ESTIMATING PREVIOUS DIAMETER FOR INGROWTH TREES ON REMEASURED HORIZONTAL POINT SAMPLES

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Abstract—The purpose of this study was to develop an improved procedure to estimate a previous diameter for ongrowth and nongrowth trees on plots sampled by variable radius plot sampling. However, the models built can be used whenever a previous diameter is required and the appropriate independent variables are available. Data for this study were from 1,965 remeasured forest inventory plots in the 1989 inventory of West Virginia conducted by the Northeastern Research Station's Forest Inventory and Analysis (NEFIA) unit. The investigation focused on three areas. First, we investigated whether breaking the data into six groups based on the rank average diameter growth was superior to NEFIA's procedure of breaking the data into seventeen species groups. Second, we investigated whether basal-area increment (BAI) was superior to diameter increment (DI) as a dependent variable. Finally, we investigated additional independent variables. Based on the R^2, mean residual, mean absolute error, and root mean square error, the best model had six species groups, BAI as the dependent variable, and a slightly expanded set of independent variables.

INTRODUCTION

Missing observations are common in "real world" data sets. In forestry, there are many instances in which a diameter for a previous period is required but not available. The impetus for this study was a problem encountered by the Forest Service's Forest Inventory and Analysis (FIA) units which are required to produce periodic tables showing net volume change from the previous inventory as well as the components of change: ingrowth (I), survivor growth (S), mortality (M), and cut (C). Equation (1) expresses net change as the sum of these components:

$$V_2 - V_1 = I + S - M - C \qquad (1)$$

where:

V_1 = volume at time period 1;

V_2 = volume at time period 2.

Martin (1982) defined six possible categories of trees encountered on remeasured point samples. The first four, ingrowth, survivor, mortality, and cut trees were measured at the first inventory. The remaining categories, ongrowth and nongrowth trees, were alive and included only in the second inventory. Ongrowth trees were nonmerchantable (below minimum dbh) at the first inventory but of merchantable size at the second inventory. Nongrowth trees were above minimum dbh at the first inventory, but grew sufficiently to be included in the second inventory.

Most FIA units have used multiple subplot horizontal point samples (prism plots) to inventory trees larger than a specified merchantable diameter. With this type of sample, the probability of selecting a sample tree is proportional to the basal area of the individual tree and depends on the basal-area factor of the prism.

Traditional estimation procedures for total net change and its components yielded estimates for which the two sides of equation (1) do not agree because ongrowth and nongrowth trees were excluded as components of growth. Martin (1982) obtained compatible estimators by including in ingrowth both ongrowth and nongrowth trees. Van Deusen and others (1986) improved the compatible estimators by rearranging Martin's equation to include nongrowth with survival growth. They showed that the standard error of this new estimator of survival growth was smaller than Martin's traditional estimator. Also, the estimator for ingrowth used by Van Deusen and others cannot be negative. Roesch and others (1989) showed that the estimation of survival growth could be improved with additional rearrangement of some components. To distinguish between ongrowth and nongrowth trees, these new procedures require the estimation of previous diameter for trees that were not measured at the initial inventory.

NEFIA unit has used several plot designs, including a 10-point variable radius design. To obtain compatible estimates of change with these plots, we need to estimate previous diameter for trees measured only at time period 2.

To estimate previous diameter, NEFIA currently uses trees measured at both inventories to develop regression equations. The model is:

$$DI = f\,(DBH2,\ TRCLS2,\ CRNCLS2,\ CRATIO2,\ CRCC2,\ DCR2) \qquad (2)$$

where:

DI = annual diameter increment;

DBH2 = tree diameter at time period 2;

TRCLS2 = tree class at time period 2, a measure of tree quality;

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CRNCLS2 = crown class at time 2, a measure of crown position in the canopy;

CRATIO2 = crown ratio at time 2, the proportion of a tree with a live crown;

CRCC2 = CRATIO2/CRNCLS2;

DCR2 = DBH2 · CRATIO2.

The data are divided into seventeen species groups. For each group, a stepwise backward elimination regression procedure finds the functional relationship between the dependent and independent variables. Annual DI is the dependent variable as opposed to total increment between inventories because the trees are remeasured at different intervals. The procedures developed by Roesch and others (1989) and Van Deusen and others (1986) assume that good estimates of previous diameter for ongrowth and nongrowth trees are possible. With the NEFIA procedure, the R^2 for each of the seventeen species groups is low, *e.g.*, the most recent inventory from West Virginia had R^2 values ranging from 0.06 to 0.56.

PROCEDURE

The data for this study are from 1,965 remeasured forest inventory plots in West Virginia (DiGiovanni 1990). Trees were initially measured in 1975 and remeasured in 1988. Only trees larger than 5 inches in diameter at both inventories were used. The data were split into a model data set with 8,723 observations and a validation data set with 7,951 observations.

Using multiple linear regression, we investigated different sets of independent variables, different species-group compositions, and BAI as a dependent variable. Although the explanatory variables have biological meaning with respect to competitive position, size, and tree quality, the model itself is not easily interpreted biologically as is the case with models developed by Teck and Hilt (1991) and Quicke and others (1994). However, our objective was to find a better estimate of annual DI for trees alive at both inventories. It was not necessary to account for ingrowth, cut, or mortality.

Apart from the variables used by NEFIA, we investigated additional variables reported in the literature as important in modeling tree growth. The additional independent variables were restricted to those measured by NEFIA and those that could be calculated:

BA2 = total plot basal area of trees at least 5 inches in diameter at time 2;

TPA2 = total number of trees per acre of trees at least 5 inches in diameter at time 2;

MD2 = plot medial diameter of trees at least 5 inches in diameter at time 2;

QD2 = plot quadratic diameter of trees at least 5 inches in diameter at time 2;

BAL2 = total basal area of trees larger than the sample tree at time 2;

RLD2 = relative diameter; ratio of tree diameter to medial diameter of plot, MD2;

RLQD2 = relative diameter; ratio of tree diameter to quadratic diameter of plot, QD2.

The plot variables, BA2, TPA2, MD2, and QD2, are expressions of site occupancy or total competition and size of the trees on the plot. The tree variables, BAL2, RLD2, and RLQD2, are measures of the competitive position of the sample tree relative to other trees on the plot.

The basal area larger (BAL2) variable is used in the potential growth times modifier type of model found in the NC and NE-TWIGS growth simulators (Hilt and Teck 1989: Miner and others 1988: Quicke and others 1994). Marquis (1991) used relative diameter to model diameter growth.

Variables not considered are tree or stand age and a measure of site productivity. NEFIA does not determine the age of individual trees, and stand age was excluded because many of the plots are classified as uneven-aged. Also, experience has shown a poor relationship between diameter growth and the site-productivity measure determined by NEFIA.

We investigated both DI and BAI as the dependent variable. Because of unequal number of years between plot measurements, annual increment was modeled as:

$$DI = \frac{(DBH2 - DBH1)}{N} \qquad (3)$$

or

$$BAI = \frac{(BA2 - BA1)}{N} \qquad (4)$$

where:

N = number of years between measurements on the plot;

DBH1 = tree diameter at time period 1;

DBH2 = tree diameter at time period 2;

BA1 = tree basal area at time period 1;

BA2 = tree basal area at time period 2.

The seventeen species groups used by NEFIA are based on form class and are used in our volume equations (Scott 1979). To investigate whether another grouping of species is more appropriate to model diameter growth, we formed six species groups based on rank of the average diameter growth for a species. Mean annual diameter growth ranged from 0.068 for the lowest ranked group to 0.203 for the highest.

RESULTS

We compared the R^2 values, mean residuals, mean absolute residuals, and root mean square errors using the NEFIA independent variables for the total sample of both the model and validation data sets (Table 1). The comparison is for six procedures: 1) the stepwise procedure on each of seventeen species groups using DI as the dependent variable; 2) all variables for each group using DI; 3) all variables ignoring species group using DI;

Table 1—R^2s, mean residuals, mean absolute residuals, and root mean square errors (MSE) for the stepwise procedure using NEFIA variables with three species groupings

Procedure	R^2	Mean residual	Mean abs. residual	Root MSE
		Model data set		
Diameter increment				
Stepwise, 17 groups	0.348	0.0	0.0530	0.0692
Nonstepwise, 17 groups	0.349	0.0	0.0529	0.0691
Nonstepwise, no groups	0.211	0.0	0.0584	0.0762
Nonstepwise, 6 groups	0.351	0.0	0.0530	0.0691
Basal-area increment				
Nonstepwise, 17 groups	0.557	0.0033	0.0440	0.0571
Nonstepwise, 6 groups	0.565	0.0013	0.0436	0.0566
		Validation data set		
Diameter increment				
Stepwise, 17 groups	0.312	0.0017	0.0544	0.0721
Nonstepwise, 17 groups	0.312	0.0017	0.0545	0.0721
Nonstepwise, no groups	0.207	0.0022	0.0585	0.0775
Nonstepwise, 6 groups	0.322	0.0005	0.0545	0.0716
Basal-area increment				
Nonstepwise, 17 groups	0.536	0.0032	0.0451	0.0592
Nonstepwise, 6 groups	0.553	0.0017	0.0448	0.0582

4) all variables using six species groups and DI; 5) all variables using seventeen species groups with BAI as the dependent variable; and 6) all variables using six species groups and BAI. Equation (5) and (6) give the formulas for calculating the R^2 and root mean square errors, respectively, for the combined data.

$$R^2 = 1 - \left(\frac{SSRES}{SSTOT}\right) \qquad (5)$$

$$RMSE = \sqrt{\frac{SSRES}{n_i}} \qquad (6)$$

where:

$$SSRES = \sum_{i=1}^{g}\sum_{j=1}^{n_i}\left(Y_{ij} - \hat{Y}_{ij}\right)^2;$$

$$SSTOT = \sum_{i=1}^{g}\sum_{j=1}^{n_i}\left(Y_{ij} - \overline{Y}\right)^2;$$

Y_{ij} = observed DI;

$\hat{Y}_{ij}$ = predicted DI;

$\overline{Y}$ = overall mean DI;

n_i = number of trees in group i;

g = number of species in a group, 17, 6, or 1.

The predicted value for the validation data set, $\hat{Y}_{ij}$, was obtained using the coefficients estimated with the model data set.

The stepwise procedure for the seventeen species groups offers little advantage over the nonstepwise procedure for each group. There is little difference in any of the statistics between these two procedures. Each of the NEFIA independent variables was significant in several of the groups, indicating that eliminating one or more of the variables would be inappropriate. Yet there is an advantage in grouping the species. All of the statistics for both the model and validation data sets show improvement when the data are divided into species groups. However, there is little difference between the results using the seventeen groups based on form class and the six groups based on rank of mean DI. As seen in Table 1, there is a marked improvement when BAI replaces DI as the dependent variable. For the BAI model, all results are expressed in reference to DI using the translation:

$$\hat{DI}(BA) = \frac{\sqrt{\frac{(BA1 + N \cdot \hat{BAI})}{K}} - DBH1}{N} \qquad (7)$$

where:

$\hat{BAI}$ = predicted BAI;

K = 0.005454154, a conversion factor from diameter in inches to basal area in square feet.

All of the statistics use $\hat{DI}(BA)$ as the predicted DI for the basal-area model.

Table 2 presents a more complete analysis of the differences between results using DI versus BAI for the

Table 2—Comparison statistics for the diameter-increment and basal-area increment models for model and validation data sets

		Diameter-increment model					Basal-area increment model			
Species group	No. of trees	Mean DI	R^2	Mean residual	Mean abs. residual	Root MSE	R^2	Mean residual	Mean abs. residual	Root MSE
					Model data set					
1	247	0.0681	0.045	0.0	0.0322	0.0408	0.235	0.0011	0.0288	0.0365
2	947	0.0929	0.085	0.0	0.0415	0.0525	0.315	0.0004	0.0359	0.0454
3	2489	0.1145	0.157	0.0	0.0434	0.0562	0.370	0.0012	0.0374	0.0486
4	2371	0.1434	0.201	0.0	0.0556	0.0714	0.471	0.0010	0.0452	0.0582
5	783	0.1595	0.219	0.0	0.0564	0.0720	0.473	0.0021	0.0464	0.0591
6	1876	0.2026	0.212	0.0	0.0689	0.0880	0.513	0.0022	0.0540	0.0692
All groups	8723	0.1416	0.351	0.0	0.0528	0.0691	0.565	0.0013	0.0435	0.0566
					Validation data set					
1	97	0.0700	0.001	0.0018	0.0337	0.0413	0.204	0.0022	0.0300	0.0369
2	616	0.0953	0.079	0.0024	0.0425	0.0546	0.315	0.0029	0.0368	0.0471
3	2417	0.1146	0.183	0.0001	0.0436	0.0557	0.392	0.0017	0.0373	0.0481
4	2448	0.1435	0.161	0.0016	0.0561	0.0732	0.450	0.0018	0.0455	0.0592
5	509	0.1662	0.115	0.0043	0.0598	0.0788	0.401	0.0027	0.0497	0.0649
6	1864	0.2030	0.201	-0.0021	0.0700	0.0898	0.514	0.0014	0.0551	0.0701
All groups	7951	0.1455	0.322	0.0005	0.0545	0.0716	0.552	0.0018	0.0447	0.0582

procedure with six-species groups. Although the bias (as expressed by the mean residual) is slightly greater for the BAI model, the R^2, mean of absolute residuals, and root mean square error are substantially smaller. Mean annual diameter growth (mean DI) for each species group also is included in Table 2.

Table 3 gives the results for different combinations of independent variables for the six species groups. Some of the independent variables discussed previously were added to the NEFIA variables. The models listed are the best 1, 2, and 3 variable combinations. The additional variables express plot occupancy and tree competitive position. In the remaining three models, CRCC2 and CRNCLS2 were deleted. In each of the three models, BAL2, RLD2, or RLQD2 was substituted for CRNCLS2. These variables measure a tree's competitive position relative to the other trees on a plot.

The results for the models with variables substituted for or added to the FIA variables are mixed. Each of the four statistics presented in Table 3 indicate a different "best" model. All differences are small and no model represents a substantial improvement over the others.

Table 3—Comparison statistics for models with variables substituted for or added to NEFIA variables using six-group BAI model and model data sets

Variable	R^2	Mean residual	Mean abs. residual	Root MSE
NEFIA[a]	0.565	0.0013	0.0436	0.0566
DBH2, TRCLS2, CRATIO2, DCR2, BAL2	0.575	0.0014	0.0431	0.0559
DBH2, TRCLS2, CRATIO2, DCR2, RLD2	0.562	0.0010	0.0438	0.0567
DBH2, TRCLS2, CRATIO2, DCR2, RLQD2	0.567	0.0010	0.0435	0.0564
NEFIA + BAL2	0.578	0.0016	0.0429	0.0557
NEFIA + BAL2 + BA2	0.581	0.0015	0.0428	0.0555
NEFIA + BAL2 + BA2+TPA2	0.581	0.0015	0.0428	0.0555

[a] NEFIA= DBH2, TRCLS2, CRNCLS2, CRATIO2, CRCC2, DCR2.

We used the BAI model with six species groups and NEFIA variables to determine how the model performed over the range of tree sizes. The mean residual, mean absolute residual, and root mean square error were determined for 2-inch diameter classes (Table 4). As indicated by the mean residual, for the validation data set there is a small average over prediction for diameters less than 9 inches and greater than or equal to 23 inches. For the diameter classes in between there is a small underprediction.

DISCUSSION AND CONCLUSIONS

With our method there was a marked improvement over current NEFIA procedures in all model valuation statistics except for the mean residual. For the validation data set, the change in the dependent variable from annual DI to annual BAI produced a 72 percent increase in R^2, a 17 percent decrease in mean absolute residual, and an 18 percent decrease in root mean square error.

Grouping by species was better than not grouping them. The grouping based on ranked diameter growth showed only a slight improvement over the seventeen NEFIA form class groups for both the DI and BAI models. Other groupings were not investigated.

There was only a slight improvement when even the best of the other procedures and models was used. As a result, there seems little reason to find a “best” set of independent variables for each group using a stepwise procedure as the outcome using all variables for all groups is essentially the same.

The variables considered as substitutions for or additions to the NEFIA variables produced little improvement. Approximately the same results could be obtained using any of the different combinations of NEFIA and the other plot and tree variables considered here.

Quicke and others (1994) developed a biologically interpretable model using a function with potential growth multiplied by a growth modifier for a single species with plots chosen for specified characteristics. For this study, trees are located on plots chosen at random from a wide range of forest conditions. Disturbance on the plot was not taken into account. There were 78 species and a wide range in size. The results presented here closely approximate those obtained by Quicke and others (1994).

Data used by Teck and Hilt (1991) are from the same type of unstructured design as those presented in this study. They also developed and used a biologically interpretable DI model. With their validation data set, the overall mean prediction error was 0.013 and the root mean square error was 0.085. These statistics are substantially higher than ours.

The coefficients of the models developed here can be used for purposes other than that of NEFIA if the independent variables included in the model are measured. Table 5 contains coefficients for the six-species-group BAI model using NEFIA variables plus BAL2. Table 6 lists the tree species assigned to each of six species groups.

The coefficients developed from this study should be applied to other regions with caution.

Table 4—Comparison statistics by diameter class for six-group basal-area increment model

		Model data set					Validation data set			
Dbh class	No. of trees	Mean DI	Mean Residual	Mean abs. residual	Root MSE	No. of trees	Mean DI	Mean residual	Mean abs. residual	Root MSE
In.										
5 - 6.9	529	0.0573	-0.0043	0.0231	0.0293	490	0.0565	-0.0034	0.0230	0.0289
7 - 8.9	1353	0.0930	-0.0018	0.0313	0.0394	1136	0.0965	-0.0010	0.0323	0.0398
9 - 10.9	1371	0.1246	0.0025	0.0412	0.0518	1202	0.1259	0.0028	0.0401	0.0508
11 - 12.9	1316	0.1401	0.0011	0.0458	0.0577	1179	0.1466	0.0042	0.0454	0.0592
13 - 14.9	1250	0.1587	0.0049	0.0468	0.0614	1087	0.1576	0.0026	0.0487	0.0636
15 - 16.9	946	0.1676	0.0008	0.0483	0.0615	918	0.1741	0.0046	0.0506	0.0651
17 - 18.9	702	0.1787	0.0027	0.0532	0.0680	643	0.1901	0.0089	0.0543	0.0696
19 - 20.9	470	0.1903	0.0064	0.0532	0.0686	480	0.1845	0.0006	0.0564	0.0719
21 - 22.9	296	0.1909	0.0007	0.0554	0.0711	263	0.1882	0.0003	0.0525	0.0665
23 - 24.9	180	0.2018	0.0056	0.0532	0.0698	222	0.1921	-0.0044	0.0580	0.0709
> 25	310	0.1913	-0.0044	0.0499	0.0614	331	0.1851	-0.0103	0.0532	0.0641
All trees	8723	0.1416	0.0013	0.0436	0.0566	7951	0.1455	0.0017	0.0448	0.0582

Table 5—Regression coefficients for six-group basal-area increment model using NEFIA variables and BAL2

Species group	Intercept	DBH2	TRCLS2	CRATIO2	CRNCLS2	DCR2	CRCC2	BAL2
1	0.00107	0.00065	0.00011	0.00028	-0.00043	0.00005	-0.00059	-0.000014
2	0.01448	0.00028	-0.00127	0.00053	-0.00225	0.00020	-0.00367	-0.000033
3	0.02058	0.00064	-0.00309	-0.00115	-0.00272	0.00025	-0.00301	-0.000042
4	0.02239	0.00048	-0.00104	-0.00159	-0.00365	0.00041	-0.00486	-0.000048
5	-0.00150	0.00160	-0.00132	-0.00306	0.00041	0.00021	0.00582	-0.000055
6	0.00132	0.00214	-0.00405	-0.00458	0.00219	0.00016	0.00957	-0.000110

Table 6—Tree species assigned to one of six species groups based on rank of mean diameter increment

Species group	Common name	Scientific name
1	Hawthorn	*Crataegus sp.*
	Shortleaf pine	*Pinus echinata*
	Flowering dogwood	*Cornus flordia*
	European alder	*Alnus glutinosa*
	Table mountain pine	*Pinus pungens*
	Black willow	*Salix nigra*
	Chinkapin oak	*Quercus muehlenbergii*
	Eastern hophornbeam	*Ostrya virginiana*
	Silver maple	*Acer saccharinum*
	Blackgum	*Nyssa sylvatica*
	Willow oak	*Quercus phellos*
	Sourwood	*Oxydendrum arboreum*
2	Maple sp.	*Acer sp.*
	Quaking aspen	*Populus tremuloides*
	Pitch pine	*Pinus rigida*
	Overcup oak	*Quercus lyrata*
	Post oak	*Quercus stellata*
	Red spruce	*Picea rubens*
	Pin cherry	*Prunus pennsylvanica*
	Am. hornbeam, Musclewood	*Carpinus caroliniana*
	Osage-orange	*Maclura pomifera*
	River birch	*Betula nigra*
	Yellow birch	*Betula alleghaniensis*
	Basswood sp.	*Tilia sp.*
	Black walnut	*Juglans nigra*
	Virginia pine	*Pinus virginiana*
	Mockernut hickory	*Carya tomentosa*
	Sweet birch	*Betula lenta*
	Common persimmon	*Diospyros virginiana*
3	Bur oak	*Quercus macrocarpa*
	Ohio buckeye	*Aesculus glabra*
	Pignut hickory	*Carya glabra*
	Hickory sp.	*Carya sp.*
	American beech	*Fagus grandifolia*
	Magnolia sp.	*Magnolia sp.*
	Chokecherry	*Prunus virginiana*
	Buckeye, Horsechestnut	*Aesculus sp.*
	Chestnut oak	*Quercus prinus*
	Sassafras	*Sassafras albidum*
	White basswood	*Tilia heterophylla*
	Shagbark hickory	*Carya ovata*
	Black locust	*Robinia pseudoacacia*
4	Hackberry	*Celtis occidentalis*
	White oak	*Quercus alba*
	Apple sp.	*Malus sp.*
	Eastern hemlock	*Tsuga canadensis*
	Bitternut hickory	*Carya cordiformis*
	Yellow buckeye	*Aesculus octandra*
	Slippery elm	*Ulmus rubra*
	Swamp white oak	*Quercus bicolor*
	Cucumbertree	*Magnolia acuminata*
	American basswood	*Tilia americana*
	Red maple	*Acer rubrum*
	American elm	*Ulmus americana*
	Sugar maple	*Acer saccharum*
	Shellbark hickory	*Carya laciniosa*
	Striped maple	*Acer pennsylvanicum*
	Eastern red-cedar	*Juniperus virginiana*
5	Black oak	*Quercus velutina*
	White ash	*Fraxinus americana*
	Butternut	*Juglans cinerea*
	Elm sp.	*Ulmus sp.*
	Sycamore	*Platanus occidentallis*
	Green ash	*Franxinus pennsylvanica*
	Boxelder	*Acer negundo*
	Black cherry	*Prunus serotina*
	Eastern white pine	*Pinus strobus*
	Scarlet oak	*Quercus coccinea*
6	Northern red oak	*Quercus rubra*
	Bigtooth aspen	*Populus grandidentata*
	Yellow-poplar	*Liriodendron tulipifera*
	Pin oak	*Quercus palustris*
	Prunus sp.	*Prunus sp.*
	Ailanthus	*Ailanthus altissima*
	Black maple	*Acer nigrum*
	Eastern redbud	*Cercis canadensis*
	Southern red oak	*Quercus falcata v. falcata*

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