THINNING GUIDELINES FOR LOBLOLLY PINE PLANTATIONS IN EASTERN TEXAS BASED ON ALTERNATIVE MANAGEMENT CRITERIA

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Abstract—Separate thinning guidelines were developed for maximizing land expectation value (LEV), present net worth (PNW), and total sawlog yield (TSY) of existing and future loblolly pine (*Pinus taeda* L.) plantations in eastern Texas. The guidelines were created using data from simulated stands which were thinned one time during their rotation using a combination row (20 percent) and selection thin to achieve a residual stocking target. LEV for future stands which had been thinned indicated optimal rotation ages of 20, 24, 28, and 32 years for site index 85, 75, 65, and 55 feet, respectively. For existing stands at optimal rotation age, PNW increased with both thinning age and site index. However, for future stands at optimal rotation age, LEV reaches maximum values at thinning ages that vary by site index. TSY reached maximum values at residual stocking levels, which increased with increasing site index for both existing and future stands. Marginal analyses indicated that when thinning older stands of poorer site quality, economic criteria (PNW or LEV) increased with higher initial stocking, and total sawlog yield (TSY) decreased with lower initial stocking. Regression tree analysis used thinning age and residual stocking for predicting PNW, LEV, and TSY by initial stand attributes (site index, and initial stand age and stocking). Thinning guidelines were derived from the tree-based models. A four step procedure is provided for applying the guidelines.

INTRODUCTION

Thinning has been a commonly employed silvicultural tool for achieving landowner financial and raw material goals. Despite its wide spread application, there is still much disagreement between professional foresters and forestry organizations with respect to the efficacy, timing, and intensity of mid-rotation thinning treatments. Guidelines are needed so that thinning can be implemented in a manner consistent with a landowner's specific goals.

Various dynamic programming (DP) formulations have been utilized to determine optimal thinning guidelines and/or rotation ages for even-aged stands (Amidon and Akin 1968; Brodie and Kao 1979). Although DP has been widely advocated for determining thinning and rotation ages, implementation is difficult due to the lack of commercial DP software. In most cases, software to perform the analysis must be custom coded. Additionally, as the number of state descriptors increase, so does the magnitude of the problem, thereby increasing computer memory requirements and the time required to solve the problem.

Using DP at the stand level will provide optimal estimates of thinning prescriptions and rotation ages. However, utilization of constrained forest-wide harvest scheduling will often deviate from the DP solution. In the current study, good feasible guidelines were desired in lieu of developing an optimal solution for thinning future and existing stands. Such guidelines would provide a range of options to incorporate into the forest-wide harvest scheduling and permit field operations some flexibility to account for stand and stem quality considerations.

The objectives of this study were to utilize simulated stand data to: (1) investigate the relationship between site quality, initial stand age and stocking, thinning age and intensity, and alternative management criteria for assessing the effectiveness of thinning treatments; and (2) develop separate thinning guidelines for maximizing present net worth (PNW), land expectation value (LEV), and total sawlog yields (TSY) for existing and future loblolly pine stands on forest lands owned by Temple-Inland Forest Products Corporation in eastern Texas.

METHODS

Stand Simulator

A stand simulator was developed for predicting growth and yield in both unthinned and thinned loblolly pine plantations. User-specified inputs include: site index (average total height in feet of dominant and co-dominant loblolly pine at index age 25 years); initial stand age (Age_0 in years, minimum 3 years), stocking (TPA_0 in trees per acre), and basal area (BAA_0 in square feet per acre); thinning type (no thin, row thin, selection thin, and combination row/selection thin) and age (Age_{Thin} in years); percent row thin; post-thin stocking (TPA_{res} in trees per acre); rotation age (Age_{Rot} in years); site preparation, planting, annual management, and harvesting costs (dollars per acre); product specifications (minimum and maximum outside bark d.b.h., and minimum outside bark top diameter) for pulp, chip-n-saw, and sawlogs; and percent interest rate for discounting costs and revenues. The simulator can project growth for both future ($Age_0 = 3$ years) and existing ($Age_0 > 3$ years) loblolly pine stands.

Stand-level dominant height (feet), basal area (square feet per acre), survival (trees per acre), and diameter percentile models were fitted with East Texas Pine Plantation Research Project (ETPPRP) data. The ETPPRP, initiated in 1982 by David Lenhart at Stephen F. Austin State University, installed 256 permanent plots in 1- to15-year-old loblolly (175 plots) and slash (81 plots) pine plantations between 1982-1984. The plots have been remeasured on a 3-year cycle, with the seventh measurement cycle completed during the

[1] Biometrician and Biometrician, Temple-Inland Forest Products Corporation, P.O. Drawer N, 207 N. Temple Drive, Diboll, TX 75941, respectively.

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summer of 2000. The plots received minimal site preparation (mechanical, chemical, and burning) and no mid-rotation treatments (thinning, hardwood release, and fertilization). Component growth and yield models, therefore, provide only baseline predictions with no explicit adjustments for cultural treatments. Dominant height, basal area, and survival are predicted with the stand-level models for the duration of each simulation.

Thinning model—Prior to thinning, a current stand table is generated using predicted diameter percentiles and a parameter recovery procedure for the three parameter Weibull distribution. The residual trees per acre remaining after thinning is specified, and the corresponding post-thin residual basal area (BAA_{res} in feet^2 per acre) predicted. A thinning algorithm performs preliminary row and/or selection removals, and the post-thin stand table is then adjusted so that the trees per acre and basal area per acre summed over all diameter classes equal the whole stand TPA_{res} and BAA_{res}.

The simulator predicts basal area (square feet per acre) in a thinned stand to converge on that of an unthinned counterpart stand of the same age and site index, with the same number of surviving trees per acre (Pienaar 1979, Pienaar and Rheney 1996, Pienaar and Shiver 1984). The basal area of the unthinned stand is adjusted using a locally calibrated index of suppression model, which describes the degree of suppression that existed in the thinned stand relative to the unthinned counterpart (fig. 1).

Stand table projection—A constrained least squares method is used for projecting the post-thin stand table for the remainder of the rotation (Cao and Baldwin 1999, Matney and others 1990). Individual tree models, fitted with ETPPRP data, predict the number of surviving trees in each diameter class and the diameter growth of the minimum and maximum diameter classes in the current stand table by 3-year projection period. The projected stand table is constrained each period so that the trees per acre and basal area per acre summed over all diameter classes equal the predicted whole stand basal area (feet^2 per acre) and trees per acre.

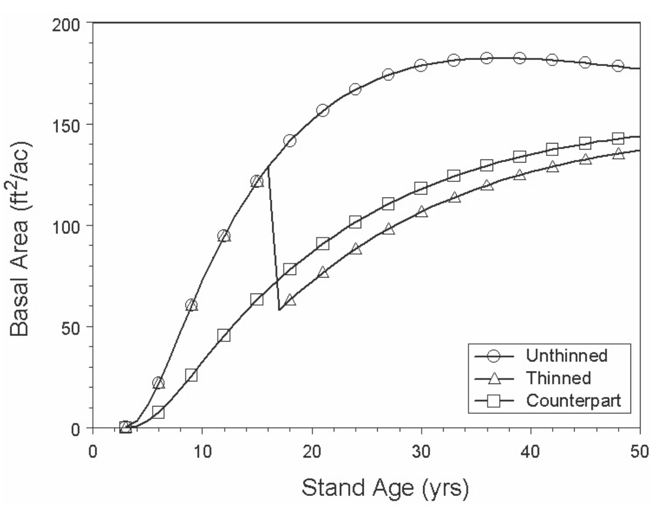

Figure 1—Predicted basal area (square feet per acre) in unthinned and thinned stands.

Economic management criteria—Pulp, chip-n-saw, and sawlog stumpage values (dollars per cord) for thinning and clear-cutting harvest operations are calculated using net converted values (NCV) and user-defined harvest costs (dollars per cord), which are adjusted for the timber source and mill destination. Thinning and clear-cutting revenues (dollars per acre), along with site preparation, planting, and management costs are appreciated to rotation age using the user-defined interest rate, and a net revenue is calculated in future dollars per acre. For future stands, a land (LEV) or soil (SEV) expectation value is calculated as the future net revenue discounted as a perpetual periodic series:

$$LEV = \frac{NR}{(1+i)^t - 1} \quad (1)$$

where, NR is the future net revenue (dollars per acre) or amount of periodic payment, "i" is the annual interest rate (percent per 100), and "t" is the rotation length (years) or interval between periodic payments; and for existing stands, a present net worth (PNW) is calculated as:

$$PNW = \frac{NR}{(1+i)^n} + \frac{LEV}{(1+i)^n} \quad (2)$$

where, "n" is the number of years until the clear-cutting harvest.

Site preparation and planting costs are included in net revenue calculations for future stands but excluded as sunk costs for existing stands.

Simulated Stand Data

The simulator generated stand attribute data for all combinations of the following input values, where Age_{Thin} was greater than Age_0: site index (55-85 by 10 foot increments); initial stand age (3 and 9-19 by 2 year increments); initial stocking (100-1000 by 100 trees per acre increments); thinning age (no thin and 10-20 by 2-year increments); post-thin residual stocking (100-300 by 25 trees per acre increments); and rotation lengths (20-36, 20-30, 20-30, and 16-30 by 2-year increments for site index values of 55, 65, 75, and 85 feet, respectively).

Stands were thinned one time during their rotation using a combination row and selection thin to achieve a residual stocking target. The row thin removed 20 percent of the standing trees from each diameter class. A selection only thin was used if the post-thin stand resulting from the row thin had less than the residual tree target.

Product merchantability specifications (table 1), site preparation costs, stand management costs, timber source, and mill destinations were held constant for all simulations. Thinning and clear-cutting harvest operations included only basic costs, with no supplements for equipment type and fuel costs. Regeneration costs included the cost of planted

Table 1—Product merchantability specifications for loblolly pine

Product	Minimum d.b.h.	Maximim d.b.h.	Minimum top diameter
	- - - - - - - - - - - - *inches*	- - - - - - - - - - - -	
Pulp	5.0	8.0	2.0
Chip-n-saw	8.0	10.0	5.0
Sawlog	10.0	—	6.0

trees (assuming 80 percent survival at age 3 years in future stands and 550 planted trees per acre in existing stands) and a graduated planting cost per acre. A constant 9 percent interest rate was used for all revenue and cost calculations.

RESULTS

The stand simulator produced response data for 52,765 (1,600 existing and future unthinned stands; 11,668 future thinned stands; and 39,497 existing thinned stands) successfully completed simulations with only a 0.2 percent unexplained failure rate.

Variance Component Analysis

All rotation ages—A variance component analysis (SPLUS 6 2001a, SPLUS 6 2001b) was used to see how much variability in the predicted management criteria (PNW, LEV, and TSY) was explained by current stand attributes (site index, Age_0, TPA_0). This preliminary analysis included data from all rotation ages.

Current stand attributes explained over 99 percent of the variability in the economic management criteria (PNW and LEV) for both existing and future stands. Site index and interaction terms with site index accounted for 96.6 and 84.5 percent of the variability in the future and existing stand data, respectively (table 2). PNW and LEV were larger for increasing site index values.

For both existing and future stand data, site index and rotation age explained approximately 55 and 40 percent of the variability in TSY, respectively. The residual error was 1.7 and 2.3 percent for existing and future stand data (table 2). TSY increased with both increasing rotation ages and site index values.

Since TSY increases with longer rotation lengths, thinning is a necessary activity to increase sawlog yield only for specified rotation ages. Rotation ages that maximize LEV by site index class are consistent with the management goals of industrial forestry landowners. LEV for future stands which had been thinned indicated optimal rotation ages of 20, 24, 28, and 32 years for site index 85, 75, 65, and 55 feet, respectively (fig. 2).

Optimal rotation ages—Simulated stand data associated with optimal rotation ages were used for subsequent analyses and for developing thinning guidelines.

Table 2—Estimated maximum likelihood variance components by management criteria and stand attributes (factors) for future and existing loblolly pine stands

Stand data		Mgmt. criteria	Percent variance explained by factor[a]: Site index	Age_0	TPA_0	Age_R	Residual error
	no.		*feet*	*years*		*years*	
Future	11,668	LEV	96.6	—	1.0	0.0	0.6
		TSY	55.1	—	0.4	40.8	2.3
	1,626	LEV	98.5	—	1.1	—	0.4
		TSY	85.0	—	3.7	—	10.6
Existing	39,497	PNW[b]	74.1	14.5	0.0	0.0	0.2
		TSY	55.5	0.0	1.2	39.6	1.7
	5,513	PNW[c]	79.2	13.0	0.0	—	0.1
		TSY[d]	79.8	1.2	9.9	—	5.9

TPA = trees per acre; LEV = land expectation value; TSY = sawlog yield; PNW = present net worth.

[a] Interaction terms account for unexplained variation in each row; future and existing variance components with larger N are calculated using all rotation ages combined, while components in rows with smaller N are based on optimal rotation ages.

[b] 10.4 percent variance explained by site index x age_0.

[c] 7.6 percent variance explained by site index x age_0.

[d] 2.1percent variance explained by site index x TPA_0.

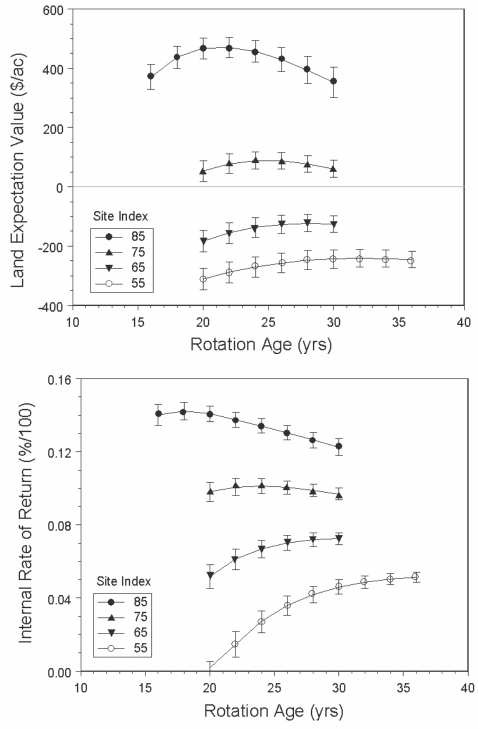

Figure 2—Mean LEV and IRR by rotation age (Age_{Rot}) and site index (feet) in future thinned stands. Error bars indicate ±1 standard deviation.

Variance components explaining the variability in PNW and LEV were similar to those calculated for all rotation ages (table 2). For both existing and future stands, however, site index and TPA_0 explained more variability in TSY, with corresponding increases in residual error. The unexplained variability (residual error) was available for developing thinning prescriptions based on thinning age and post-thin residual trees per acre.

For existing stands, PNW increases with both thinning age and site index (fig. 3). TSY also increases with site index but decreases with increasing thinning age, except for site index 85 which decreases to age 12 years and then gradually increases to age 18 years. Both PNW and TSY attain maximum values at post-thin residual stocking levels, which increase with increasing site index. Similar trends exist for future stand data, except for LEV, which reaches maximum values at thinning ages that vary by site index.

Marginal Analysis

Are there circumstances when choosing not to thin will produce greater financial (dollars per acre) and/or fiber (cords per acre) returns? That question can be quickly answered by plotting PNW, LEV, or TSY against its marginal value for both existing and future stand data. The marginal values are calculated as the difference between the no thinned and thinned management criteria values predicted for each combination of initial stand attributes (site index, Age_0, and TPA_0). If the difference is equal to or greater than zero, then do not thin. On the other hand, thinning will provide additional benefit if the difference is negative.

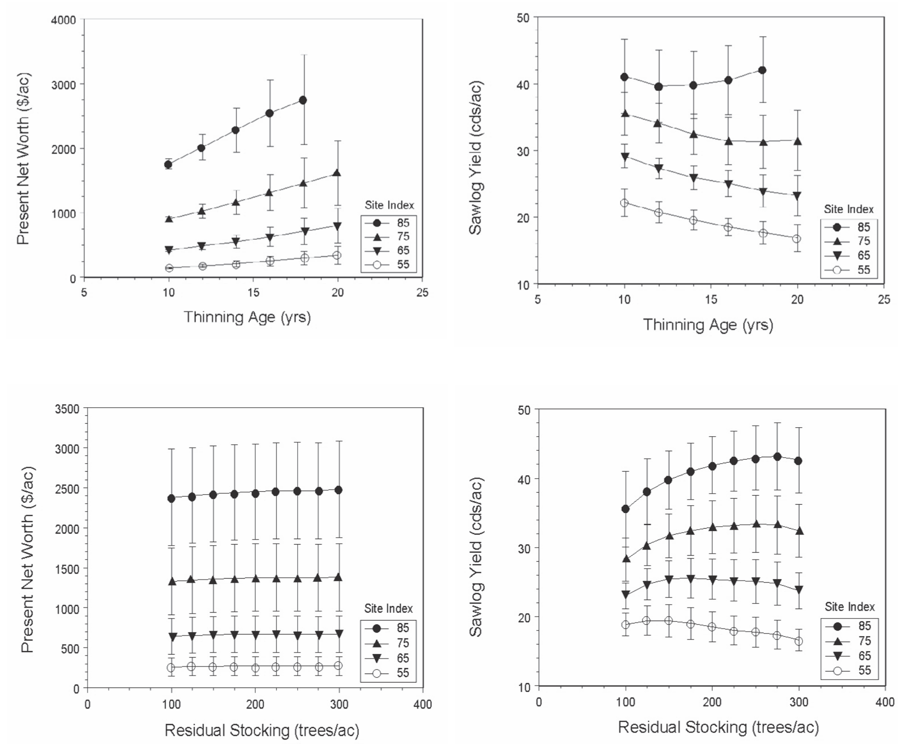

Figure 3—Mean PNW and TSY by thinning age (Age_{Thin}), residual trees per acre (TPA_{res}), and site index (feet) in existing thinned stands. Error bars indicate ±1 standard deviation.

For existing stand data, PNW and TSY were plotted against their marginal values by site index (fig. 4). The decision not to thin occurs more frequently with increasing site index for both PNW and TSY management criteria. For maximizing PNW, stands with higher site index (75 and 85 feet) have larger marginal PNW with increasing Age_0 and TPA_0; stands with lower site index (55 and 65 feet) have smaller marginal PNW. Thinning thus becomes a more viable economic treatment for older stands with poorer site quality and higher stocking. For maximizing TSY, marginal TSY increases with increasing Age_0, except for site index 85 feet, which has decreasing marginal TSY. Marginal TSY decreases with increasing TPA_0 for site index 55 feet. However, at higher site indices, TSY increases to a maximum, which occurs at progressively higher TPA_0 and then decreases. Thinning thus becomes a less viable treatment for older stands with lower initial stocking. The plotted relationships for marginal LEV and TSY are similar using future (Age_0 = 3 years) stand data.

Regression Tree Analysis

Regression tree analysis (RTA) is a method for fitting trees to predict a quantitative variable. The computer algorithm performs stepwise splitting based on a list of potential predictor variables, choosing predictor variables and their cut points that yield the smallest overall within cluster sum of squares. The fitting procedure minimizes a least squares loss function and reports the proportion of reduction in error (PRE) as a goodness-of-fit statistic (Venables and Ripley 1999).

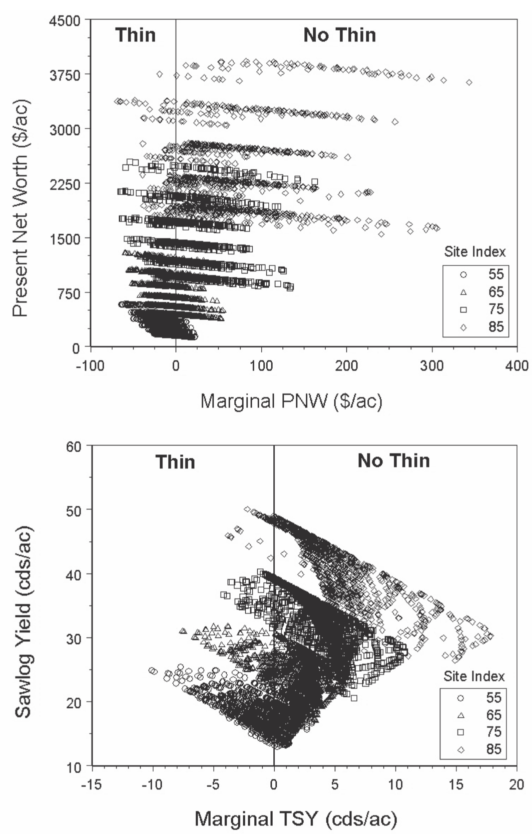

Figure 4—PNW and TSY observations versus their marginal (unthinned minus thinned) values by site index (feet) for existing thinned stands.

The advantages of tree-based models include: invariance of predictor variables to monotonic transformations; the capability to handle missing values in response and predictor variables; the flexibility to handle a broad range of response types and non-normally distributed predictor variables; the ability to depict complex interactions between predictor variables; and clearer, more easily understood interpretations of nonlinear relationships and complex interactions (De'ath and Fabricius 2000).

Existing and future stand data were first assigned to strata using site index, Age_0, and TPA_0. Trees were then fitted by strata to predict PNW and TSY in existing stands and LEV and TSY in future stands (SPLUS 6 2001). Fitted trees were constrained to have no more than four terminal nodes based on a preliminary tenfold cross validation. Thinning age and post-thin residual trees per acre were used as potential predictor variables.

Figure 5 shows a fitted regression tree for predicting PNW in an existing stand which has site index = 65 feet, Age_0 = 11 years, and TPA_0 = 500 trees per acre. The PRE was 0.74, and histograms show the distribution of observed PNW values assigned to each node. The tree has four terminal nodes, with mean PNW values ranging from $526.40 per acre to $567.80 per acre. The largest mean PNW is predicted for Age_{Thin} = 20 years and TPA_{res} = 100-300 trees per acre.

Thinning Guidelines

Thinning age and post-thin residual trees per acre were used to split the data that had been stratified by current stand conditions into more homogeneous groups, giving

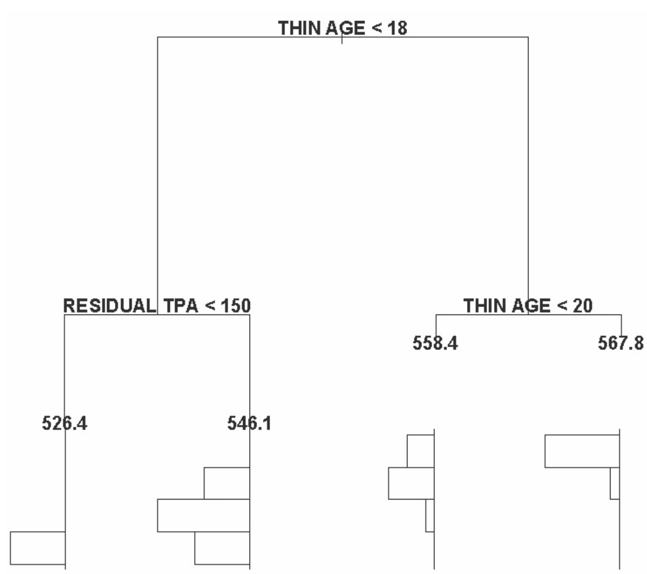

Figure 5—Fitted regression tree for predicting PNW in existing stands with site index = 65 feet, Age_0 = 11 years, and TPA_0 = 500 trees per acre. Each node is labeled with the predicted mean PNW (dollars per acre).

smaller within group and greater between group variability. The groups formed by the RTA splitting process were subsequently sorted in descending order by PNW, LEV, and TSY for future and existing stands and assigned a thinning rank or priority. Lower ranks (the highest thinning priority) were associated with current stand conditions and thinning prescriptions with either the highest mean PNW or LEV, or the largest mean TSY. Within each group there was one optimal and several sub-optimal values for each management criteria. Sub-optimal values were removed from the lists, which were then resorted in descending order and re-assigned a thinning rank. This procedure was used to create thinning guidelines by PNW and TSY for existing stands and by LEV and TSY for future stands.

Tables 3 and 4 illustrate thinning guidelines for maximizing PNW and TSY, respectively, in an existing stand. The thinning guidelines are applied using the following 4-step procedure:

Site index (feet), initial stand age (years), and initial stocking (trees per acre) are estimated using a pre-thin stand inventory (e.g., site index = 65 feet, Age_0 = 11 years, and TPA_0 = 600 trees per acre). The appropriate guideline is accessed using the estimated stand attributes. A thinning prescription (Age_{thin} and TPA_{res}) is given, along with the mean and standard deviation of the predicted management criteria (PNW or TSY). In addition, the guideline lists the predicted value of the management criteria for a no-thin scenario. For example: The thinning prescription for maximizing PNW using table 3 is Age_{thin} = 18-20 years and TPA_{res} = 175-300 trees per acre with a predicted PNW of \$571.70 ± \$4.97 per acre and a no-thin PNW of \$553.74 per acre. The thinning prescription for maximizing TSY using table 4 is Age_{thin} = 12-13 years and TPA_{res} = 100-300 trees per acre with a predicted mean TSY of 27.89 ± 1.54 cords per acre and a no-thin TSY of 27.82 cords per acre.

These prescriptions provide a recommended range of thinning ages and post-thin residual stocking, which would give good feasible solutions requiring professional judgment for field application.

The stand simulator uses the recommended prescription to search for a thinning age and post-thin residual stocking

Table 3—Thinning guidelines for maximizing PNW in an existing stand with site index = 65 feet, Age_0 = 11 years, and Age_{Rot} = 28 years

Existing stand	Thinning prescription		Thin PNW		No thin
TPA_0	Thin age	TPA_{res}	Mean	Std. dev.	PNW
	years		- - - - - *dollars per acre* - - - - -		
250 – 349	18 – 20	100 – 200	552.58	6.66	542.21
350 – 449	18 – 20	100 – 300	557.58	8.64	550.09
450 – 549	20	100 – 300	567.77	8.42	553.62
550 – 649	18 – 20	175 – 300	571.70	4.97	553.74
650 – 749	18 – 20	175 – 300	575.24	3.94	551.24
750 – 849	14 – 20	150 – 300	564.76	4.50	548.87
850 – 949	16 – 20	150 – 300	562.97	4.17	545.79
950 – 1,049	14 – 20	150 – 300	564.47	5.27	542.34

TPA = trees per acre; PNW = present net worth.

Table 4—Thinning guidelines for maximizing TSY in an existing stand with site index = 65 feet, Age_0 = 11 years, and Age_{Rot} = 28 years

Existing stand	Thinning prescription		Thin sawlog yield		No thin
TPA_0	Thin age	TPA_{res}	Mean	Std. dev.	Sawlog
	years		- - - - *cords per acre* - - - -		
250 – 349	12 – 20	250 – 300	30.07	0.19	30.66
350 – 449	12 – 20	150 – 300	27.30	0.54	29.96
450 – 549	12 – 17	125 – 250	27.10	0.52	28.96
550 – 649	12 – 13	100 – 300	27.89	1.54	27.82
650 – 749	12 – 13	100 – 300	27.49	1.57	26.60
750 – 849	12 – 13	100 – 300	26.99	1.50	25.43
850 – 949	12 – 13	100 – 300	26.49	1.43	24.30
950 – 1,049	12 – 13	100 – 300	26.54	1.70	23.21

TSY = sawlog yield; TPA = trees per acre.

with maximum values for the management criteria (PNW or TSY). For example,

- For maximizing PNW, Age_{thin} = 18 years and TPA_{res} = 225 trees per acre with a predicted PNW of $581.56 per acre.
- For maximizing TSY, Age_{thin} = 12 years and TPA_{res} = 200 trees per acre with a predicted TSY of 29.48 cords per acre.

If the predicted value for the management criteria (PNW or TSY) is greater than the no thin value, then thin the stand; otherwise, do not thin the stand. The existing stand in the example would be thinned using both the PNW and TSY management criteria.

DISCUSSION AND CONCLUSION

Different landowner objectives require different thinning regimes. In this analysis we examined the timing and intensity of a single entry thinning utilizing an optimal rotation age. The subsequent guidelines provide direction to field and planning personnel for planning and implementing two common landowner objectives. As suggested by the thinning guidelines, maximization of TSY requires early thinning when compared to maximization of PNW or LEV. Maximization of PNW is relatively insensitive to the residual stocking levels (fig. 3). In contrast, if maximizing TSY is the management goal, lower residual stocking should be the target with decreasing site index. However, individual stands will deviate from these trends based on initial stand attributes.

Despite landowner objectives, innate site productivity is the most important predictor for financial and raw material production. In general, there exists greater opportunity to increase PNW, LEV, and TSY by preferentially thinning lower site quality stands. Higher site quality lands provide less opportunity for thinning, as opportunity costs exist for reduced stocking.

Thinning guidelines were developed using regression tree analysis. This methodology provided an efficient means of summarizing data and identified feasible regions of thinning intensities and ages for maximizing a particular landowner objective. The regions identified with regression tree analysis provide field personnel with flexibility for determining residual trees per acre. This flexibility can be utilized to address stand specific stem quality considerations when selecting leave trees. Similarly, the feasible regions provide a range of thinning ages, which give wider latitude for achieving operational thinning constraints such as minimum stem length. The flexibility can also be beneficial in developing good and feasible thinning regimes for constrained harvest scheduling that are aligned with the objective function of the planning problem. Finally, if additional analysis is desired to determine the optimal thinning age and residual trees per acre, the thinning guidelines can define the boundaries of the search, thus reducing the time required to identify the optimal solutions.

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