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Evaluation of mapped-plot variance estimators across a range of partial nonresponse in a post-stratified national forest inventory

James A. Westfall, Andrew J. Lister, and Charles T. Scott

Abstract: When conducting a forest inventory, sometimes portions of plots cannot be measured due to inaccessibility. Two primary methods have been presented to account for partial nonresponse in the estimation phase: (*i*) use a ratio-to-size estimator or (*ii*) apply an adjustment factor to all plot observations in proportion to the missing area. Both approaches provide identical estimates of the population mean, but the estimates of variance differ when partial nonresponse is present. The performance of variance estimators was examined for a range of population forest area and partial nonresponse proportions in the sample. The ratio-to-size variance estimator performed unbiasedly with respect to simulation results, but the adjustment factor variance estimates were biased, with magnitude and direction dependent upon the forest area proportion and amount of partial nonresponse. The bias is relatively small when the partial nonresponse is small, which is often the case; however, the ratio-to-size method is preferred to ensure accurate variance estimation for a wide range of circumstances.

Key words: nonforest, ratio estimate, bias, correlation.

Résumé : Lors de la réalisation d'un inventaire forestier, il arrive parfois que des portions de placettes ne puissent pas être mesurées parce qu'elles sont inaccessibles. Deux méthodes principales ont été présentées pour tenir compte de l'absence partielle de résultat pendant la phase d'estimation : (*i*) utiliser un estimateur quotient selon la taille, ou (*ii*) appliquer un facteur d'ajustement proportionnel à la zone manquante à toutes les observations de la placette. Les deux approches produisent une estimation identique de la moyenne de la population, mais l'estimation de la variance est différente lorsqu'il y a une absence partielle de résultat. La performance de l'estimateur de la variance a été examinée en fonction d'une gamme de superficies forestières et de proportions d'absence partielle de résultat dans l'échantillon. L'estimateur quotient selon la taille de la variance produisait des résultats de simulation non biaisés, mais les estimations de la variance provenant du facteur d'ajustement étaient biaisées en fonction de l'amplitude et de la direction des résultats qui variaient selon la proportion de la superficie forestière et de la quantité d'absence partielle de résultat. Le biais est relativement petit lorsque l'absence partielle de résultat est de petite taille, ce qui est souvent le cas. Malgré tout, la méthode de l'estimateur quotient selon la taille est préférée pour s'assurer d'une estimation précise de la variance pour une large gamme de circonstances. [Traduit par la Rédaction]

Mots-clés : absence de forêt, estimation quotient, biais, corrélation.

Introduction

Various factors affect the specification of field plot measurement protocols in forest inventories. For large-area inventories in which many different land characteristics are likely to be encountered, a mapped-plot protocol is used to describe the amount and type of various conditions that may be present (Wakelin et al. 2016; Bechtold and Scott 2005; Schreuder and Gregoir 1995). In addition to the flexibility for mapping differing conditions, the mapped-plot protocol also allows for delineation of portions of plots not sampled due to lack of access permission or hazardous conditions. In general, there are two primary estimation approaches for handling the partial nonresponse in the estimation phase: (*i*) use a ratio-to-size method whereby the area actually sampled is used instead of the full sample plot area (Van Deusen 2004), or (*ii*) apply an average adjustment to all plot observations based on the amount of partial nonresponse in the sample (Scott et al. 2005). Both approaches provide identical estimates of the population mean for a given sample; however, empirical results for the estimated variance differ. Van Deusen (2005) used a simulated population to show that the average adjustment method overestimated the variance proportionally to the amount of partial nonresponse. Velasco-Bautista et al. (2020) found similar results using an artificial mapped tree population based on national forest inventory (NFI) data in Mexico. These studies showed the two approaches to be equal when no partial nonresponse was present and nearly so when partial nonresponse was rarely encountered. Thus, use of the average adjustment method may have small practical impact for NFIs having little partial nonresponse.

Although it is not specifically stated in either the Van Deusen (2005) or Velasco-Bautista et al. (2020) studies, it appears that most (if not all) plot observations were >0. In the case of many NFIs, the population structure can differ substantially from this configuration, as the population definition includes all lands

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J.A. Westfall and A.J. Lister. US Forest Service, Northern Research Station, 3460 Industrial Drive, York, PA 17402, USA.
C.T. Scott.* US Forest Service, West Chester, PA 19382, USA.

Corresponding author: James Westfall (email: james.westfall@usda.gov).

*Retired.

within the country borders, and thus, nonforest plots contribute a valid observation of zero for estimation of forest attributes. Thus, further investigation is warranted to evaluate the partial nonresponse estimation approaches in a typical NFI application. Specifically, the performance of the variance estimators needs to be broadly assessed across a range of forest land area proportions in the population and across various amounts of partial nonresponse that may be realized in a NFI sample to better understand the factors that affect estimation accuracy.

Materials and methods

Data

The data used to evaluate estimator performance were collected during 2013 to 2019 across an area of approximately 9 755 000 ha in the state of Pennsylvania, USA, by the Forest Inventory and Analysis (FIA) program of the US Forest Service. FIA employs a quasi-systematic sampling design with a sampling intensity of approximately one plot per 2428 ha (5937 acres) (Reams et al. 2005). The field data used in this study were collected using a 0.067 ha (0.166 acre) cluster plot design, in which each plot is composed of four circular subplots having a 7.32 m (24 ft) radius with one subplot located at plot center and the remaining three subplots centered at azimuths 120°, 240°, and 360° and distance of 36.58 m (120 ft) from plot center (Fig. 1; Bechtold and Scott 2005). In forested areas, trees having a diameter at breast height (DBH) of ≥12.70 cm (5.0 in) are measured and their locations relative to subplot center are recorded. Observed tree data are subsequently used in statistical models to estimate tree wood volume (Scott 1981). In this study, wood volume is the attribute of interest used to evaluate different estimation approaches.

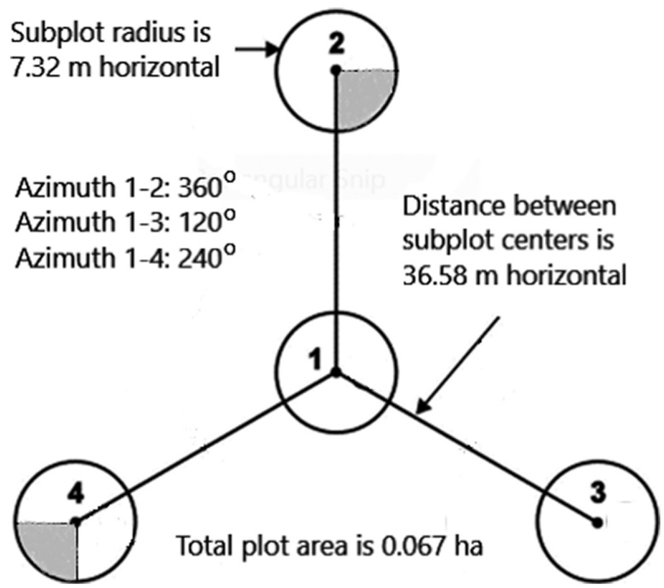
In some cases, portions of plots are nonsampled due to hazardous situations or lack of access permission, and the boundaries of these areas are mapped such that the actual sampled area proportion can be calculated (US Forest Service 2018). The FIA program gains considerable efficiency via pre-screening of plot locations using high-resolution remotely sensed imagery, such that entirely nonforest plots are not field visited and by definition have no trees or associated wood volume. These "office" plots are still part of the inventory sample and, unlike field plots, are not subject to nonresponse. This study only used 2328 fully sampled plots that were entirely forest or nonforest, of which 905 were forest (i.e., forest area ≈ 39% of population area).

It is common in many NFIs to increase the precision of estimates through the use of auxiliary data. In the FIA program, this is accomplished via post-stratification whereby the post-strata are constructed from the National Land Cover Dataset (NLCD) tree canopy cover map (Homer et al. 2015) consisting of 30 m pixel based estimates of canopy cover proportion between 0 and 1 across the US. To be consistent with the NFI implementation in the study area, five strata were defined by map canopy cover classes of 0%–5% (stratum 1), 6%–20% (stratum 2), 21%–50% (stratum 3), 51%–80% (stratum 4), and 81%–100% (stratum 5). Plots were assigned to strata based on the pixel value corresponding with plot center. The stratum weights (W_h) were determined as the proportion of pixels in the canopy cover class divided by total number of pixels in the population and the weights were considered known quantities due to the complete spatial coverage of the NLCD data.

Post-stratified estimation

Given the interest in estimator performance under various amounts of partial nonresponse, the estimators to be studied include (*i*) the partial nonresponse correction method as implemented by the FIA program (denoted FIA approach; Scott et al. 2005), (*ii*) the mapped-plot estimator (VD) presented by Van Deusen (2004), and (*iii*) the ratio-to-size estimator (RS) as described in an NFI context by Korhonen and Salmensuu (2014) with further

Fig. 1. Forest Inventory and Analysis plot design and example of randomly generated 90° sections of partial nonresponse occurring on two subplots (shaded).



elucidation by Scott and Westfall (2022). A mean of ratios (MOR) approach is not considered due the potentially high bias (Salas and Gregoire 2010) and conceivable implementation issues (Zarnoch and Bechtold 2000). A key difference between the FIA approach and the others is the modification of the plot-level observations to account for the partial nonresponse using

$$y_{hi} = \frac{y'_{hi}}{\bar{a}_h}$$

where y'_{hi} is the observed value expressed on a per hectare basis from the accessible portion of plot i in stratum h, $\bar{a}_h = \left(\sum_i^{n_h} a_{hi}\right)/n_h$ is the mean proportion of accessible plot areas in stratum h, a_{hi} is the proportion of accessible area of plot i in stratum h ($0 \le a_{hi} \le 1$), and y_{hi} is the ith plot observation in stratum h used in estimation to account for partial nonresponse. Plots that are wholly nonresponse ($a_{hi} = 0$) are not included in the calculation of $\bar{a}_h$ and are not used in estimation calculations. This adjustment approach is used to avoid potential bias resulting from excluding plots that are only partially observed. Within stratum h, all sampled plot areas are adjusted equally regardless of whether partial nonresponse exists on a plot (Scott et al. 2005). Implementation of VD and RS methods requires no plot-level adjustments, thus

$$y_{hi} = y'_{hi}$$

For FIA, $\bar{y}_h = \left(\sum_i^{n_h} y_{hi}\right)/n_h$ is the estimated mean for attribute y in stratum h, and n_h is the sample size for stratum h. The estimated mean under both VD and RS methods is $\bar{y}_h = \sum_i^{n_h} y_{hi} \Big/ \sum_i^{n_h} a_{hi}$. Given that $\sum_i^{n_h} a_{hi} = n_h\bar{a}$, the estimated $\bar{y}_h$ is identical for all three methods. However, this is not the case for estimated variances, where the FIA method uses $v_{\mathrm{FIA}}(\bar{y}_h) = \left(\sum_i^{n_h} (y_{hi} - \bar{y}_h)^2\right)/n_h(n_h - 1)$ to obtain the estimated variance of the mean for attribute y in stratum h, where N_h is the total number of population units in stratum h. The variance estimators differ accordingly, with the estimated

Table 1. Stratum size (N_h), weight (W_h), and summary statistics for forest area and volume in a post-stratified population of FIA plots in Pennsylvania, USA.

			Forest area proportion			Volume (m^3/ha)		
Stratum	N_h	W_h	Min.	Mean	Max.	Min.	Mean	Max.
1	1204	0.375	0.00	0.01	1.00	0.00	0.33	293.86
2	174	0.085	0.00	0.13	1.00	0.00	12.09	436.02
3	84	0.055	0.00	0.57	1.00	0.00	66.18	369.58
4	286	0.190	0.00	0.90	1.00	0.00	135.61	427.62
5	580	0.295	0.00	0.97	1.00	0.00	158.19	410.90
Total	2328	1.000	0.00	0.39	1.00	0.00	59.53	436.02

Note: Note that each stratum contains nonforest plots (forest area proportion = 0) and plots having volume = 0 due to being either entirely nonforest or where forestland exists but no trees larger than 12.70 cm diameter at breast height are present.

variances for VD and RS methods being, respectively, $v_{\mathrm{VD}}(\bar{y}_h) = \sum_i^{n_h}(y_{hi} - a_{hi}\bar{y}_h)^2 \Big/ \sum_i^{n_h} a_{hi}\left(\sum_i^{n_h} a_{hi} - 1\right)$ and $v_{\mathrm{RS}}(\bar{y}_h) = \frac{n_h}{n_h - 1} \sum_i^{n_h}(y_{hi} - a_{hi}\bar{y}_h)^2 \Big/ \left(\sum_i^{n_h} a_{hi}\right)^2$. Although finite population correction factors are usually unnecessary in NFI applications due to small sampling fractions, inclusion in the preceding formulae ($1 - n_h/N_h$ for FIA and $1 - \left(\sum_i^{n_h} a_{hi}\right)/N_h$ for VD and RS) may be needed when large sampling fractions are used and were included in the simulations described below. The respective overall estimated mean and variance (Cochran 1977) for all methods are $\bar{y} = \sum_h^H W_h\bar{y}_h$ and $v(\bar{y}) = 1/n\left[\sum_h^H W_h n_h v(\bar{y}_h) + \sum_h^H (1 - W_h)(n_h/n)v(\bar{y}_h)\right]$, where n is the total sample size.

Simulation

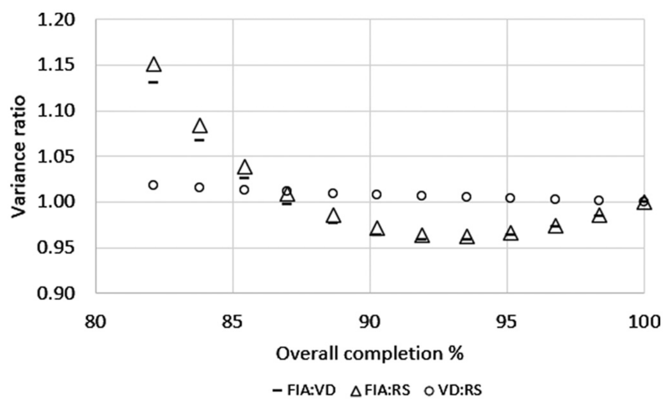
The performance of the estimators described above was evaluated for estimates of mean wood volume per hectare in the population. A Monte Carlo simulation was conducted, with each replication taking the following steps:

1) Select a 12% sample in each stratum proportional to stratum population size N_h. This sampling intensity was chosen to attain a minimum of $n_h = 10$ in each stratum (Westfall et al. 2011). The resultant stratum sample sizes were $n_1 = 143$, $n_2 = 21$, $n_3 = 10$, $n_4 = 34$, and $n_5 = 69$.
2) On forested plots, randomly choose 0–4 subplots for partial nonresponse.
3) For each subplot selected for nonresponse, randomly choose an azimuth from 0° to 359°. To this azimuth, add the appropriate number of degrees for the level of nonresponse being investigated (30°–330° in increments of 30) (Fig. 1).
4) Calculate the plot volume, excluding the volume of any trees located within the nonresponse zone. Nonforest plots have zero volume by definition.

Two thousand replications were conducted for each partial nonresponse proportion class and the estimated means and variances for each method were calculated for comparative purposes.

A broader simulation study was also conducted to examine a wider range of forestland area and partial nonresponse scenarios. This simulation used a synthetic population of 500 FIA plots where the forestland area proportions were controlled to span from 0.01 to 1.00 and the average partial nonresponse proportion on forested plots ranged from 0.0 to 0.5. Based on the specified combination of forestland area proportion and accessible proportion class, the following steps were taken at each iteration: (*i*) plots were randomly classified as either forest or nonforest, (*ii*) total tree volume on forested subplots was randomly generated from a uniform distribution (0, 50) and nonforest subplots were assigned an observation of zero volume, (*iii*) a subset of forested subplots were randomly chosen to be inaccessible and subplot volume a missing value, and (*iv*) means and variances were calculated based on $v_{\mathrm{FIA}}(\bar{y}_h)$ and $v_{\mathrm{RS}}(\bar{y}_h)$. Upon completion of 10 000 iterations, the mean values of the respective variances were calculated to evaluate the difference between the two methods.

Fig. 2. Ratios of estimated variances arising from $v_{\mathrm{FIA}}(\bar{y})$, $v_{\mathrm{VD}}(\bar{y})$, and $v_{\mathrm{RS}}(\bar{y})$ at various levels of accessible area proportions. Differences between $v_{\mathrm{VD}}(\bar{y})$ and $v_{\mathrm{RS}}(\bar{y})$ are small and both perform well regardless of completion rate.



Results

Given that each method provides an identical and unbiased estimate of the mean, the primary focus is on the performance of the variance estimators. For the population being considered (39% forest land), nonforest plots being excluded from partial nonresponse, and a post-stratified estimation framework, it was found that the FIA approach resulted in a smaller estimated variance compared with $v_{\mathrm{VD}}(\bar{y})$ and $v_{\mathrm{RS}}(\bar{y})$ when the total accessible area proportion within the sample plots was greater than ~0.86 (Fig. 2). The FIA approach estimates a higher variance when the accessible area proportion is less than ~0.86 and the difference between FIA and the other approaches increases with decreasing accessibility proportion. Understanding this phenomenon requires examination of variances arising from each stratum as this is the level at which the estimators are applied.

A primary difference among strata are minimum accessible proportions attainable given the proportion of forested plots present. For example, stratum 1 contains about 1% forest area and therefore only about 1% of the plots are subject to partial nonresponse. Thus, even at high partial nonresponse rates, the minimum accessible proportion will be near 0.99. As strata become increasingly forested (Table 1), the potential minimum accessible proportion decreases accordingly. This point is relevant because

Fig. 3. Ratio of $v_{\mathrm{FIA}}(\bar{y}_h)/v_{\mathrm{RS}}(\bar{y}_h)$ by stratum and accessible proportion.

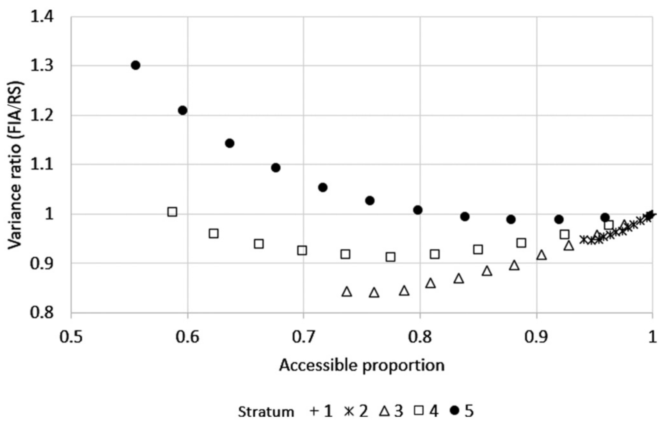


Fig. 4. Ratio of $v_{\mathrm{FIA}}(\bar{y}_h)/v_{\mathrm{RS}}(\bar{y}_h)$ for various levels of forest area proportion and accessible proportion.

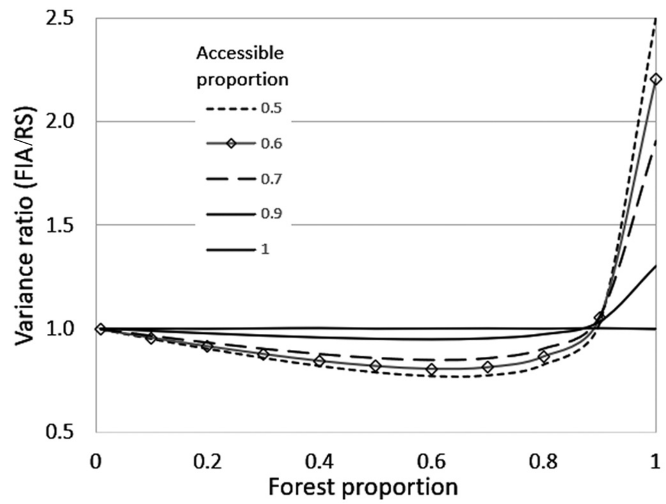
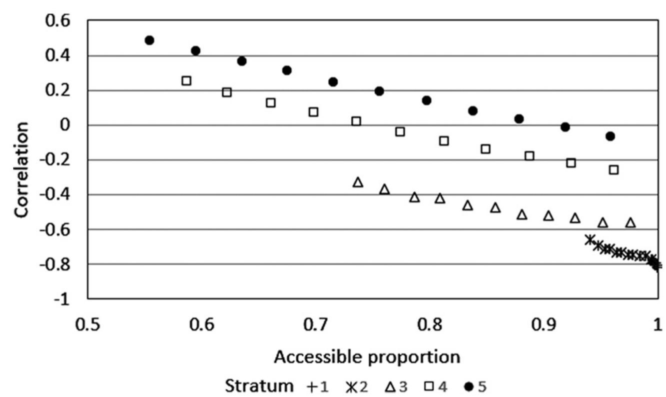


Fig. 5. Correlation between a_{hi} and y_{hi} by stratum and accessible proportion.



the methods perform identically when the accessible proportion is 1.0, and thus, deviations between the two approaches will be small within strata where completion rates are slightly less than 1.0 (e.g., stratum 1). A second outcome was that, similar to the findings of Velasco-Bautista et al. (2020), $v_{\mathrm{RS}}(\bar{y})$ and $v_{\mathrm{VD}}(\bar{y})$ produce variance estimates that are nearly identical (Fig. 2); however, $v_{\mathrm{VD}}(\bar{y})$ becomes slightly larger as accessible proportion diminishes. A third factor of considerable importance is that the Monte Carlo variances for each stratum and accessible proportion matched very closely with $v_{\mathrm{RS}}(\bar{y}_h)$ and thus deviations from $v_{\mathrm{RS}}(\bar{y}_h)$ are considered as either over- or underestimates of the true variance. These differences varied by stratum and accessible proportion, with those due to $v_{\mathrm{FIA}}(\bar{y})$ being of primary interest where underestimation occurred in strata 1–4, with an initial slight underestimation shifting to considerable overestimation in stratum 5 (Fig. 3).

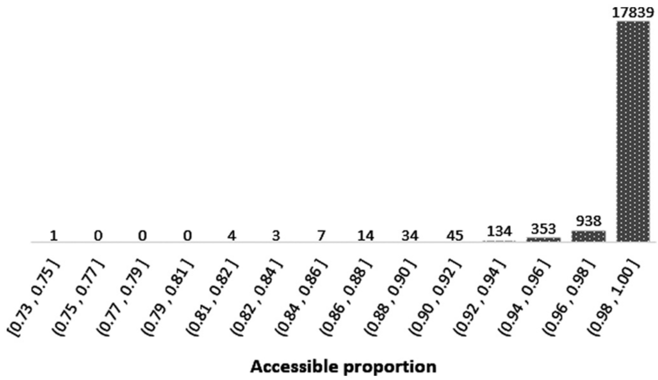
The results show that for strata composed of approximately 90% forestland or less, partial nonresponse produces $v_{\mathrm{FIA}}(\bar{y}_h)$ estimates that are too small. When forestland area exceeds this threshold, a shift occurs whereby $v_{\mathrm{FIA}}(\bar{y}_h)$ overestimates the actual variance, with the overestimation increasing with increasing amounts of partial nonresponse. In this study, stratum 2 (13% forest) and stratum 3 (57% forest) show an asymptotic effect of accessible proportion on the underestimation of $v_{\mathrm{FIA}}(\bar{y}_h)$ and stratum 4 (90% forest) exhibits a reversal in bias direction occurring at the accessible proportion of 0.77 (Fig. 3). Stratum 5 (97% forest) initially has a small negative bias for $v_{\mathrm{FIA}}(\bar{y}_h)$, but shifts to a positive bias near the 0.82 accessible proportion, with this positive bias increasing as accessible proportion continues to decline. This latter outcome is consistent with the results of Van Deusen (2004) and Velasco-Bautista (2020) pertaining to 100% (or nearly so) forestland area. It is suspected that the slight initial negative bias in $v_{\mathrm{FIA}}(\bar{y}_h)$ in stratum 5 is due to the presence of a few nonforest plots that produce a weak negative covariance between a_i and y_i.

The observed outcomes using the FIA plot population led to a broader assessment of variance estimation behavior over a systematic range of forest area proportion and accessible proportion scenarios. The outcome aligned with the findings based on the FIA plot population, particularly the interaction that occurs between the proportion of forest area and the accessible proportion that results in a shift from under- to overestimation of variance using $v_{\mathrm{FIA}}(\bar{y}_h)$ (Fig. 4). The results also agree with the result using $v_{\mathrm{FIA}}(\bar{y})$ (Fig. 2), where an overall accessible proportion lower than about 0.88 results in underestimation of the variance, with overestimation occurring at accessible proportions higher than ~0.88. This study confirms the findings of both Van Deusen (2005) and Velasco-Bautista et al. (2020) using $v_{\mathrm{FIA}}(\bar{y}_h)$ for fully forested populations and provides a more complete picture of variance estimator behavior across a wide range of population structures common to national forest inventory applications.

Discussion

Generally, the FIA approach does not consider the correlation between a_i and y_i, i.e., $\bar{a}_h$ is a constant for a given sample. However, it is often the case that this assumption does not hold. Consider stratum 1, which is only 1% forest and is therefore subject to only a small amount of partial nonresponse. This stratum is composed mostly of nonforest plot observations where $a_i = 1$ and $y_i = 0$, with a few forest plots having $a_i = 1$ and $y_i > 0$ (no partial nonresponse) or $a_i < 1$ and $y_i > 0$ (partial nonresponse). This type of data structure results in a negative correlation between a_i and y_i. This same phenomenon occurs in other strata having $a_i = 1$ and $y_i = 0$ plots (primarily nonforest plots but could occur with forest plots with no tree volume). Correlations <0 are present for strata 1–3 at all the partial nonresponse levels evaluated, and strata 4 and 5 only have correlation >0 for relatively large inaccessible proportions (Fig. 5). As the strata tend to become more forested (e.g., strata 4 and 5) and as the accessible proportions become smaller, the nonforest phenomenon that promotes negative correlations between a_i and y_i is diminished and positive correlations are realized because as a_i increases then y_i also tends to increase.

Fig. 6. Distribution of number of strata within accessibility proportion classes across 24 states in the northern United States from 2003 to 2019.



The circumstance that FIA does not account for any correlation between a_{hi} and y_{hi} contributes to the inaccurate estimation of the stratum variance. Expansion of the numerator from the $v_{\mathrm{RS}}(\overline{y}_h)$ estimator into $\sum_{i=1}^{n} y_{hi}^2 - 2\overline{y}_h \sum_{i=1}^{n} y_{hi}a_{hi} + \overline{y}_h^2 \sum_{i=1}^{n} a_{hi}^2$ explicitly reveals the covariance component as $-2\overline{y}_h \sum_{i=1}^{n} y_{hi}a_{hi}$. This term increases the overall variance when a negative covariance is present, but $v_{\mathrm{FIA}}(\overline{y}_h)$ does not account for the covariance and thus is too small. Likewise, a positive within-stratum covariance should produce a commensurate decrease in the variance, but $v_{\mathrm{FIA}}(\overline{y}_h)$ then overestimates the variance in this situation.

Given that the $v_{\mathrm{RS}}(\overline{y}_h)$ estimated variances aligned with the Monte Carlo variances across all strata and accessible proportions, the $v_{\mathrm{RS}}(\overline{y}_h)$ approach appears to be the preferred method for accurately estimating variance in the presence of partial nonresponse. This study examined a wide range of stratum accessibility proportions, many of which are unlikely to be encountered in practical application. An examination of stratum accessibility proportions across 24 states within the Northern FIA region (www.nrs.fs.fed.us/fia) showed a highly skewed distribution resulting from a few strata with relatively low accessible proportions (Fig. 6). However, from a practical standpoint the mean accessible proportion over nearly 20 000 strata constructed from 2003 to 2019 was 0.995, with over half of the strata having 1.0 accessibility proportion. Thus, bias in stratum variances due to partial nonresponse using $v_{\mathrm{FIA}}(\overline{y}_h)$ are generally very small and have minimal practical impact in creating bias in overall variance estimates produced by the FIA program. However, the results indicate that using $v_{\mathrm{FIA}}(\overline{y}_h)$ when large partial nonresponse proportions are present can produce considerably inaccurate estimations of variance.

It is noteworthy that $v_{\mathrm{RS}}(\overline{y}_h)$ is equivalent to the variance of a traditional ratio estimate (Cochran 1977, p. 155) under the conditions that the variance of the ratio is (*a*) calculated with y'_{hi} (or, equivalently, y_{hi} with $\overline{a}_h = 1$), and (*b*) calculated only for the subpopulation consisting of accessible land. In this case, the ratio of means estimator expresses the result in terms of per unit area of the subpopulation; in our example, this would be *per hectare of accessible land*. The ratio is calculated as

$$\hat{R}_h = \frac{\overline{y}'_h}{\overline{x}'_h}$$

where $\overline{y}'_h$ is calculated as

$$\overline{y}'_h = \frac{\sum_i^{n_h} y'_{hi}}{n_h}$$

and $\overline{x}'_h$ is the average proportion of the stratum that is accessible and $\overline{x}'_{hi}$ is the proportion of plot i that is accessible, a_{hi}. The average of x'_{hi} is calculated in the same manner as for y'_{hi}. The variance of $\hat{R}_h$ is calculated as

$$v(\hat{R}_h) = \frac{1}{\overline{X}_h'^2}\left[v(\overline{y}'_h) + \hat{R}_h^2 v(\overline{x}'_h) - 2\hat{R}_h \ \mathrm{cov}(\overline{y}'_h, \overline{x}'_h)\right]$$

where $v(\overline{y}'_h)$ is calculated as

$$v(\overline{y}'_h) = \frac{\left(1 - {}^{n_h}/_{N_h}\right)\sum_i^{n_h}\left(y'_{hi} - \overline{y}'_h\right)^2}{n_h(n_h - 1)}$$

and $v(\overline{x}'_h)$ is calculated likewise. The covariance is calculated as

$$\mathrm{cov}(\overline{y}'_h, \overline{x}'_h) = \frac{\left(1 - {}^{n_h}/_{N_h}\right)\sum_i^{n_h}\left(y'_{hi} - \overline{y}'_h\right)\left(x'_{hi} - \overline{x}'_h\right)}{n_h(n_h - 1)}.$$

It is evident that $v_{\mathrm{RS}}(\overline{y}_h)$ can be rewritten as

$$v_{\mathrm{RS}}(\overline{y}_h) = \frac{n_h}{n_h - 1}\left(1 - \sum_i^{n_h} a_{hi} \Big/ N_h\right)\frac{\left(\sum_i^{n_h} y_{hi}^2 - 2\overline{y}_h \sum_i^{n_h} y_{hi}a_{hi} + \overline{y}_h^2 \sum_i^{n_h} a_{hi}^2\right)}{\left(\sum_i^{n_h} a_{hi}\right)^2}$$

which is equivalent to Cochran's alternate expression of the traditional ratio estimate (Cochran 1977, p. 155, eq. 6.10), when subpopulation estimation is involved, i.e., when summations are done using y'_{hi} and x'_{hi}, which corresponds with the subpopulation of accessible land only and substituting $\sum_i^{n_h} a_{hi}$ for n_h.

Identifying this equivalency helps reveal the nature of a key difference between $v_{\mathrm{FIA}}(\overline{y}_h)$ and $v_{\mathrm{RS}}(\overline{y}_h)$, i.e., the way the partial plot correction ($\overline{a}_h$) is applied at the level of y_{hi}. This impacts the variance in the ways described above. In other words, $v_{\mathrm{FIA}}(\overline{y}_h)$ applies $\overline{a}_h$ to each observation in the sample as a method of accounting for missing portions of plots to obtain an unbiased estimate of the mean for the entire population. However, this approach has undesirable properties in the context of variance estimation. Using $v_{\mathrm{RS}}(\overline{y}_h)$ or $v(\hat{R}_h)$ implicitly narrows the scope of inference to the subpopulation of accessible land and relies on the data consumer to evaluate the validity of the assumption that nonsampled portions of the population are similar in status to the sampled portions. This assumption of similarity would be necessary to extend the results to the entire population that includes inaccessible areas. While there are pros and cons of both $v_{\mathrm{FIA}}(\overline{y}_h)$ and $v_{\mathrm{RS}}(\overline{y}_h)$ paradigms, the latter approach is more rigorous across a wide range of partial nonresponse outcomes.

Further study may be needed to extend the applicability to other forest inventory applications in which nested fixed plot sizes or Bitterlich sampling methods are used to obtain plot observations. One possible approach when nested plot sizes are used is to scale the observations from smaller plot sizes j to the largest plot size J. The plot attribute becomes $y_{hi} = \sum_j^J y_{hij}\left(\sum_i^{n_h} a_{hiJ} \Big/ \sum_i^{n_h} a_{hij}\right)$, where within stratum h, y_{hij} is the observation for plot i on plot size j, and a_{hij} is the plot size j area proportion measured on plot i. For variable size (Bitterlich) plots, the situation becomes more complex, but in cases for which full assessment of trees to be included in the sample is possible (but some physical tree locations cannot be accessed), a ratio based on accessible tally trees to total tally trees could be calculated on a basal area (or tree count) basis, or if diameters of inaccessible trees can be adequately estimated, then a similar ratio could be obtained based on sums of individual tree inclusion zone areas. When inaccessible tally trees cannot be assessed for sample inclusion, other methods would need to be devised. The performance of any given approach should be tested for validity prior to actual implementation in operational inventories.

Conclusion

This study extended the analytical realm of partial nonresponse scenarios to pertain more broadly to large-area forest inventories in which more heterogeneous populations are often encountered. The results showed how interactions between forestland area proportion and sample plot accessibility proportion affect correlations between a_i and y_i, and the resultant effects when using the $v_{\mathrm{FIA}}(\bar{y}_h)$ estimator. Bias in the overall variance estimate $v_{\mathrm{FIA}}(\bar{y})$ depends both on the magnitude and direction of the stratum biases, as well as the stratum weights. This study did not evaluate directly the role of stratum weights in over- or underestimation of the variance by $v_{\mathrm{FIA}}(\bar{y})$; however, broad generalizations can be made, e.g., heavily forested populations will likely have larger weights in the high canopy cover strata (i.e., strata 4 and 5) and thus $v_{\mathrm{FIA}}(\bar{y})$ will be more likely to overestimate the overall variance, whereas populations with little forest would be dominated by low canopy cover strata (i.e., strata 1 and 2) and presumably underestimation would result.

Practitioners currently using a sample observation adjustment factor to account for partial nonresponse should evaluate their particular forest inventory circumstances. It was shown that $v_{\mathrm{FIA}}(\bar{y})$ performs very similarly to $v_{\mathrm{RS}}(\bar{y})$ when partial nonresponse is small (and identically when partial nonresponse is absent). Forest inventories such as those conducted on industry- or state-owned lands where entire plot access is essentially certain can use $v_{\mathrm{FIA}}(\bar{y})$ with little concern. However, other types of inventories in which plot access is dependent on various external factors that may result in nontrivial partial nonresponse should consider using $v_{\mathrm{RS}}(\bar{y})$ to ensure accurate variance estimation across a range of potential nonresponse outcomes.

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