitude of the cyclic stress, Ω is the circular frequency, and $\sigma_c > \sigma_o$.

From Eqs. (5) and (6), we obtain

$$\int_1^{f_s} \frac{df}{f} = -\omega_b \exp \left[-\frac{E}{RT} \right] \int_0^{t_s} \exp \left[\frac{\beta}{f} (\sigma_c + \sigma_o \sin \Omega \tau) \right] d\tau \quad (7)$$

in which the initial value off is equal to 1 by definition, and f_s is the value off at time t_s when failure is imminent. Between t_s and the time t when failure occurs, there is a final cascade of bond ruptures. We shall treat t_s as being essentially t , the time at failure. Because f is essentially equal to 1 until failure is imminent, it is a good approximation to set $f = 1$ under the integral sign on the right side of Eq. (7). The phenomenon of a final cascade was borne out by the fatigue tests of wind turbine blades by Tsai and Ansell (1990), who observed no reduction in strength until the final stage of damage development.

Let $\alpha \equiv \beta \sigma_o$. Then,

$$\ln f_s = -\omega_b \exp \left[-\frac{E}{RT} + \beta \sigma_c \right] Q(\alpha) \quad (8)$$

in which

$$Q(\alpha) \equiv \int_0^t \exp[\alpha \sin \Omega \tau] d\tau. \quad (9)$$

Let $x \equiv \Omega \tau$ and write

$$Q(\alpha) = \frac{1}{\Omega} \int_0^{\Omega t} \exp[\alpha \sin x] dx. \quad (10)$$

The integrand of $Q(\alpha)$ is periodic, with period 2π . If we measure time in whole cycles, then $\Omega t = 2\pi n$ where n is the number of whole cycles. Therefore,

$$\begin{aligned} Q(\alpha) &= \frac{1}{\Omega} \int_0^{2\pi n} \exp[\alpha \sin x] dx \\ &= \frac{n}{\Omega} \int_0^{2\pi} \exp[\alpha \sin x] dx \\ &= \frac{t}{2\pi} \int_0^{2\pi} \exp[\alpha \sin x] dx \\ Q(\alpha) &\equiv tI_0(\alpha) \end{aligned} \quad (11)$$

in which $I_0(\alpha)$ is the modified Bessel function of the first kind, of order zero (Rainville 1960).

Hence we have from Eqs. (8) and (11)

$$\begin{aligned} \ln\left(\frac{1}{f_s}\right) &= \omega_b \exp\left[-\frac{E}{RT} + \beta\sigma_c\right] tI_0(\beta\sigma_o) \quad (12) \\ \ln \ln\left(\frac{1}{f_s}\right) &= \ln(\omega_b) - \frac{E}{RT} + \\ &\quad + \beta\sigma_c + \ln[tI_0(\beta\sigma_o)] \end{aligned} \quad (13)$$

or

$$\ln[tI_0(\beta\sigma_o)] = \frac{E}{RT} - \ln(\omega_b) - \beta\sigma_c + \ln \ln\left(\frac{1}{f_s}\right). \quad (14)$$

For $\sigma_o = 0$, $I_0(0) = 1$, and Eq. (14) reduces to the same form as the constant load case. We note Eq. (14) is considerably simpler than the fatigue life expressed in terms of an integral derived by Moghe (1971) and applied by Pai et al. (1991) for the same loading condition in Eq. (6).

If we write

$$t = t(\sigma_c, \sigma_o, f_s) \quad (15)$$

we can put Eq. (14) in the following form

$$t(\sigma_c, \sigma_o, f_s) = \frac{t(\sigma_c, 0, f_s)}{I_0(\beta\sigma_o)} \quad (16)$$

in which f_s is to be determined from cyclic fatigue tests to be consistent with Eq. (14).

Note that Eq. (14) does not contain the stress frequency Ω . That is, according to the theory in this study, the time at fracture is independent of stress frequency. This contradicts the usual view that fatigue life can be measured in cycles (Coleman and Knox 1957).

MODEL CORRELATION

Because there are few existing cyclic fatigue test data that can be used to correlate with the model in Eq. (14), we used existing data from constant load tests to correlate with the following model developed by Liu and Schaffer (1991):

$$\sigma_c = \frac{1}{\beta} \left[\frac{E}{RT} - \ln(\omega_b t) + \ln \ln\left(\frac{1}{f_c}\right) \right] \quad (17)$$

in which σ_c is the constant stress at fracture; f_c is the value off when fracture is imminent, and the other parameters are as in Eq. (14) and are independent of loading conditions.

According to Gerhards (1977), the test data on bending strength of small, clear Douglas-fir beams under constant load reported by, Wood (1951) are more comprehensive than any of those reported by others and were analyzed by most duration-of-load researchers of wood structural members. Although the reaction rate theory was derived for tensile strength of solids (Tobolsky and Eyring 1943; Coleman 1956; Zhurkov 1965; Hsiao et al. 1968), the theory was shown to be applicable for bending strength of wood (Caulfield 1985; Liu and Schaffer 1991). In air-dry wood of structural dimensions, final bending failure is always in the tension side (Tsai and Ansell 1990).

The data by Wood (1951) covering a time span from 0.1 h to 10 yr are shown in Fig. 1. Although the specimens are of clear wood, the data scatter is seen to be much more ex-

cessive than those of other materials reported by Zhurkov (1965). However, Wood's data are usually represented by the following equation that shows a linear relationship between strength ratio and the logarithm of time-to-fracture.

$$\frac{\sigma_c}{\sigma_u} \times 100 = 112.8 - 2.736 \ln(t) \quad (18)$$

in which σ_u is the short-term ultimate strength of Douglas-fir, and t is in seconds.

For stress to vary linearly with the logarithm of time t , we may set $\ln \ln(1/f_c) = 0$ in Eq. (17), as suggested by Hsiao (1966), so that $f_c = 1/e = 0.3679$. From the literature, we also have the following values for the parameters in Eq. (17): $E = 1.73 \times 10^5$ J/mol and $R = 8.314$ J/mol K (Caulfield 1985); $T = 300$ K (Clouser 1959), and $\sigma_u = 53.1$ MPa (Liska 1950). With these data and by comparing Eq. (17) with Eq. (18), we obtain $\beta = 0.6885$ MPa⁻¹ and $\omega_b = 1.6470 \times 10^{12}$ sec⁻¹.

The value of ω_b so obtained is of the same order of magnitude as the values obtained for silver chloride, aluminum, and polymethyl methacrylate (Zhurkov 1965).

RESULTS AND DISCUSSION

With the parameter values determined, we can proceed to solve Eq. (14). We set $\ln \ln(1/f_s) = 0$ as we did in the constant load case, although we could retain it as another parameter to fit our model to cyclic fatigue test data. When we set $f_s = 1/e$ [i.e., $\ln \ln(1/f_s) = 0$], we note that our Eq. (14) agrees in form with the model obtained by Coleman (1956). Coleman's model has the form

$$t(\sigma_c, \sigma_o) = \frac{t(\sigma_c, 0)}{I_o \left(\frac{\sigma_o \delta}{2kT} \right)} \quad (19)$$

in which $t(\sigma_c, \sigma_o)$ is the fatigue lifetime corresponding to a mean stress σ_c and a stress amplitude σ_o , δ is the volume of the moving element or force center (defined as the product of the effective cross section area per force center and the separation between the positions

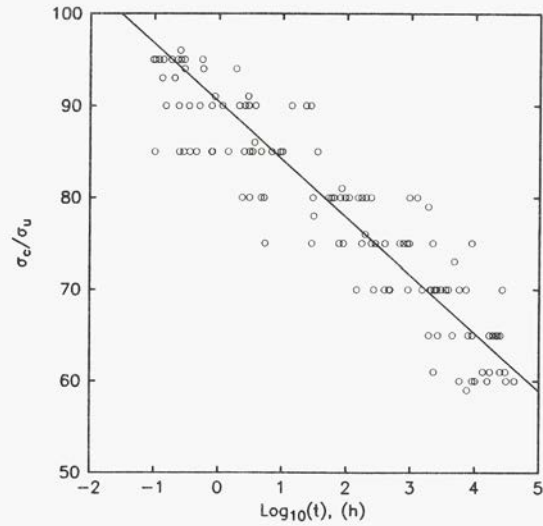


FIG. 1. Relation of strength to time of loading (Wood 1951), based on constant load bending tests of clear Douglas-fir.

of minimum potential of mean force), k is Boltzmann's constant, and T is the absolute temperature.

The resulting models agree exactly if we identify Coleman's $\delta/2kT$ with our stress coefficient, p . In that case, we obtain $6 = 5.70 \times 10^{-27}$ m³. This agrees closely with the estimate of $6 = 5.6 \times 10^{-27}$ m³ obtained by Caulfield (1985) using Douglas-fir data of Youngs and Hilbrand (1963). Because both δ and β are unknown microscale parameters and must be inferred by fitting to macroscopic data, the two models are functionally identical. The values for each of the other parameters in Eq. (14) are assigned as follows:

$$\frac{\sigma_c}{\sigma_u} = \begin{cases} 0.6 \\ 0.65 \\ 0.7 \end{cases} \quad \frac{\sigma_o}{\sigma_u} = \begin{cases} 0.065 \\ 0.13 \\ 0.26 \end{cases} \quad (20)$$

With these input data, results from Eq. (14) are shown in Fig. 2. The nonlinear portions of the curves for small stress amplitude ratio values are due to the characteristics of the Bessel function. The horizontal distances between any two curves are constant. For comparison purposes, we also show in Fig. 2 results from Eq. (17) with σ_c replaced by $\sigma_c + \sigma_o$. Clearly, the

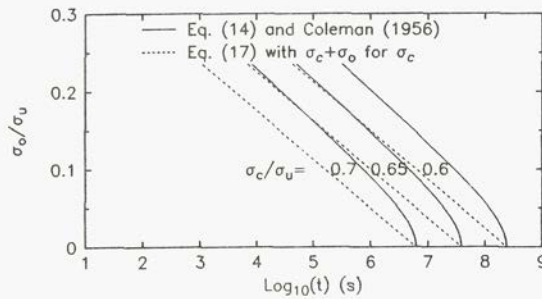


FIG. 2. Stress amplitude ratio, σ_o/σ_u , versus fatigue life in seconds. In this figure, σ_c is the mean stress, and the static ultimate strength, σ_u , has been taken equal to 53.1 MPa.

lifetime resulting from a mean stress, a , plus a sinusoidal stress, σ_o , is longer than that resulting from a static stress equal to $\sigma_c + \sigma_o$ and shorter than that resulting from a static stress equal to a .

However, results in Fig. 2 are based on the assumption that $f_s = f_c = 1/e$ to be consistent with Coleman's model, Eq. (19). These results are not in agreement with the cyclic fatigue data of rayon reported by Moghe and Skolnik (1986), showing that the curve for cyclic fatigue should fall below that for static stress. In any case, f_s in Eq. (14) and f_c in Eq. (17) need not be numerically equal. They can take different values between 1 and 0 because of the catastrophic rapidity with which failure occurs. If we replace t in Eq. (14) by mt , with $m = 1$ corresponding to $\ln \ln(1/f_s) = 0$, and keep the other terms unchanged we obtain

$$\ln(m) = \ln \ln \left(\frac{1}{f_s} \right) \quad (21)$$

or

$$m = \ln \left(\frac{1}{f_s} \right). \quad (22)$$

The relation between m and f_s is presented in Fig. 3. By choosing $m < 1$, we can describe the same trend as observed by Moghe and Skolnik (1986). Equation (14) can fit any fatigue failure data corresponding to the loading condition in Eq. (6) by adjusting the values for f_s . This flexibility of Eq. (14) makes it more

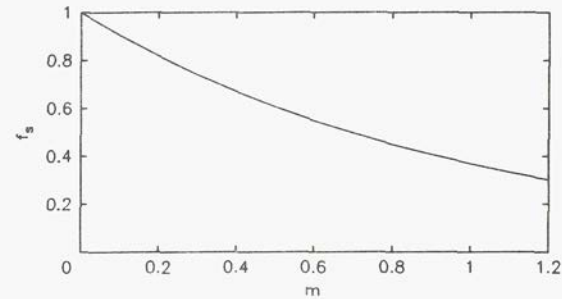


FIG. 3. Value of f_s , that results from choice of m .

attractive than other similar models in the literature.

Even if the lifetime resulting from a mean stress, a , and a sinusoidal stress, σ_o , is longer than that resulting from a static stress equal to $a + \sigma_o$, the failure mode for the former may be more definite than that for the latter as reported by Burton (1968) on metals. A less definite failure mode may account for some of the data scatter in Fig. 1.

Note that the volume of the moving element δ in Eq. (19) is the same as the constant coefficient modifying the applied stress in Zhurkov (1965) and is effectively equivalent to the stress coefficient β in Eq. (14) or Eq. (16). According to Zhurkov (1965), that coefficient is very sensitive to various structural changes in solids, and a precise physical meaning of it is difficult to formulate. Regel and Leksovsky (1967) emphasized that the condition of loading should definitely affect this structure-sensitive coefficient. Kozin and Bogdanoff (1990) concluded that β must be estimated from test data and may vary for different loading conditions. We found from solving Eq. (19) that β seems to be dependent on temperature as suggested by Schaffer (1973). To evaluate all the parameters in Eq. (14) with acceptable accuracy, an extensive testing program is required, using methods such as those described by Tsai and Ansell (1990) and Bonfield and Ansell (1991).

Although a fatigue life independent of stress frequency seems to be an innate feature of reaction rate theory, it is not physically indisputable. It is well known that frequency of loading will increase the specimen temperature

in a cyclic fatigue test (Regel and Leksovsky 1967; Tsai and Ansell 1990; Kozin and Bogdanoff 1990) and a temperature increase will result in a decrease in lifetime (Schaffer 1973, 1982). Therefore, temperature should be a function of stress frequency, unless the stress frequency is low and the temperature increase is small. In deriving Eqs. (14) and (19), the isothermal condition was assumed to exist, leading to a lifetime that is independent of stress frequency. If, in Eq. (7), temperature is expressed as a function of stress frequency, the mathematical developments that follow would be much more complex. However, the actual physical phenomenon can only become clear when the functional relationship between temperature and stress cycle is established.

CONCLUSIONS

We applied the statistical theory of the absolute reaction rate to describe the fatigue life of wood structural members under sinusoidal load. The parameters in the derived mathematical model are evaluated from a corresponding model for constant load, for which test data of Douglas-fir beams under constant bending load are available. The models for sinusoidal load and constant load are of the same mathematical form.

Our results predict that the fatigue life under cyclic bending load is independent of stress frequency for a polymeric material such as wood. It depends only on the mean stress and a function of the stress amplitude of the applied load if the isothermal condition is assumed to exist. This agrees with the earlier predictions of Coleman (1956), whose model is functionally identical to ours, and those of Moghe and Skolnik (1986), whose model shows independence of frequency at frequencies greater than 1 Hz.

REFERENCES

- BONFIELD, P. W., AND M. P. ANSELL. 1991. Fatigue properties of wood in tension, compression and shear. *J. Mater. Sci.* 26(17):4765-4773.
- BURTON, R. 1968. *Vibration and impact*. Dover Publications, Inc., New York, NY. Pp. 292-293.
- CAULFIELD, D. F. 1985. A chemical kinetics approach to the duration-of-load problem in wood. *Wood Fiber Sci.* 17(4):504-521.
- CLOUSER, W. S. 1959. Creep of small wood beams under constant bending load. Report No. 2 150, USDA, Forest Service, Forest Products Laboratory, Madison, WI.
- COLEMAN, B. D. 1956. Application of the theory of absolute reaction rates to the creep failure of polymeric filaments. *J. Polymer Sci.* 20:447-455.
- , AND A. G. KNOX. 1957. The interpretation of creep failure in textile fibers as a rate process. *Textile Res. J.* 5:393-399.
- GERHARDS, C. C. 1977. Effect of duration and rate of loading on strength of wood and wood-based materials. Research Paper FPL 283, USDA, Forest Service, Forest Products Laboratory, Madison, WI.
- GRAHAM, P. H., C. N. ROBINSON, AND C. B. HENDERSON. 1969. Analysis of dilatational failure of heterogeneous materials by reaction rate theory. *Intl. J. Fracture Mech.* 5(1): 57-62.
- HANSEN, A. C., AND J. BAKER-JARVIS. 1990. A rate-dependent kinetic theory of fracture for polymers. *Intl. J. Fracture* 44(3):221-231.
- HENDERSON, C. B., P. H. GRAHAM, AND C. N. ROBINSON. 1970. A comparison of reaction rate models for the fracture of solids. *Intl. J. Fracture Mech.* 6(1):33-40.
- HSIAO, C. C. 1966. Fracture. *Physics Today*. Pp. 49-53.
- , AND C. S. TING. 1966. On deformation and strength. *Proceedings, 1st International Conference on Fracture*, Japan Society for Strength and Fracture 1:449-457.
- , S. R. MOGHE, AND H. H. KAUSCH VON SCHMELING. 1968. Time-dependent mechanical strength of oriented media. *J. Appl. Phys.* 39(8):3857-3861.
- KOZIN, E., AND J. L. BOGDANOFF. 1990. Cumulative damage model for mean fatigue crack growth based on the kinetic theory of thermally activated fracture. *Eng. Fracture Mech.* 37(5):995-1010.
- LISKA, J. A. 1950. Effect of rapid loading on the compressive and flexural strength of wood. Report No. 1767, USDA, Forest Service, Forest Products Laboratory, Madison, WI.
- LIU, J. Y., AND E. L. SCHAFER. 1991. Time-dependent mechanical strength of wood structural members. *Proceedings, International Timber Engineering Conference*, London. TRADA 4: 164-171.
- MOGHE, S. R. 1971. *International Conference on Mechanical Behavior of Materials*. Society of Material Science, Japan 3:541.
- , AND L. SKOLNIK. 1986. Fatigue failure of fiber assemblies. *J. Macromol. Sci. Phys.* B25(4):487-504.
- PAI, G. A., S. K. BATRA, AND S. P. HERSH. 1991. Interpreting creep and fatigue failure of fibers via deformation kinetics. *J. Appl. Polymer Sci.: Appl. Polymer Symp.* 47: 127-141.
- RAINVILLE, E. D. 1960. *Special functions*. The MacMillan Co., New York, NY. Pp. 109, 114.

- REGEL, V. R., AND A. M. LEKSOVSKY. 1967. A study of fatigue within the framework of the kinetic concept of fracture. *Intl. J. Fracture Mech.* 5:99-109.
- SCHAFFER, E. L. 1973. Effect of pyrolytic temperatures on the longitudinal strength of dry Douglas-fir. *J. Testing Eval. JTEVA* 1(4):319-329.
- SCHAFFER, E. L. 1982. Influence of heat on the longitudinal creep of dry Douglas-fir. Pages 20-52 in R. W. Meyer and R. M. Kellogg, eds. *Structural uses of wood in adverse environments*. Society of Wood Science and Technology, Van Nostrand Reinhold Co., New York, NY.
- TOBOLSKY, A., AND H. EYRING. 1943. Mechanical properties of polymeric materials. *J. Chem. Phys.* 11:125-134.
- TSAI, K. T., AND M. P. ANSELL. 1990. The fatigue properties of wood in flexure. *J. Mater. Sci.* 25:865-878.
- WOOD, L. W. 1951. Relation of strength of wood to duration of load. Report No. R-1916, USDA, Forest Service, Forest Products Laboratory, Madison, WI.
- YOUNGS, R. L., AND H. C. HILBRAND. 1963. Time-related flexural behavior of small beams under prolonged loading. *Forest Prod. J.* 13(6):227-232.
- ZHURKOV, S. N. 1965. Kinetic concept of the strength of solids. *intl. J. Fracture Mech.* 1:311-323.