

Wood Diaphragm Deflections. I: Generalizing Standard Equations Using Mechanics-Based Derivations for Panel Construction

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Abstract: Horizontal wood diaphragm systems, whether decked with conventional or mass timber panels, transfer wind and seismic loads to vertical elements of a lateral force-resisting system (LFRS), in flexible, rigid, or semirigid fashion. Characterizing and calculating the resulting diaphragm deflections determines the distribution of forces to critically loaded components and a significant portion of lateral building translations and rotations. Deflection equations for sheathed wood structural panel (WSP) diaphragms are well established in US design standards in a four-term expression that models flexural, shear, and fastener-slip deformations, and its full derivation using principles of mechanics is provided herein. Derivations of similar equations for cross-laminated timber (CLT) diaphragms have yet to unfold, despite growing industry consensus that CLT panels make efficient slabs and decks. In this first of two companion papers, the corrected full derivation of the current four-term WSP diaphragm deflection expression is provided and assessed, and two ways to quantify the cumulative contribution of fastener slip are presented to expand its usage to a wider variety of WSP and CLT configurations in current use. Building on this generalized mechanics-based derivation, the authors are able to propose and assess in the companion paper a unified diaphragm deflection model to compute both WSP and CLT diaphragm deflections as implemented under current practice and guide further development. DOI: [10.1061/JAEIED.AEENG-1573](https://doi.org/10.1061/JAEIED.AEENG-1573). © 2023 American Society of Civil Engineers.

Introduction

Engineered wood building diaphragms, using plywood structural panels, date back nearly three-quarters of a century. Since the 1970s, other composite products, such as oriented strand board (OSB), gained market share and gave rise to the general category of wood structural panel (WSP) sheathing (Peterson 1983), which typically measures $1,220 \times 2,440$ mm (4×8 ft) of rectilinear area and up to 28.5 mm ($1\frac{1}{8}$ in.) of thickness in nominal unitized dimensions. The WSP diaphragm type has extensively served conventionally framed residential and commercial wood buildings up to six stories tall. Given the importance of horizontal wood diaphragms in resisting a variety of loads (e.g., earth, wind, and seismic) acting transversely to wall planes (Cobeen et al. 2014), design procedures for WSP diaphragms are well established in US building code standards. Deflection equations for WSP diaphragms in the latest edition of the American Wood Council's *Special Design Provisions for Wind and Seismic* (SDPWS) (AWC 2021) were derived using principles of engineering mechanics, but past corrections and simplifications have obscured their origins over time.

Over the last decade, cross-laminated timber (CLT) has emerged as a new mass timber product, with a majority of engineers expressing that they are likely to adopt CLT construction in a future project if given availability (Laguarda-Mallo and Espinoza 2018). The most common use of CLT in building construction is as floor and roof decking, where direct use of the CLT deck as a diaphragm system is economically attractive. Like WSPs, CLT members are rectangular, layered-composite wood panels, typically fastened around their perimeters and designed to primarily span flat in one direction; however, CLT panels are much larger and have larger dimensional aspect ratios than WSP panels. As with WSP diaphragms, CLT panel edges are fastened to provide stiffness and resist in-plane shears. For CLT diaphragms, SDPWS states that deflections shall be determined using principles of engineering mechanics (AWC 2021), yet no deflection equations are directly provided. The motivation to develop similarly established diaphragm deflection models for CLT panel diaphragms, therefore, leads us to document the principles of engineering mechanics underlying WSP diaphragm deflection equations.

Diaphragm deflections determine whether a diaphragm may be idealized as flexible (ASCE 2022; AWC 2021) or rigid (AWC 2021; ICC 2021), or what stiffness factor to use in a semirigid approach. Deformation compatibility, torsional irregularities, and building separations, are additionally influenced by this computation. Relative stiffness of the diaphragm and vertical elements, furthermore, calibrates the results of numerical earthquake response time-history analyses used for detailed evaluation of structures. Overestimation of diaphragm deflection may lead to conservative sizing of seismic joints but underestimates period-based seismic forces, owing to diaphragm stiffness (Lawson 2019). Large single-story buildings for warehousing or “big-box” retailers in the US typically comprise stiff in-plane concrete or masonry walls with a flexible diaphragm, resulting in a dynamic seismic response dominated more by the diaphragm's period than that of the shear walls. Because of the trend of using period-based seismic forces

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in the diaphragms of these building types (FEMA 2021; ASCE 2017), the Building Seismic Safety Council (BSSC 2021) identified the computation of period-based diaphragm stiffness as an important current research need. This paper re-establishes the derivation of the four-term WSP deflection equations, including two approaches that quantify the cumulative contribution of nail slip. Despite the size disparity between WSP and CLT panels, both systems share mechanical characteristics that make a unified approach to modeling diaphragm deflections adaptable to a wide variety of diaphragm configurations.

Standard Deflection Equations of Diaphragms Sheathed with WSPs

Current US practice of calculating wood-sheathed diaphragm deflections is based on modified forms of

$$\delta_{\text{dia,US}} = \frac{5vL^3}{8EAW} + \frac{vL}{4G_v t_v} + 0.188Le_n + \frac{\sum (x\Delta_c)}{2W} \quad (1)$$

published in the SDPWS commentary (AWC 2021). For definitions of the parameters appearing in the equations and figures, throughout this paper, see the notation section.

Eq. (1), developed for customary units used by US designers, includes implicit unit conversions in the first and third terms. This equation was developed specifically for uniformly loaded simple span diaphragms, to quantify the four sources of deflection schematically illustrated in Fig. 1.

In respective order of the terms, Eq. (1) estimates the contributions from flexural deformation, shear-induced panel deformation, diaphragm deformation associated with nail slip, and the effects of the flexural chord connection slip. The next section reviews the chronological development of the four-term deflection model to uncover the origins of these mechanical terms and track evolution of the equation from fundamental mechanics to the handy equations provided in building code references. Subsequently in the paper, Eq. (1) will be expanded for generalized use with any standard system of measure and application to a broader variety of WSP and CLT diaphragms.

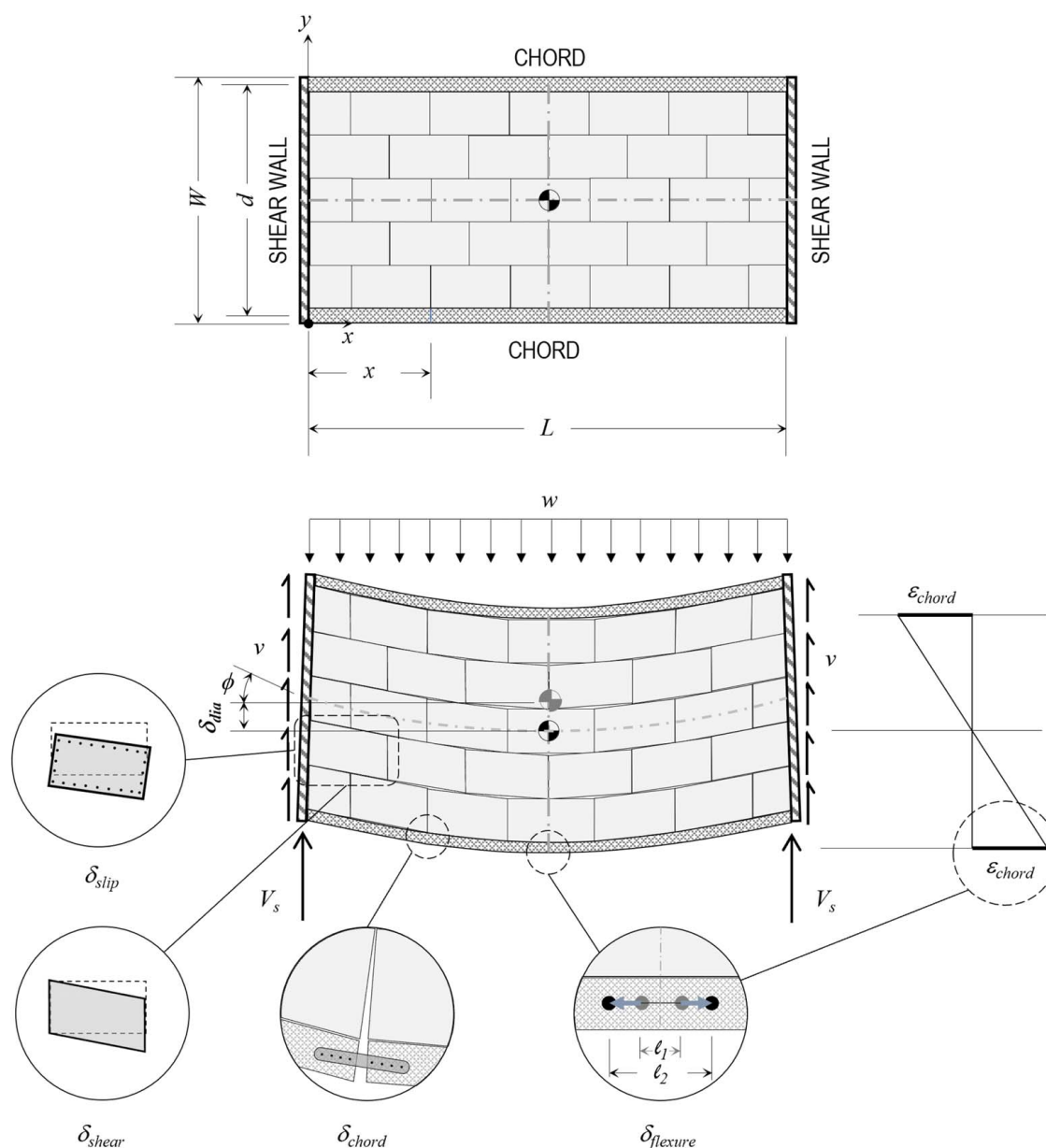


Fig. 1. Deflection parameters of simple span diaphragm.

History of Diaphragm Deflection Modeling

The Douglas Fir Plywood Association developed an original deflection equation for wood-sheathed diaphragms, based on testing of four full-scale and six quarter-scale diaphragms that, respectively, measured 12.19 m (40 ft) and 3.05 m (10 ft) in length. Countryman (1952) used principles of engineering mechanics to develop the three terms of

$$\delta_{\text{dia,US}} = \frac{5vL^3}{8EAW} + \frac{vL}{4G_v t_v} + 0.094Le_n \quad (2)$$

which estimated the flexural, shear, and nail-slip contributions to the observed cumulative deformations.

Although this three-term deflection model was a pivotal development in analyzing panel-sheathed diaphragms, Eq. (2), derived originally by Countryman (1952) and subsequently by ATC-7 (ATC 1981), erroneously underestimated the coefficient of the third term, by a factor of two. Walford, of the New Zealand Forest Research Institute, noted the error and provided a correction (Walford 1980; Walford 1981), which the USDA Forest Service Forest Products Laboratory (Liu 1981), and American Plywood Association (Tissell 1981) confirmed. Ever since, the nail-slip coefficient has been accepted as $0.188Le_n$, as written in Eq. (1).

In addition to underestimating nail-slip deflections, Eq. (2) did not account for the deflection contributions of slip in the chord splices of the diaphragm. After testing long-span diaphragms, Tissell (1966) observed that diaphragms directly connected to adhesively laminated and effectively continuous tension chords deflected less than diaphragms with mechanically spliced tension chords. To account for this additional source of deflection, the American Plywood Association (APA) added the fourth term of Eq. (1) to the diaphragm deflection model (Bower 1974; Carney 1970). Based on testing of 11 diaphragms, Tissell and Elliott (1977, 2004) developed design recommendations and compared a four-term deflection model, matching Eq. (1) with observed deflections.

Many structural engineering documents, such as FEMA 273 (ATC 1997), the *Uniform Building Code* (ICBO 1997), and the *International Building Code* (ICC 2000), reference a version of this four-term expression for diaphragm deflections of WSP systems. Additionally, the Canadian Standard *Engineering Design in Wood* (CSA 2019) contains an equivalent of Eq. (1), and the New Zealand *Timber Structures Standard* commentary clauses (NZS 1993) provide a similar expression. The four-term diaphragm deflection equation, rooted in the original work of Countryman (1952), therefore applies internationally.

Since the 2001 edition of SDPWS, a three-term expression that combines the effects of panel shear deformation and fastener slip in one term has been the standardized deflection equation for WSP diaphragms:

$$\delta_{\text{dia,US}} = \frac{5vL^3}{8EAW} + \frac{0.25vL}{1,000G_a} + \frac{\sum(x\Delta_c)}{2W} \quad (3)$$

SDPWS presents the four-term Eq. (1) and the approximation used to reduce it to the three-term form in a commentary.

Eqs. (1) and (3) present terms published in US design standards, which use US customary units for each variable as a shortcut for common practice. For example, L is in feet and the deflection, $\delta_{\text{dia,US}}$ is in inches. In this paper, the subscript “US” will be used to distinguish an equation that uses US customary units and implicit unit conversions. The US customary units for each variable are listed in the notation section of this paper. Equations without the subscript “US” do not have implicit unit conversions and may be generally applied to any system of units.

To make derivations of deflection equations more accessible for engineers and researchers to build on, this paper is, it is believed,

the first publication to correctly and more fully document the principles of mechanics, assumptions, and derivation of the four-term diaphragm deflection model that accounts for flexural, shear, fastener-slip, and chord slip deformations.

Fundamental Assumptions

The physical model underlying Eq. (1) relies on several assumptions. First, distinct chords provide all resistance to flexural bending forces, and these chords may exhibit chord slip at specific locations that depend on connection detailing. Second, the shear stresses are distributed equally through the width, W or W' , of the diaphragm and are resisted by the diaphragm panels and their fastening only. Third, classic beam theory applies to the diaphragm, to estimate the magnitude and direction of flexural and shear forces. According to these assumptions, the diaphragm acts as a beam spanning between supporting vertical elements, such as shear walls and braced frames, of the lateral force-resisting system (LFRS). The main field of the diaphragm acts as an I-beam web, providing shear resistance, and chords independently act as I-beam flanges, providing flexural resistance. This engineering model is often referred to as the “deep beam analogy” for diaphragm behavior. Out-of-plane forces on the diaphragms, such as from gravity loads, are ignored and assumed not to impact the diaphragm’s in-plane behavior.

Because of common WSP construction detailing and practices, Eq. (1) and other standard variations further assume that:

1. panels measure 1.22 m (4 ft) by 2.44 m (8 ft) in size, as viewed in plan;
2. shear fasteners used on all edges of all panels (e.g., nails) are the same type and size;
3. shear fastener spacing is equal around all edges of all panels;
4. detailing follows conventions of blocked diaphragms; and
5. axial chord forces act at the edges of the diaphragm.

Observing that construction practices have grown more diverse over the last 70 years, we revisit the fundamental diaphragm deformation equation to make the four terms readily adaptable to a wider variety of construction details and structural layouts.

Derivation of the Four-Term Diaphragm Deflection Equation for WSP Diaphragms

This section provides an in-depth review of each of the four terms of Eq. (1). Presenting the derivation of each term, this paper provides generalized equations that address panelized WSP and CLT diaphragm configurations outside the assumptions listed previously. The four terms of Eq. (1) can be written as

$$\delta_{\text{dia}} = \delta_{\text{flex}} + \delta_{\text{shear}} + \delta_{\text{slip}} + \delta_{\text{chord}} \quad (4)$$

For the uniformly loaded, simple span diaphragm in Fig. 1, the following sections derive the four terms of Eq. (1). The first two terms of Eq. (1) address beam flexure and beam shear, which are assumed to act independently. The last two terms of Eq. (1) consider additional sources of deformation inherent to panelized assembly. For WSP and CLT diaphragms, each of these sources of deflection may make significant contributions to the overall diaphragm deflection.

Flexural Deformation

For a uniformly loaded simple span diaphragm, the first term of Eq. (4), δ_{flex} , may be derived using the following system of equations:

$$\delta_{\text{flex}} = \frac{5wL^4}{384EI} \quad (5a)$$

$$w = \frac{2vW}{L} \quad (5b)$$

$$I = \frac{1}{2}Ad^2 \quad (5c)$$

$$W = d \quad (5d)$$

Eq. (5a) provides the maximum bending deflection in a form commonly listed in mechanics of materials references. US standards express the diaphragm deflection as a function of the unit shear force at the supports, so Eq. (5b) converts the uniformly applied load (w) to terms of unit shear force (v) and diaphragm dimensions of width (W) and length (L).

Eq. (5c) estimates the moment of inertia of the diaphragm using only the cross-sectional area (A) of the chords and the dimension (d) between the diaphragm chords. For conventional WSP diaphragms, the chords are typically located at the very edges of the diaphragm, using such details as continuous rim joists or perimeter beams, so Eq. (5d) equates the width of the diaphragm and the distance between chord forces for a practical approximation. Solving the system of Eq. (5) leads to a general expression of maximum flexural deflection of a uniformly loaded simple span diaphragm:

$$\delta_{\text{flex}} = \frac{5vL^3}{96EAW} \quad (6)$$

Converting to US customary units results in

$$\delta_{\text{flex,US}} = \left(\frac{12 \text{ in.}}{\text{ft}} \right) \frac{5vL^3}{96EAW} = \frac{5vL^3}{8EAW} \quad (7)$$

The flexural deformation to other load patterns may be addressed by beginning with the corresponding classic beam deflections and applying similar conversions. The Appendix provides several forms of the flexural deformation term resulting from this derivation.

Shear Deformation of Wood Panels

Panel shear deformation contributes a significant amount toward the total deflection of a WSP diaphragm. For a uniformly loaded simple span beam, the cumulative shear deformation from one support to the midpoint may be expressed as

$$\delta_{\text{shear}} = \int \frac{V(x)}{GA_w} dx = \int_0^{L/2} \frac{V_s \left(1 - \frac{2}{L}x \right)}{GA_w} dx = \frac{V_s L}{4GA_w} \quad (8)$$

Using the relationships $V_s = vW$ and $GA_w = G_v t_v W$ results in

$$\delta_{\text{shear}} = \frac{vL}{4G_v t_v} \quad (9)$$

where $G_v t_v$ is published for WSPs of various thicknesses, plies, and span ratings in the SDPWS commentary. The right-hand side of Eq. (8) generally applies to any load configuration.

Deformation Due to Fastener Slip

The third term of Eq. (1) is specific to panelized diaphragms, with the deformation due to fastener slip around the panel edges. Countryman (1952) assumed a uniform distribution of shear load across the diaphragm width and that all shear load transfer is through the fasteners, with only negligible bearing contact between adjacent

panels. Countryman's approach recognizes that diaphragm deformations due to fastener slip, δ_{slip} , and panel shear, δ_{shear} , result from the same diaphragm shear and are therefore directly related proportionally to the calculated change in the panel's diagonal lengths for each respective behavior, e'_n and e'_s :

$$\frac{\delta_{\text{slip}}}{e'_n} = \frac{\delta_{\text{shear}}}{e'_s} \quad (10)$$

To find the diagonal axis elongation associated with fastener slip, e'_n , Countryman (1952) used opposing corner nails as reference points. A single rectangular sheathing panel is shown in equilibrium in Fig. 2. The parallel and perpendicular subscript notations define the directions of parameters with respect to the directions of the externally applied diaphragm loads that produce the shear. For WSP diaphragms, the edge distance from the nails to the closest panel edge, S_e , is often specified as a minimum of 10 mm ($\frac{3}{8}$ in.). Because of the small magnitude of the edge distance relative to the panel dimensions, for the diaphragm deflections the fasteners are assumed at the edges of the panels and S_e does not appear in calculations.

The panel aspect ratio causes longer panel edges to have a greater total shear, by a ratio of long-to-short panel edge lengths. In Fig. 2, the nail fasteners are equally spaced around all panel edges, and thus the shear per nail parallel to the adjacent panel edges, V_n , is assumed to have equal magnitude for all nails.

To investigate how the shear forces result in diaphragm deformation due to fastener slip, Fig. 3 shows fastener slip around the edges of a panel and transforms this slip to the panel diagonal axes. The enlarged vector diagram of Fig. 3(a) shows where orthogonal displacement components of equal magnitude, e_n , parallel to the panel edges, meet to produce a combined slip of $e_n\sqrt{2}$ at the panel corners, oriented at 45° to each panel edge.

Fig. 3(b) transforms the nail slip at the panel corners to the diagonal axes of a panel. At each corner, the fastener slip between the panel and supporting frame measured along the diagonal axes has a magnitude of $e_n(\sin\theta + \cos\theta)$.

Fig. 4 illustrates the cumulative effects of fastener slip on the deformation of the supporting framing. The supporting framing of the diaphragm panel distorts into a parallelogram resulting from the fastener slip while the panel remains rectilinear. Measured along the panel diagonal, the frame deformation is twice the corner fastener slip in the same direction, so

$$e'_n = 2e_n(\sin\theta + \cos\theta) \quad (11)$$

To find the diagonal axis elongation associated with panel shear deformation, e'_s , Countryman (1952) used relationships

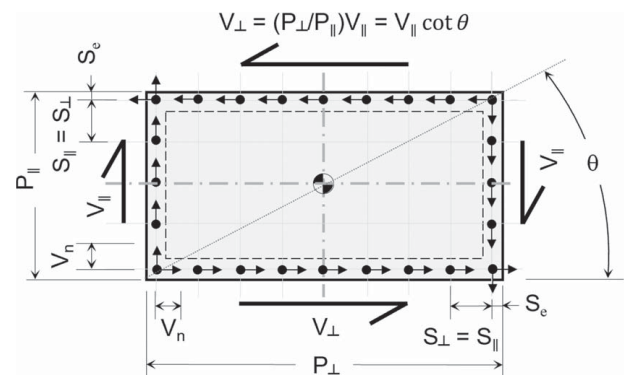


Fig. 2. Panel dimensions, fastener spacings, and shear forces.

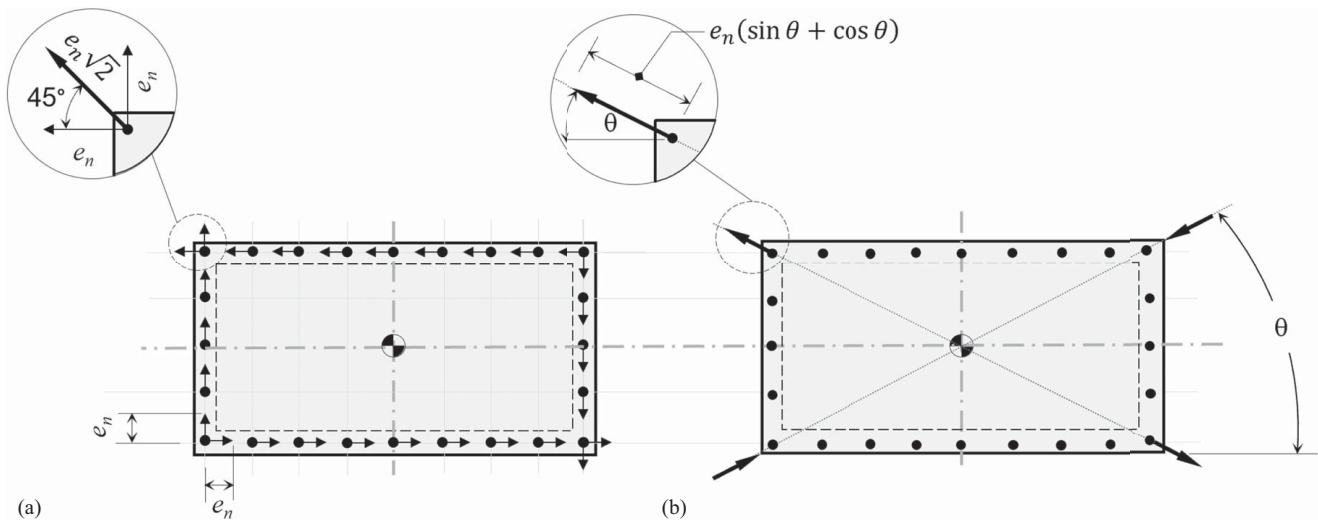


Fig. 3. Fastener slip: (a) parallel to panel edges; and (b) transformed to align with panel diagonals.

published in *Technique of Plywood* (Norris 1943). Alternatively, when using principles of plane stress (Gere and Timoshenko 1990), the panel undergoes pure shear stress, τ_{xy} , from shear force per length, v , acting along the panel edges, which causes shear strain, γ_{xy} , calculated by

$$\gamma_{xy} = \frac{\tau_{xy}}{G_v} = \frac{v}{G_v t_v} \quad (12)$$

The magnitude of net axial change in diagonal length, e'_s , elongation or shortening, may be found through the shear deformation relationships (Gere and Timoshenko 1990) of Fig. 5:

$$e'_s = \gamma_{xy} P \cos \theta = \frac{v}{G_v t_v} P \cos \theta \quad (13)$$

Therefore, using Eqs. (10), (11), and (13), the diaphragm deflection slip term for a uniformly loaded, simple span is

$$\delta_{\text{slip}} = \frac{e'_s}{e'_s} \delta_{\text{shear}} = \frac{e_n L (\sin \theta + \cos \theta)}{2P \cos \theta} \quad (14)$$

Expressed in terms of panel dimensions, Eq. (14) becomes

$$\delta_{\text{slip}} = \frac{e_n L}{2} \left(\frac{1}{P} + \frac{1}{P_{\perp}} \right) \quad (15)$$

For the common WSP measuring 4 ft (1.22 m) by 8 ft (2.44 m), in US customary units, Eq. (15) becomes

$$\delta_{\text{slip,US}} = \frac{e_n L}{2} \left(\frac{1}{4 \text{ ft}} + \frac{1}{8 \text{ ft}} \right) = \frac{3}{16} e_n L \approx 0.188 e_n L \quad (16)$$

Eq. (16) estimates the fastener-slip contribution to diaphragm deflections and matches the third term of Eq. (1) found in the SDPWS (AWC 2021). Eq. (15) can, more broadly, be applied to any size and shape of panel. This derivation of the fastener-slip term generally follows the methods of Countryman (1952), but includes a correction (Walford 1980).

Deformation due to Chord Connection Slip

The last term in Eq. (1) calculates the diaphragm deformation resulting from concentrated axial deformations, labeled chord slip

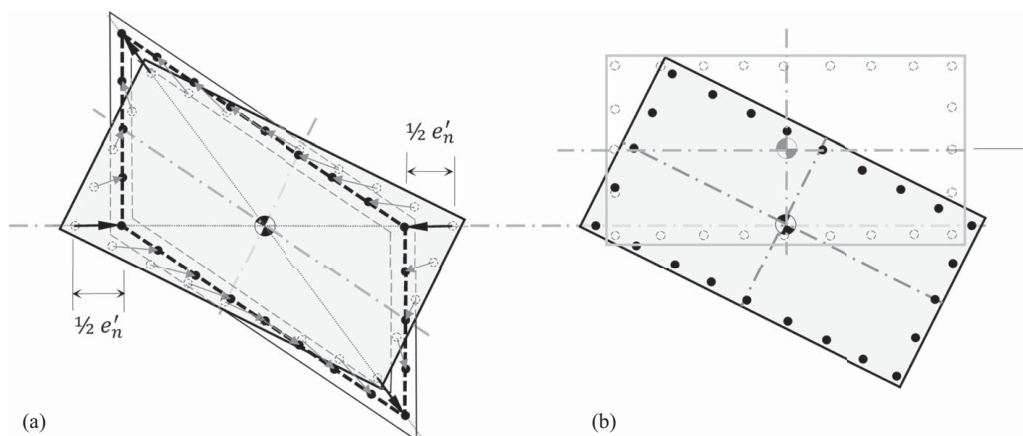


Fig. 4. Fastener-slip deformation of (a) supporting framing; and (b) sheathing panel with respect to original position.

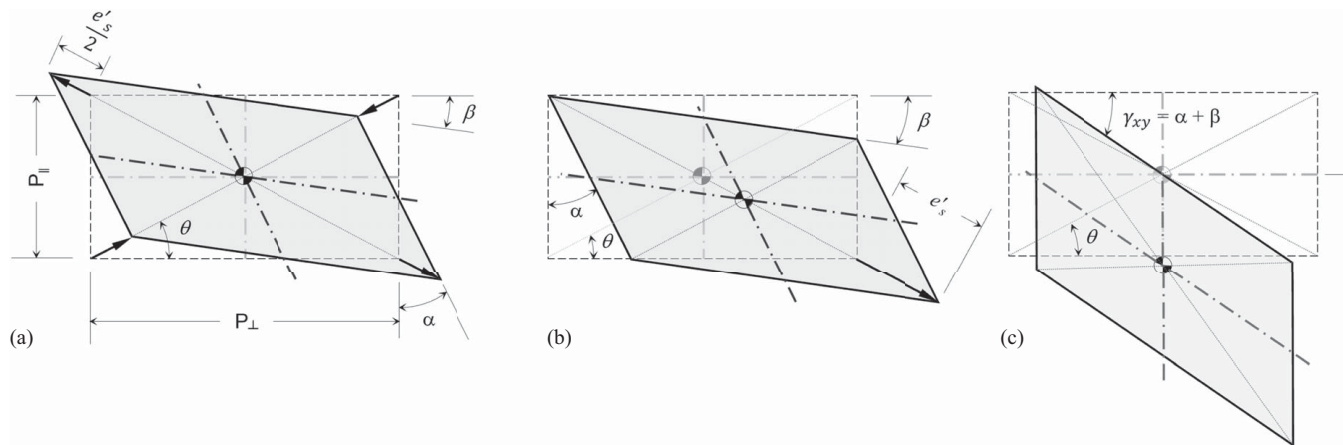


Fig. 5. Shear deformation of panel, shown with (a) strains at each corner; (b) cumulative diagonal elongation at lower right corner; and (c) net deformation relative to original position.

Δ_c , at specific locations along the chords. Often, this term addresses tension elongation of doweled splice connections, but similar principles apply to shortening of a compression chord splice. Skaggs and Martin (2004) reviewed the history of commonly used values for Δ_c in WSP diaphragms, which depend on details of the fasteners, construction assembly, and installation tolerances.

Values of Δ_c may be analytically or experimentally determined, and the resulting magnitudes of diaphragm deflections are estimated using rigid-body mechanics. For small-angle rotations of rigid diaphragm sections A and B, about the point in the compression chord opposite to the tension chord splice shown in Fig. 6, the maximum diaphragm deflection at the location of the chord slip may be calculated as

$$\delta_{c,max} = x\theta_A = (L - x)\theta_B \quad (17)$$

Solving for θ_A :

$$\theta_A = \left(\frac{L}{x} - 1\right)\theta_B \quad (18)$$

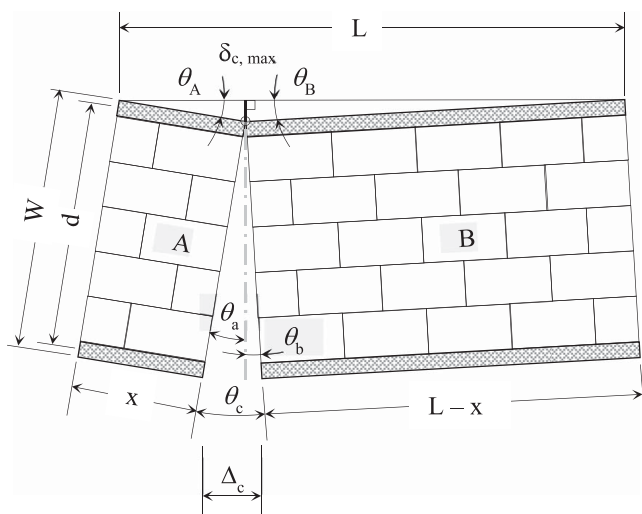


Fig. 6. Model of deformation caused by slip of chord splice.

The total angle of rotation between the diaphragm areas is related to the chord slip by

$$\theta_c = \frac{\Delta_c}{W} \quad (19)$$

The total angle of rotation is also

$$\theta_c = \theta_A + \theta_B = \left(\frac{L}{x} - 1\right)\theta_B + \theta_B = \frac{L}{x}\theta_B \quad (20)$$

Equating these and solving for θ_B yields

$$\theta_B = \frac{x\Delta_c}{LW} \quad (21)$$

The deformation at the chord slip is

$$\delta_{c,max} = (L - x)\theta_B = (L - x)\frac{x\Delta_c}{LW} \quad (22)$$

The deformation midspan due to chord slip is

$$\delta_c = \left(\frac{L}{2}\right)\theta_B = \left(\frac{L}{2}\right)\frac{x\Delta_c}{LW} = \frac{x\Delta_c}{2W} \quad (23)$$

Eq. (23) accounts for a slip at a single chord splice location. The total diaphragm deflection attributable to chord slip is simply a summation of the values calculated at each chord splice:

$$\delta_{chord} = \frac{\sum(x\Delta_c)}{2W} \quad (24)$$

where x is the distance, or x ordinate, from the chord slip location to the nearest diaphragm support. Fig. 6 and Eq. (24) model the US standard equation, which places the pivot point and tension chord at the extremities. If the pivot point and chords are located more interior, then d may be substituted for W in the preceding equations.

Alternative Derivation of Fastener-Slip Term

The following provides a more general derivation of the fastener-slip term, similar to that presented in ATC-7's Appendix A (ATC 1981) and Lawson (2018), and provides a methodology that can extend to a wider variety of construction configurations in both WSP and CLT assemblies. Note that the published ATC-7 derivation contains an error in the nail-slip term, equivalent to that of Countryman (1952), but is correctly presented here.

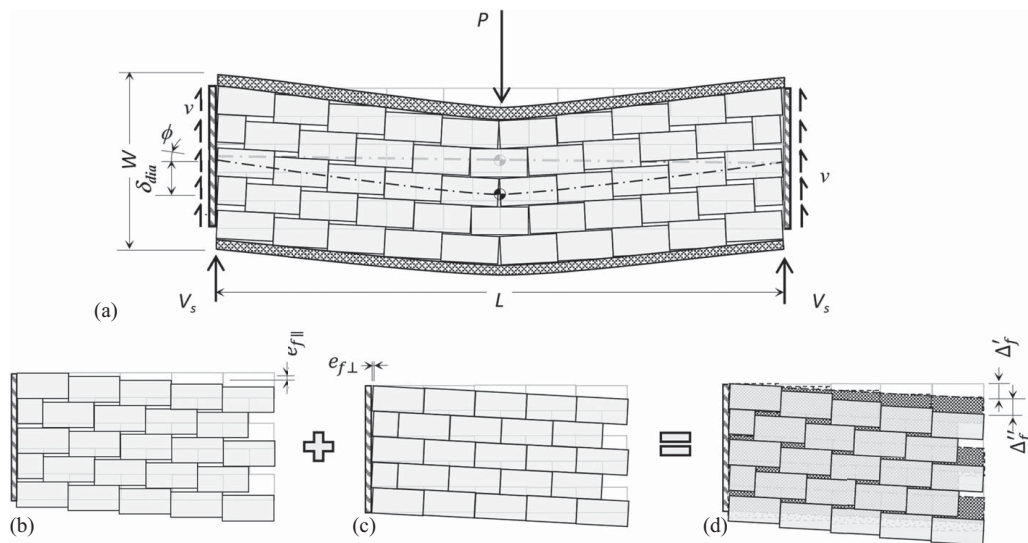


Fig. 7. Deformed diaphragm, under (a) constant shear; (b) translation; (c) rotation; and (d) superposition.

This approach divides the fastener-slip term into two separate contributions that are added as shown in Fig. 7. The translational contribution Δ'_f results from fastener slip on the panel edges parallel to the applied diaphragm load direction, and the rotational contribution Δ''_f results from fastener slip on the panel edges perpendicular to the applied diaphragm load direction. For the superposition shown in Fig. 7(d), the two contributions are added:

$$\delta_{\text{slip}} = \Delta'_f + \Delta''_f \quad (25)$$

Translational Contribution Δ'_f

Fastener slip is caused by the shear experienced per fastener. Adjacent panel edges parallel to the diaphragm's loaded direction will slip $e_{f||}$ in opposite directions, opening gaps between panels, as illustrated in Fig. 7(b). In a diaphragm configured as in Fig. 7(a), with a concentrated load at midspan that produces constant shear of magnitude V_s on each side of the applied load, the gaps have uniform size. Considering two WSPs with a common framing member below, the connection joint parallel to the load contains two translational slip planes, one at the connection of one panel to the framing, and another at the connection of the framing to the adjacent panel. This $2e_{f||}$ slip of translational movement will occur at every panel length from the support to the midspan. For this situation, Δ'_f is the slip per panel multiplied by the number of panel lengths $P_{\perp}$ from the diaphragm boundary to the midspan. The number of panel lengths to the midspan equals

$$n_{pl} = \frac{L/2}{P_{\perp}} \quad (26)$$

Thus, the translation contribution, dimensioned in Fig. 7(d), is

$$\Delta'_f = 2e_{f||} \frac{L/2}{P_{\perp}} = e_{f||} \frac{L}{P_{\perp}} \quad (27)$$

Rotational Contribution Δ''_f

Fig. 7(c) dimensions the widest part of an open gap with distance $e_{f\perp}$, that is, perpendicular to the applied load. The gaps open from rotation of the panels. The shear forces, diagrammed in Fig. 2, produce the clockwise rotation shown in Fig. 8. Although the fasteners maintain force equilibrium, slip makes rotation inevitable and produces the beam curvature of the diaphragm.

Using the theorem of similar triangles in Fig. 8, the rotational offset per panel Δ''_p is

$$\Delta''_p = e_{f\perp} \frac{P_{\perp}}{P_{\parallel}/2} = 2e_{f\perp} \frac{P_{\perp}}{P_{\parallel}} \quad (28)$$

For the diaphragm of Fig. 7(a) that experiences constant shear on each side of the applied concentrated load, the rotation of every panel is constant, ϕ , as illustrated in Fig. 7(c). Therefore, Δ''_f is the rotational offset per panel, Δ''_p , Eq. (28), multiplied by the number of panel lengths to the midspan, Eq. (26):

$$\Delta''_f = \Delta''_p \frac{L/2}{P_{\perp}} = 2e_{f\perp} \frac{P_{\perp} L/2}{P_{\parallel} P_{\perp}} = e_{f\perp} \frac{L}{P_{\parallel}} \quad (29)$$

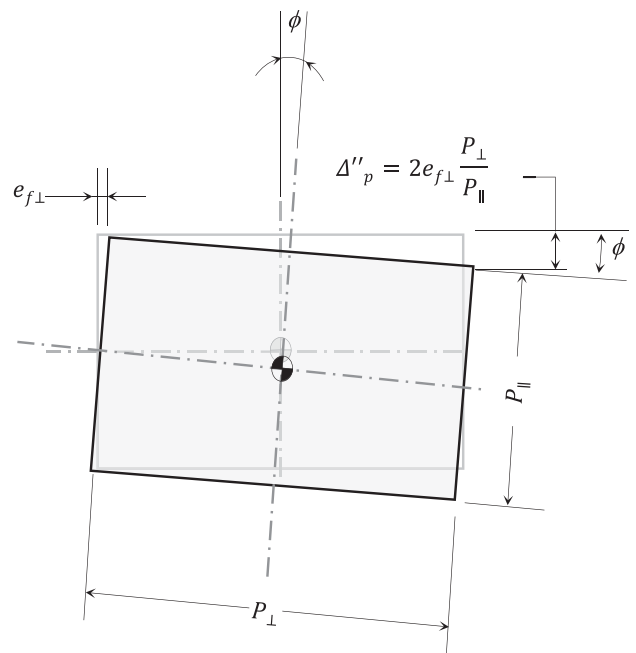


Fig. 8. Isolated diaphragm panel in rotation.

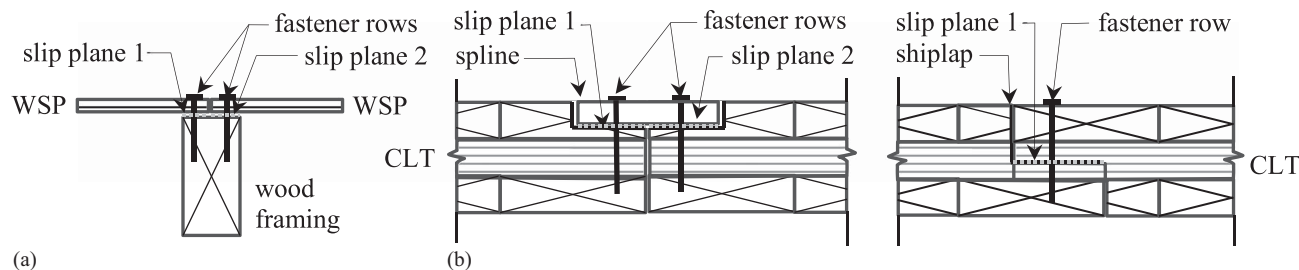


Fig. 9. Fastener-slip planes in typical panel joints of (a) WSP; and (b) CLT diaphragms.

Combining the two contributions of Eq. (27) and Eq. (29), as illustrated in Figs. 7(a and d), the total slip is obtained:

$$\delta_{\text{slip}} = \Delta'_f + \Delta''_f = e_{f\parallel} \frac{L}{P_{\perp}} + e_{f\perp} \frac{L}{P_{\parallel}} = L \left(\frac{e_{f\parallel}}{P_{\perp}} + \frac{e_{f\perp}}{P_{\parallel}} \right) \quad (30)$$

Eq. (30) provides the fastener-slip contribution specific to the Fig. 7 configuration of a diaphragm placed in constant shear by a midspan concentrated load. Other loading conditions can be evaluated by proportioning to this case, using the Euler–Bernoulli beam relationship of the loading diagram to the shear diagram. For example, uniformly distributed loads on a simply supported diaphragm produce shear that varies linearly across the diaphragm from its peak at each boundary to zero at the midspan. The average diaphragm shear, therefore, amounts to one-half of the boundary shear, so Eq. (30) may be multiplied by half to represent a uniformly distributed loaded diaphragm:

$$\delta_{\text{slip}} = \frac{L}{2} \left(\frac{e_{f\parallel}}{P_{\perp}} + \frac{e_{f\perp}}{P_{\parallel}} \right) \quad (31)$$

Eqs. (15) and (31) are similar, except that Eq. (31) can investigate different fastener types and spacings on the parallel and perpendicular oriented sides. The terms $e_{f\parallel}$ and $e_{f\perp}$ are the fastener-slip dimensions along the panel edges parallel and perpendicular to the diaphragm load direction (respectively) determined from the load per fastener induced by the maximum diaphragm shear along the diaphragm boundary. The magnitudes of $e_{f\parallel}$ and $e_{f\perp}$ are based on the shear per fastener found from static equilibrium (Fig. 2) and the fastener's stiffness.

For CLT diaphragms, Spickler et al. (2015) presented a similar modification of the fastener-slip term, as in Eq. (15), for panels other than the WSP standard 1.22×2.44 m (4×8 ft) dimensions. Eq. (31) is consistent with diaphragm connections where a separate shear-transfer element is provided at panel-to-panel shear connections, such as where adjacent panel edges are fastened to supporting framing members or where CLT panels are joined with surface spline connections (Mohammad et al. 2013). When a separate shear-transfer element is employed, adjacent panel edges will slip e_f in opposite directions because each panel slips relative to the framing below or the spline, and the result is the $2e_f$ value per panel identified earlier. In Fig. 9, for example, the WSP and CLT-spline configurations each exhibit two slip planes. The half-lapped, or shiplap, panel edge connection of CLT in Fig. 9(b), however, provides only one slip plane, so the total slip across the panel-to-panel connection is e_f . The total slip between panels can be generalized, by accounting for the number of slip planes in a connection, with parameter n . Replacing the panel slip, $2e_f$, with $n_{\parallel}e_{f\parallel}$ and $n_{\perp}e_{f\perp}$ for the number of slip planes corresponding to panel edges parallel and perpendicular to the applied load, Eq.

(31) becomes

$$\delta_{\text{slip}} = \frac{L}{4} \left(\frac{n_{\parallel}e_{f\parallel}}{P_{\perp}} + \frac{n_{\perp}e_{f\perp}}{P_{\parallel}} \right) \quad (32)$$

The generalization of Eq. (32), therefore, makes it possible to estimate fastener slip for a wider variety of construction details by identifying the number of slip planes. Several recent studies addressed the behavior of these spline and half-lap connections in CLT (Sullivan et al. 2018; Taylor et al. 2021).

Combining the four generalized expressions for each source of deflection into a unified equation for a total deflection estimate of a uniformly loaded, simple span wood panelized diaphragm:

$$\delta_{\text{dia}} = \frac{5vL^3}{96EAW} + \frac{vL}{4G_v t_v} + \frac{L}{4} \left(\frac{n_{\parallel}e_{f\parallel}}{P_{\perp}} + \frac{n_{\perp}e_{f\perp}}{P_{\parallel}} \right) + \frac{\sum (x\Delta_c)}{2W} \quad (33)$$

Extension to Cantilevered Wood Diaphragms

Cantilevered diaphragm configurations commonly occur in construction practice. SDPWS addresses cantilevered diaphragms slightly differently from the simple span case, with notations of diaphragm width W' , and length L' , particular to the cantilever. The equations developed for the simple span diaphragm can be adapted to cantilevered diaphragms, recognizing the effects from their free-end and fixed-end supports. Combining the four adapted generalized expressions for each source of deflection provides a total deflection estimate of a uniformly loaded cantilevered wood panel diaphragm:

$$\delta_{\text{cant},u} = \frac{vL'^3}{4EAW'} + \frac{vL'}{2G_v t_v} + \frac{L'}{2} \left(\frac{n_{\parallel}e_{f\parallel}}{P_{\perp}} + \frac{n_{\perp}e_{f\perp}}{P_{\parallel}} \right) + \frac{\sum (x'\Delta_c)}{W'} \quad (34)$$

Experimental researchers will find value in a concentrated end-loaded diaphragm deflection equation. Given the different moments and constant shear, a general equation is

$$\delta_{\text{cant},p} = \frac{2vW'L'^3}{3EAd^2} + \frac{vL'}{G_v t_v} + L' \left(\frac{n_{\parallel}e_{f\parallel}}{P_{\perp}} + \frac{n_{\perp}e_{f\perp}}{P_{\parallel}} \right) + \frac{\sum (x'\Delta_c)}{W'} \quad (35)$$

Conclusions

Following straightforward principles of engineering mechanics on a deep beam analogy of diaphragm behavior, this paper re-establishes the derivation of the current four-term WSP diaphragm deflection equation in SDPWS (AWC 2021), and generalizes the derivation to target a wider range of WSP and CLT applications. A derivation updated to correct previous errors has not been readily

available for US and international designers, and so the steps shown in this paper offer a framework for designers who seek to adapt the standard equations to different loading conditions and frequently encountered construction details that evolve with changing practices over time. Accurate estimates of diaphragm deflections are routinely needed to characterize diaphragms as either flexible, rigid, or semirigid, as well as to specify building separations and property line setbacks, and to evaluate the deformation compatibility of the attached building elements.

Today's WSP diaphragms frequently do not match standard assumptions used in the original derivation of the diaphragm deflection model, first developed in the early 1950s. In this paper, a series of standard diaphragm deflection equations has been generalized, allowing for panel sizes other than $1,220 \times 2,440$ mm (4×8 ft). In addition, the generalized equations track the fastener slip, e_f , on the long and short panel edges separately, allowing investigation of different fastener types and spacings on the same panel, as is often done in today's practice.

With the heightened interest in CLT floor and roof decks, designers need a diaphragm deflection model that addresses the unique aspect of this mass timber product. In line with SDPWS (AWC 2021) requirements that CLT diaphragm deflections be computed using principles of engineering mechanics, in this paper, a generalized tool is derived that unifies these computations for both WSP and CLT diaphragms, based on their commonality of

panelized construction. This paper's generalized standard diaphragm deflection equations address the high aspect ratios common with CLT panels, as well as half-lap connections with one instead of two slip planes per shear connection at panel joints.

Furthermore, variants of the deflection equations are shown that cover the uniformly loaded, simple span diaphragm, and the uniformly loaded and point-loaded cantilevered diaphragm, all of which have frequent applications in design and research. This generalized derivation of diaphragm deflections is based on a consistent set of engineering mechanics assumptions that may guide WSP and CLT diaphragm analysis and design in further research and construction practice. In the second of the two companion papers, equations are derived with specific variables for different fastener spacings and fastener stiffnesses on the short and long panel edges of WSP and CLT diaphragms, and an assessment of the unique issues associated with chord and shear deformations in current practice is given.

Appendix. Deflection Contributions for Various Diaphragm Contributions

For reference, Table 1 shows the resulting diaphragm deformation components for a range of assumptions presented in the derivations.

Table 1. Summary of diaphragm deflection contributions for various configurations

δ_{flex}	δ_{shear}	δ_{slip}	δ_{chord}
<i>Simple span, uniform loading</i>			
General form		General form	General form
$\frac{5vWL^3}{192EI}$	$\frac{vL}{4G_v t_v}$	$\frac{L}{4} \left(\frac{n_{\parallel} e_{f\parallel}}{P_{\perp}} + \frac{n_{\perp} e_{f\perp}}{P_{\parallel}} \right)$	$\frac{\sum (x \Delta_c)}{2d}$
Two equal chords		Two slip planes at edges	Chords located at edge
$\frac{5vWL^3}{96EA d^2}$		$\frac{L}{2} \left(\frac{e_{f\parallel}}{P_{\perp}} + \frac{e_{f\perp}}{P_{\parallel}} \right)$	$\frac{\sum (x \Delta_c)}{2W}$
Chords located at edge		Equal fasteners and spacing	
$\frac{5vL^3}{96EAW}$		$\frac{e_f L}{2} \left(\frac{1}{P_{\perp}} + \frac{1}{P_{\parallel}} \right)$	
<i>Cantilever span, uniform loading</i>			
General form		General form	General form
$\frac{vW'L^3}{8EI}$	$\frac{vL'}{2G_v t_v}$	$\frac{L'}{2} \left(\frac{n_{\parallel} e_{f\parallel}}{P_{\perp}} + \frac{n_{\perp} e_{f\perp}}{P_{\parallel}} \right)$	$\frac{\sum (x' \Delta_c)}{d'}$
Two equal chords		Two slip planes at edges	Chords located at edge
$\frac{vW'L^3}{4EA d^2}$		$L' \left(\frac{e_{f\parallel}}{P_{\perp}} + \frac{e_{f\perp}}{P_{\parallel}} \right)$	$\frac{\sum (x' \Delta_c)}{W'}$
Chords located at edge		Equal fasteners and spacing	
$\frac{vL'^3}{4EAW'}$		$e_f L' \left(\frac{1}{P_{\perp}} + \frac{1}{P_{\parallel}} \right)$	
<i>Cantilever span, point loading</i>			
General form		General form	General form
$\frac{vW'L^3}{3EI}$	$\frac{vL'}{G_v t_v}$	$L' \left(\frac{n_{\parallel} e_{f\parallel}}{P_{\perp}} + \frac{n_{\perp} e_{f\perp}}{P_{\parallel}} \right)$	$\frac{\sum (x' \Delta_c)}{d'}$
Two equal chords		Two slip planes at edges	Chords located at edge
$\frac{2vW'L^3}{3EA d^2}$		$2L' \left(\frac{e_{f\parallel}}{P_{\perp}} + \frac{e_{f\perp}}{P_{\parallel}} \right)$	$\frac{\sum (x' \Delta_c)}{W'}$
Chords located at edge		Equal fasteners and spacing	
$\frac{2vL'^3}{3EAW'}$		$2e_f L' \left(\frac{1}{P_{\perp}} + \frac{1}{P_{\parallel}} \right)$	

Data Availability Statement

All data, models, and code generated or used during the study appear in the submitted article.

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Notation

The following symbols are used in this paper. For parameters referenced in US design standards, US customary units are provided in parentheses. All parameters may generally be applied consistently with a standardized system of units.

A = area of diaphragm chord (in.²);
 A_w = cross-sectional web area of diaphragm;
 d = distance between axial tension and compression of diaphragm chords;
 E = modulus of elasticity of diaphragm chords (lb_f/in.²);
 e_f = fastener slip;
 $e_{f\parallel}$ = fastener slip along panel edge parallel to diaphragm load direction based on maximum diaphragm shear along diaphragm boundary;
 $e_{f\perp}$ = fastener slip along panel edge perpendicular to diaphragm load direction based on maximum diaphragm shear along diaphragm boundary;
 e_n = nail slip measured parallel to nearest panel edge based on shear along diaphragm boundary (in.);
 e'_n = change in panel framing diagonal length due to nail slip (in.);
 e'_s = change in panel diagonal length due to shear strain (in.);
 G = panel shear modulus (psi);
 G_a = apparent diaphragm shear stiffness (kips/in.);
 G_{vt} = panel in-plane, through-the-thickness, shear stiffness (lb_f/in. of depth) of WSP panels;
 I = moment of inertia of diaphragm chords;
 L = length of diaphragm span, perpendicular to diaphragm load direction (ft);
 L' = length of cantilevered diaphragm span, perpendicular to diaphragm load direction (ft);
 n_{pl} = number of panel lengths from edge of diaphragm to midspan of diaphragm;
 $n_{\parallel}$ = number of slip planes at panel-to-panel connections parallel to diaphragm load direction;
 $n_{\perp}$ = number of slip planes at panel-to-panel connections perpendicular to diaphragm load direction;
 $P_{\parallel}$ = panel dimension parallel to diaphragm load direction;
 $P_{\perp}$ = panel dimension perpendicular to diaphragm load direction;
 $S_{\parallel}$ = fastener spacing on panel edges parallel to diaphragm load direction;
 $S_{\perp}$ = fastener spacing on panel edges perpendicular to diaphragm load direction;
 $V(x)$ = total shear over width of diaphragm at location x ;
 V_n = shear force per nail;
 V_s = total diaphragm shear at line of support;
 $V_{\parallel}$ = total shear on panel edge parallel to diaphragm load direction;
 $V_{\perp}$ = total shear on panel edge perpendicular to diaphragm load direction;

v = induced shear per unit length typically at diaphragm boundary/support line (lb/ft);
 W = width of diaphragm, parallel to diaphragm load direction (ft);
 W' = width of cantilevered diaphragm, parallel to diaphragm load direction (ft);
 w = uniformly distributed applied diaphragm load;
 x = distance from nearest diaphragm support to location of interest (ft);
 x' = distance from free end of cantilever to location of interest (ft);
 α = angle of shear strain component measured with respect to panel edges oriented parallel to applied diaphragm load direction, in geometric context;
 β = angle of shear strain component measured with respect to panel edges oriented perpendicular to applied diaphragm load direction;
 γ_{xy} = shear strain in panel or supporting framing;
 Δ_c = diaphragm chord splice slip at induced unit shear (in.);
 Δ'_f = diaphragm slip deformation resulting from panel translation;
 Δ''_f = diaphragm slip deformation resulting from panel rotation;
 Δ''_p = panel slip deformation resulting from panel rotation;
 δ_c = diaphragm deformation at midspan due to chord slip at single location;
 $\delta_{c,max}$ = maximum diaphragm deformation due to chord slip at single location;
 δ_{chord} = diaphragm deformation component from chord slip;
 δ_{dia} = diaphragm deformation;
 δ_{flex} = diaphragm deformation component from bending;
 δ_{shear} = diaphragm deformation component from panel shear deformation;
 δ_{slip} = diaphragm deformation component from panel fastener slip;
 θ = angle of diaphragm rotation due to chord slip; and
 τ_{xy} = shear stress per unit area in panel.

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