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A NATIONAL-SCALE TREE VOLUME, BIOMASS, AND CARBON MODELING SYSTEM FOR THE UNITED STATES

Corrigendum (February 2026): An incorrect reference (Nowacki and Brock 1995) was replaced with the correct reference (Nowacki et al. 2003). The corresponding citations on pages 3 and 16 were also corrected.

Cover photos: Weighing branches from a lodgepole pine in the Willamette National Forest, OR. USDA Forest Service photo by Andrew N. Gray. **Inset:** Wood disc samples from a lodgepole pine from Willamette National Forest, OR. USDA Forest Service photo by Andrew N. Gray.

This publication was previously released as a postprint after undergoing technical and policy review. This final version includes a completed layout, changes to wording and grammar, and limited corrections to content. The most current version of the publication will always be available from the Digital Object Identifier (DOI): <https://doi.org/10.2737/WO-GTR-104>.

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ABSTRACT

The Forest Inventory and Analysis (FIA) Program of the U.S. Department of Agriculture, Forest Service conducts the national forest inventory of the United States. Although FIA assembles a myriad of forest resource information, many analyses rely on the fundamental attributes of tree volume, biomass, and carbon content. Due to the chronological development of the FIA Program, numerous models and methods are currently used across the country, contingent upon the tree species and geographic location. Thus, an effort to develop nationally consistent methods for prediction of tree volume, biomass, and carbon content was undertaken. A key component of this study was amassing existing data in conjunction with the collection of new data to fill information gaps related to tree size and species frequency and spatial distributions. These data were used in a modeling framework that provides compatible predictions of tree volume, biomass, and carbon content across the

entire United States. National-scale comparisons to currently used methods show that only a small increase in volume occurs, but substantial increases in biomass and carbon are realized due to relatively large increases in predicted tree top/limbs biomass and carbon. Changes in tree carbon were also affected by use of newly developed species carbon fractions instead of the current constant conversion factor of 0.5. Examples of the calculations required to predict tree volume, biomass, and carbon content for commonly encountered tree conditions provide step-by-step implementation details. An appendix lists supplemental data tables of values needed to calculate results, which are available as comma-separated values (CSV) files at <https://doi.org/10.2737/WO-GTR-104-Supp1>.

Keywords: carbon fraction, ecodivision, forest inventory, specific gravity, volume ratio, whole stem

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INTRODUCTION

The practice of forestry in the United States has a long history of quantifying individual tree volume to characterize the amount and type of wood resources. Because obtaining direct, exact measurements of tree volume is impractical, various methods for estimating volumes of standing trees have been developed. Pioneering efforts to predict tree volume included freehand curves (Mulloy and Beale 1937) and statistical models (Schumacher and Hall 1933). Regardless of the underlying method, it was common to develop volume tables for ease of application (Gevorkiantz and Olsen 1955, Mesavage and Girard 1946). Eventually, the direct use of prediction models became more favorable than the use of tabular methods (Avery and Burkhart 1983). Increases in computer usage, software capability, and advancements in statistical methods led to more sophisticated and flexible modeling approaches (Max and Burkhart 1976, Van Deusen et al. 1981). This trend continued to evolve as data and statistical capabilities increased (Burkhart and Tomé 2012, Garber and Maguire 2003, Gregoire and Schabenberger 1996, Zhao et al. 2019).

The appearance of corresponding tables and statistical models to directly assess tree weight or biomass began decades later (Schlaegel 1975, Wiant et al. 1977). Subsequently, many studies on tree biomass prediction appeared in the scientific literature (Baldwin 1987, Smith 1985, Tritton and Hornbeck 1982), including national-scale tree biomass models for the United States (Jenkins et al. 2003). As with tree volume, tree biomass modeling has continually evolved and has become a focal point for quantifying tree carbon storage and sequestration (Hoover and Smith 2021, McRoberts et al. 2018, Temesgen et al. 2015).

The progression of volume and biomass prediction methods has been an important facet of the national forest inventory of the United States, which began with the passage of the McNary-McSweeney Act (P.L. 70–466) in 1928. The Forest Inventory and Analysis (FIA) Program of the U.S. Department of Agriculture (USDA), Forest Service originated at that same time, with a primary focus on quantifying timber volume. Because the work was initially done sporadically and primarily on a State-by-State basis, tree volumes

were usually obtained from available sources of information for species common to the area being inventoried (Cowlin and Moravets 1938, Flanary et al. 2016). As FIA became more geographically diverse and eventually nationwide, tree volume and biomass predictions across the country arose from numerous unrelated studies (Woodall et al. 2011). Nonetheless, use of these diverse models allowed for the compilation of forest resource assessments at State, regional, and national scales. This capability was highly relevant for FIA to fulfill its mission, meet reporting requirements, and accommodate a large and diverse user community that conducts independent analyses via online availability of data and analytical software. However, models were often developed from small and geographically limited datasets using a variety of model forms and predictor variables (Temesgen et al. 2015, Weiskittel et al. 2015). Due to the wide-ranging uses of FIA data and the need to improve consistency across the country, a standard method for calculating tree biomass and carbon was adopted nationally circa 2010 (Woodall et al. 2011). While the method was nationally consistent, the underlying basis relied on the numerous regional volume models still in use. Further, the spatial application of volume models was often defined by administrative boundaries instead of any meaningful ecological basis. For tree biomass prediction, the accuracy and precision of models were essentially unknown due to the pseudo-data approach used in the original research. Thus, efforts were undertaken to develop a national methodology for compatible predictions of tree volume, biomass, and carbon content (Radtke et al. 2015, 2017; Weiskittel et al. 2015) for species commonly occurring on U.S. forest land. Specifically, the targeted species are inclusive of those identified by FIA species code (*SPCD*) ≤999, except for those designated as woodland species (USDA Forest Service 2022). The resulting methodology is hereafter referred to as the national-scale volume and biomass (NSVB) framework. This document serves as the primary reference for the outcome of those efforts and describes all the relevant aspects of the data, statistical modeling methods, and results.

METHODS

DATA

In the NSVB study, two primary efforts were undertaken to maximize data availability: (1) engage in felled-tree work to fill information gaps in tree species, size, and location, and (2) find existing data from previous studies, convert the data into electronic format, if necessary, and assimilate the data into a common database structure. Several universities were engaged in the felled-tree data collection effort, where tree volume, biomass, and wood density information were measured on over 3,000 trees nationally. The primary emphasis for this effort was to target the top 20 species (by cubic-foot volume) in the Eastern United States and top 10 species (by cubic-foot volume) in the Western United States, which represented 67 and 81 percent of total live tree volume, respectively. These studies encompassed measuring diameter of inside and outside bark along boles, obtaining branch weights, cutting wood disks from bole sections and branches to examine wood properties, and collecting foliage for biomass analysis. The focus was on cubic-foot volume, so no effort was made to quantify volume in board-foot units. Protocols were modified as necessary to accommodate landowner requirements (e.g., keeping merchantable log lengths intact). Substantial effort was also invested in obtaining legacy data from numerous sources, including peer-reviewed journal articles, master's theses, doctoral dissertations, Forest Service publications and field surveys, forest industry studies, and other miscellaneous resources. This effort compiled records from nearly 280,000 trees—most destructively sampled—for use in this study, and data are available at www.legacytreedata.org (also see Radtke et al. 2023). Construction of the database entailed standardizing tree component definitions for compatibility across studies (e.g., total stem was defined as groundline to tree tip; merchantable cubic volume was from a 1-foot stump height to a 4.0-inch top diameter outside bark). The minimum criteria for inclusion of a tree record in the modeling dataset were measurements of diameter at breast height (4.5 feet), total height, and one or more measurements of tree taper or biomass

components. The actual model fitting data consisted of 234,823 destructively sampled trees from 339 species across 23 ecoregions (Cleland et al. 2007). These data are available in a permanent open repository (Radtke et al. 2023), with the exception of some confidential proprietary data. Supplemental data tables of values needed to calculate results are available as comma-separated values (CSV) files and are listed in the appendix.

MODEL DEVELOPMENT

Due to the wide range of species and ecological conditions, it was assumed a single model form may not deliver optimal predictions for all trees in the fitting dataset. Four candidate allometric models were initially considered for evaluation:

Schumacher-Hall model

$$(1) \quad y_i = a \times D_i^b \times H_i^c + \varepsilon_i$$

Segmented model

$$(2) \quad y_i = \begin{cases} a \times D_i^b \times H_i^c + \varepsilon_i; & D_i < k \\ a \times k^{(b-b_1)} \times D_i^{b_1} \times H_i^c + \varepsilon_i; & D_i \geq k \end{cases}$$

Continuously Variable model

$$(3) \quad y_i = a \times D_i^{a_1 \times (1 - \exp(-b \times D_i))^{c_1}} \times H_i^c + \varepsilon_i$$

Modified Wiley model

$$(4) \quad y_i = a \times D_i^b \times H_i^c \times \exp^{-(b_1 \times D_i)} + \varepsilon_i$$

where for each tree i , y_i is the observed value of the component to be estimated (weight or volume), D_i is diameter (inches) at breast height (4.5 feet), H_i is total tree height (feet), k is a set segmentation point that is 9.0 inches for softwoods ($SPCD < 300$) and 11.0 inches for hardwoods ($SPCD \geq 300$), $\exp$ is the base of the natural logarithm, ε_i is a random residual error, and all other variables are coefficients estimated from regression. Note here that the models were fit to various assemblages of species and spatial domains as needed. Also, for slash pine (*Pinus*

elliottii) (SPCD = 111) and loblolly pine (*P. taeda*) (SPCD = 131), stand origins for planted (stand origin code (STDORGCD) = 1) and natural (STDORGCD = 0) stands may be fitted separately. All candidate models were evaluated, but the Schumacher-Hall model was considered the default formulation due to the parsimonious formulation and consistency in performance across a wide range of data sources. A different model was chosen only if the Akaike information criteria (AIC) score (Akaike 1974) was lower and all estimated coefficients were significant at the $\alpha = 0.05$ level.

Preliminary investigations showed that the relationship between tree size and volume (or biomass) within a species or species group frequently varied across ecodivisions. Therefore, models were fit for species and species groups by ecodivision (fig. 1). Within-ecodivision biomass models for total aboveground, stem wood, stem bark, branch, and foliage biomass were developed for any species groups with at least 50 trees. Within-ecodivision volume models for stem wood, stem bark, and volume ratio were developed for species groups with at least 80 trees. These thresholds were chosen to balance the tradeoff between the number of species-specific models that could be presented while still maintaining a sufficient number

of observations (n) for those species. (Note that large samples are often described as $n > 30$.) The threshold was higher for volume models because a relatively larger number of trees in the database had volume information. Species-level models were also fit across ecodivisions because the FIA database, hereafter FIADB (see Burrill et al. 2023 for documentation), contained species and ecodivision combinations that were not represented in the fitting dataset.

The species-level models, either within ecodivisions or across ecodivisions, accounted for 89 percent of standing volume in the FIADB and 72 percent of standing aboveground biomass. To produce estimates for the remaining species in the FIADB, models were also estimated for the species groups

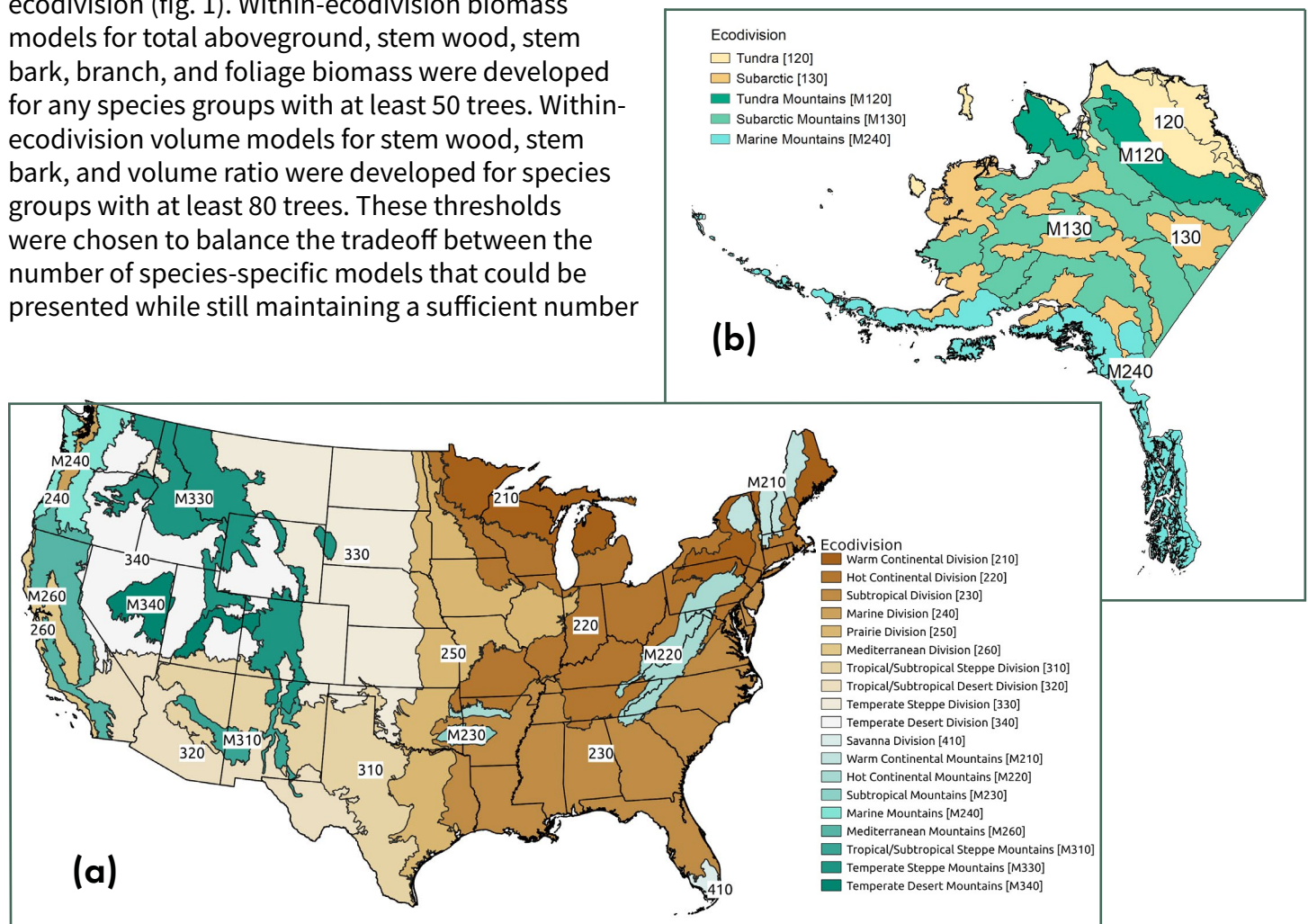


Figure 1.—Ecodivisions used by Forest Inventory and Analysis for (a) 48 of the 50 United States (Source: Cleland et al. 2007) and (b) Alaska (Source: Nowacki et al. 2003).

described in Jenkins et al. (2003). The Jenkins groups are already in use by FIA and consist of species assemblages based on phylogenetic relationships and wood specific gravity. Models were estimated for 8 of the 10 Jenkins groups. Two Jenkins groups, Douglas-fir because it was a single species and woodland groups due to lack of data, were excluded from this study. For species with fewer than five trees, model 5, which incorporates published species-level wood specific gravity (*WDSG*) values (Miles and Smith 2009), was estimated for total aboveground and branch biomass by Jenkins group:

Modified Schumacher-Hall model

$$(5) \quad y_i = a \times D_i^b \times H_i^c \times WDSG_i + \varepsilon_i$$

For species with 5 to 50 biomass trees (or 80 volume trees), mixed-effects model techniques were used at the Jenkins group level to fit model 1 for bark and foliage biomass and the modified Schumacher-Hall model 5 for total aboveground and branch biomass. For these models, species was used as a random effect to account for differences among species within a given Jenkins group. The random effect was associated with the b parameter, i.e., the coefficient is a mixed parameter ($b + \theta$) where θ is the random species effect.

Allometric models were developed for the following volume and biomass measurements: total stem wood volume, total stem bark volume, total branch wood and bark biomass, total aboveground biomass (without foliage), and total foliage biomass. Additionally, inside-bark and outside-bark volume ratio models were estimated to predict the proportion of volume to any height along the stem for all possible species and Jenkins groups:

Volume Ratio model

$$(6) \quad R_i = \left[1 - \left(1 - \frac{h_i}{H_i} \right)^\alpha \right]^\beta + \varepsilon_i$$

where R_i is the proportion of total stem volume from groundline to h_i as a height along the stem with α and β as estimated parameters. Although no formal statistical tests were performed, heteroscedastic residual patterns were visually apparent in initial modeling analyses. Subsequent weighting of

observations by $1/D_i^2$ for models 1–5 and $1/\left[\frac{h_i}{H_i} \times \left(1 - \frac{h_i}{H_i}\right)\right]$ for model 6 displayed satisfying homoscedastic residual behaviors that were deemed to sufficiently address the assumption of constant error variance (Crow and Laidly 1980).

Model 6 can also be combined with model 1 to estimate the height h_i to any diameter d_i . This is accomplished by recognizing that the stem volume or biomass from groundline to h_i can be constructed as the product of a total volume model and a volume ratio model:

$$y_i = a \times D_i^b \times H_i^c \times \left[1 - \left(1 - \frac{h_i}{H_i} \right)^\alpha \right]^\beta$$

The implied taper function is then specified as (Zhao et al. 2019):

$$d_i^2 = a \times D_i^b \times H_i^c / 0.005454154 / H_i \times \alpha \times \beta \times \left(1 - \frac{h_i}{H_i} \right)^{(\alpha-1)} \times \left[1 - \left(1 - \frac{h_i}{H_i} \right)^\alpha \right]^{(\beta-1)}$$

The height along the stem (h_i) at a specified diameter on the stem (d_i) can be obtained by iteratively solving (i.e., numeric optimization or minimization per Nocedal and Wright (2006)) equation 7 for h_i :

$$(7) \quad d_i - \left\{ a \times D_i^b \times H_i^c / 0.005454154 / H_i \times \alpha \times \beta \times \left(1 - \frac{h_i}{H_i} \right)^{(\alpha-1)} \times \left[1 - \left(1 - \frac{h_i}{H_i} \right)^\alpha \right]^{(\beta-1)} \right\}^{0.5}$$

where d_i is the desired top diameter; h_i is the height to desired top diameter; a , b , and c are coefficients from the outside-bark volume coefficient tables (tables S3a and S3b); and α and β are coefficients from the outside-bark volume ratio coefficient tables (tables S4a and S4b).

Modifications for standing dead trees to wood density and for bark and branch losses based on the observed level of decay as indicated by the FIA decay class code (*DECAYCD*) variable (Burrill et al. 2023) and hardwood or softwood species classification are incorporated into the NSVB framework by adopting the findings of Harmon et al. (2011) as shown in table 1. Note that these values account

for differences between hardwood and softwood species, unlike the values presented in Domke et al. (2011). The values for wood density proportion for *DECAYCD* = 3 are also used to account for the fact that rotten wood cull still maintains a weight greater than zero even though rotten cull is entirely deducted to obtain sound cubic volume amounts. In this case, the observed cull is assumed to be entirely rotten wood, and the density of that wood

is reduced accordingly. In addition, model 6 is used to implement a standardized approach to estimate volume and biomass reductions from missing stem tops. Belowground coarse root biomass is calculated using the approach described in Heath et al. (2009) but by using merchantable stem wood volume as calculated here and applying the wood density proportions from table 1 for standing dead trees.

Table 1.—Wood density proportions and remaining bark and branch proportions for dead trees by species classification (hardwood vs. softwood) and Forest Inventory and Analysis (FIA) decay code (*DECAYCD*) classification.

Species classification	FIA decay code (<i>DECAYCD</i>)	Wood density proportion (<i>DensProp</i>)	Remaining bark proportion (<i>BarkProp</i>)	Remaining branch proportion (<i>BranchProp</i>)
Hardwood	1	0.99	1.0	1.0
Hardwood	2	0.80	0.8	0.5
Hardwood	3	0.54	0.5	0.1
Hardwood	4	0.43 ^a	0.2	0
Hardwood	5	0.43 ^a	0	0
Softwood	1	0.97	1.0	1.0
Softwood	2	1.00	0.8	0.5
Softwood	3	0.92	0.5	0.1
Softwood	4	0.55 ^a	0.2	0
Softwood	5	0.55 ^a	0	0

^a Decay class 4 values from Harmon et al. (2011) are used for FIA *DECAYCD* = 4 and 5.

RESULTS

Due to the large number of species and ecodivision combinations and the numerous volume and biomass models required, tables of coefficients are provided to address the prediction requirements for all species included in the study (tables S1a–S9b in the appendix). Two basic types of tables are included and are labeled with either a “spcd” name suffix or a “jenkins” name suffix. Tables with the spcd suffix provide the models 1–4 form and associated coefficients for species-ecodivision-stand origin combinations. If a species occurs in an ecodivision not explicitly listed, the entry having no ecodivision noted is used. For species not included in the spcd tables, the jenkins suffix tables are used with model 5 and associated coefficients for the Jenkins group that represents the species of interest. Species assignments to Jenkins groups are in FIADB REF_SPECIES table as variable name *JENKINS_SPGRPCD*. Note that Jenkins group coefficients incorporate the predicted random effect into the reported coefficients, i.e., in some cases, the value is a sum of the fixed and random effects. Tables of coefficients for predicting volume ratios (model 6) are also included. New carbon content fractions based on Doraisami et al. (2022) are provided in tables S10a and S10b, where species-specific values are given for live trees and values for dead trees are based on hardwood vs. softwood classification and level of wood decay (*DECAYCD*) (Martin et al. 2021). Mean crown ratios of live trees based on FIA data are provided in table S11 for making branch and foliage weight deductions for dead trees with broken tops. Example 3 in the Results section provides additional information on using table S11.

In addition to the tables needed for calculations, key modeling statistics such as sample sizes (n), tree diameter distributions (minimum, mean, and maximum), fit index (FI ; analogous to R^2), root mean squared error ($RMSE$), prediction error mean ($Mean(PE)$) and standard deviation ($SD(PE)$), percent prediction error mean ($Mean(PE\%)$) and standard deviation ($SD(PE\%)$), absolute prediction error mean ($Mean(APE)$) and percent ($Mean(APE\%)$), and diameter at breast height-weighted prediction error variability ($Sigma$) may be of primary interest

to inventory practitioners and data users. These statistics are defined as follows:

$$FI = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n}}$$

$$Mean(PE) = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)}{n} = \bar{\epsilon}$$

$$SD(PE) = \sqrt{\frac{\sum_{i=1}^n (\hat{y}_i - \bar{\epsilon})^2}{n-1}}$$

$$Mean(PE\%) = \frac{1}{n} \sum_{i=1}^n \frac{(y_i - \hat{y}_i)}{y_i} 100 = \frac{1}{n} \sum_{i=1}^{n_j} PE_i\% = \bar{\epsilon}\%$$

$$SD(PE\%) = \sqrt{\frac{\sum_{i=1}^n (PE_i\% - \bar{\epsilon}\%)^2}{n-1}}$$

$$Mean(APE) = \frac{\sum_{i=1}^n |y_i - \hat{y}_i|}{n}$$

$$Mean(APE\%) = \frac{1}{n} \sum_{i=1}^n \left| \frac{(y_i - \hat{y}_i)}{y_i} \right| 100$$

$$Sigma = \sqrt{\frac{\sum_{i=1}^n \left[(y_i - \hat{y}_i)^2 \left(\frac{1}{D_i^2} \right) \right]}{n-1}}$$

where $\hat{y}_i$ is the predicted value of the weight or volume component to be estimated for tree i , $\bar{y}$ is the mean of the y_i , and n is the sample size. Supplemental tables listed in the appendix report the relevant statistics for the entire suite of models 1–5. For example, supplemental tables S12–S20 provide statistics for various aggregations of ecodivision, species, FIA region, State, and national perspectives. As expected, various outcomes were realized across attributes (volume or biomass)

and the attribute components (e.g., wood, bark, branches). Readers are encouraged to consult the tables for their specific ecdivisions and species of interest.

Typically, biomass is converted to carbon by applying a carbon fraction value. Previously, FIA used the generic approximation of 0.5 as the ratio of carbon to dry wood weight for all species. For the species addressed in this study, the NSVB framework introduces more rigorous carbon content predictions. Species-specific carbon fractions were developed for 100 species using the Global Woody Tissue Carbon Concentration Database (GLOWCAD) (Doraisami et al. 2022) and were established for the remaining 321 species as a linear model prediction based on specific gravity (Martin et al. 2018). On average, the carbon fraction is 0.477 across all species, with a minimum value of 0.420 and a maximum value of 0.538. Thus, the general expectation will likely be that carbon content will decline for a given amount of biomass because the overall average is less than the previous carbon fraction of 0.5. However, realized differences in carbon amounts will depend on various interrelated factors, including changes in the tree biomass basis, species composition, and tree size distributions for a specified area of interest.

EXAMPLES OF TREE-LEVEL CALCULATIONS

A number of calculations are required to obtain the full suite of volume and biomass components for each tree. An outline of the necessary calculations is provided here to familiarize readers with the general conceptual approach, followed by a series of examples. The general approach requires the following steps:

1. Predict gross total stem wood volume as a function of diameter at breast height (D) and total height (H).
2. Predict gross total stem bark volume as a function of D and H .
3. Obtain gross total stem outside-bark volume as the sum of wood and bark gross volumes.
4. Estimate heights to merchantable (4.0-inch) top diameter and, if present, sawlog top diameter (7.0 inches for softwoods ($SPCD < 300$) and 9.0

inches for hardwoods ($SPCD \geq 300$)). Adjust values as needed for trees with a broken top.

5. Estimate stem component gross volumes (stump; merchantable stem; sawlog, if present; and stem top) using a ratio function.
6. Estimate stem component sound volumes to account for any cull present or dead tree density reductions.
7. Convert total stem wood gross volume to biomass weight using published wood density values (Miles and Smith 2009). Reduce stem wood weight due to broken top, cull deductions (accounting for nonzero weight of cull), and dead tree wood density reduction.
8. Predict total stem bark biomass as a function of D and H . Reduce the prediction if necessary for missing bark due to a broken top or dead tree structural loss, if either is present.
9. Predict total branch biomass as a function of D and H . Reduce the prediction if necessary for missing branches due to a broken top or dead tree wood density reduction and structural loss, if present.
10. Predict total aboveground biomass as a function of D and H . Reduce the prediction if necessary using the overall proportional reduction obtained from the stem wood, bark, and branch component reductions. This biomass value is considered the “optimal” biomass estimate.
11. Sum total stem wood biomass, total stem bark biomass, and total branch biomass (with each component reduced for broken tops, cull, and dead tree density loss as appropriate) to obtain a second total aboveground biomass.
12. Proportionally distribute the difference between the directly predicted total biomass and the total from the component estimates across total stem wood, total stem bark, and total branch weights to create an adjusted total stem wood weight, an adjusted total stem bark weight, and an adjusted total branch weight.
13. Calculate an adjusted wood density by dividing the adjusted total stem wood weight by the predicted total stem wood volume. This adjusted wood density can be used to convert any subsection of the main stem wood volume to biomass.

14. Calculate an adjusted bark density by dividing the adjusted total stem bark weight by the predicted total stem bark volume. This value can be used to convert any subsection of the main stem bark volume to biomass.
15. Directly predict total foliage dry weight as a function of D and H .
16. Estimate total aboveground carbon using total aboveground biomass (excluding foliage) and the appropriate carbon fraction.

In the following examples, the model forms are referred to by the number listed in the Methods section. For all examples, units for volume and biomass predictions are cubic feet and pounds, respectively. The calculations retain many digits only to minimize the compounding of rounding error effects throughout the prediction system. This is not intended to imply a level of accuracy in the predictions, and users can choose to round the final predictions for their attributes of interest to the extent desired.

Example 1

Assume the following measurements were taken for a Douglas-fir (*Pseudotsuga menziesii*; $SPCD = 202$) tree having $D = 20.0$ inches and $H = 110$ feet with no cull growing in the Marine Division ($DIVISION = 240$). The first step is to predict total stem wood volume in cubic feet using the appropriate model form and coefficients. The inside-bark wood volume coefficient table (table S1a) indicates trees in the group 202/240 (i.e., $SPCD = 202$ and $DIVISION = 240$) use model 2 with the appropriate coefficients:

$$V_{tot_{ib}} Gross = a \times k^{(b-b1)} \times D^{b1} \times H^c$$

$$\begin{aligned} V_{tot_{ib}} Gross &= 0.001929099661 \\ &\times 9^{(2.162413104203 - 1.690400253097)} \times 20^{1.690400253097} \\ &\times 110^{0.985444005253} = 88.452275544288 \end{aligned}$$

Total bark volume is predicted next. Consulting the bark volume coefficient table (table S2a) indicates the use of model 1 with the appropriate coefficients:

$$V_{tot_{bk}} Gross = a \times D^b \times H^c$$

$$\begin{aligned} V_{tot_{bk}} Gross &= 0.000031886237 \times 20^{1.21260513951} \\ &\times 110^{1.978577263767} = 13.191436232306 \end{aligned}$$

Total outside-bark volume is then calculated via addition:

$$V_{tot_{ob}} Gross = V_{tot_{ib}} Gross + V_{tot_{bk}} Gross$$

$$\begin{aligned} V_{tot_{ob}} Gross &= 88.452275544288 + 13.191436232306 \\ &= 101.643711776594 \end{aligned}$$

Note that table S3a provides the information needed to directly obtain model predictions of $V_{tot_{ob}} Gross$. However, this table is not intended to be used in this manner as it does not facilitate maintaining additive properties nor enable proper treatment of the stem wood and bark components in terms of reductions for wood cull or dead tree decay and loss. The primary use of table S3a is for calculating merchantable and sawlog stem volumes. Merchantable volumes are defined as the volume from a 1-foot stump to a 4.0-inch outside-bark top diameter. Sawlog volumes are defined as being between a 1-foot stump and a 7.0-inch top diameter for softwood species ($D \geq 9.0$ inches) and 9.0-inch top diameter for hardwood species ($D \geq 11.0$ inches). Equation 7 can be used to find the height (h_i) to any top diameter (d_i); however, it cannot be inverted or algebraically rearranged to be solved directly. Therefore, iterative methods must be used (i.e., numerical optimization or minimization). For the merchantable height to a 4.0-inch top (h_m), inserting the correct coefficient values for a , b , and c from the outside-bark volume coefficient table (table S3a) and values for θ and θ from the outside-bark volume ratio coefficient table (table S4a) results in the following calculation:

$$\begin{aligned} &|4 - \{0.002916157874 \times 20^{1.778795704183} \\ &\times 110^{1.08526548472} / 0.005454154\} / 110 \\ &\times 2.386864288974 \times 0.907607415992 \\ &\times (1 - h_m / 110)^{(2.386864288974 - 1)} \\ &\times [1 - (1 - h_m / 110)^{2.386864288974}]^{(0.907607415992 - 1)}\}^{0.5} \end{aligned}$$

Iterative minimization results in $h_m = 98.28126765402$. To determine merchantable volume, use model 6 and the coefficients from the inside-bark volume ratio table (table S5a) to find the proportion of total stem volume for both the 1-foot stump height and the 4.0-inch top diameter height:

$$R_1 = [1 - (1 - h_i/H)^a]^\beta$$

$$\begin{aligned} R_1 &= [1 - (1 - 1/110)^{2.220714200464}]^{0.952218706779} \\ &= 0.024198309503 \end{aligned}$$

$$R_m = [1 - (1 - h_m/H)^{\alpha}]^{\beta}$$

$$R_m = [1 - (1 - 98.28126765402/110)^{2.220714200464}]^{0.952218706779} = 0.993406175350$$

where h_1 is stump height (1 foot), h_m is the merchantable height, R_1 is the proportion of volume to 1 foot, and R_m is the proportion of volume to the merchantable height.

Then, multiply the ratios by the already estimated total stem wood volume and subtract the stump volume to obtain the merchantable stem inside-bark volume:

$$Vmer_{ib} Gross = (R_m \times Vtot_{ib} Gross) - (R_1 \times Vtot_{ib} Gross)$$

$$Vmer_{ib} Gross = (0.993406175350 \times 88.452275544288) - (0.024198309503 \times 88.452275544288) = 85.728641209612$$

The same procedure can be used to estimate the merchantable stem outside-bark volume:

$$Vmer_{ob} Gross = (R_m \times Vtot_{ob} Gross) - (R_1 \times Vtot_{ob} Gross)$$

$$Vmer_{ob} Gross = (0.99340617535 \times 101.643711776594) - (0.024198309503 \times 101.643711776594) = 98.513884967785$$

Note that the same volume ratio coefficients are used for both inside-bark and outside-bark ratios to ensure consistency. Merchantable stem bark volume is then calculated via subtraction:

$$Vmer_{bk} Gross = Vmer_{ob} Gross - Vmer_{ib} Gross$$

$$Vmer_{bk} Gross = 98.513884967785 - 85.728641209612 = 12.785243758174$$

Calculating cubic-foot volume in the sawlog portion of the stem (1-foot stump height to 7.0-inch top diameter for softwoods ($SPCD < 300$; $D \geq 9.0$ inches) and 9.0-inch top diameter for hardwoods ($SPCD \geq 300$; $D \geq 11.0$ inches)) proceeds similarly, with sawlog height (h_s) being obtained from the following calculation:

$$\begin{aligned} & [7 - \{0.002916157874 \times 20^{1.778795704183} \times 110^{1.085526548472} \\ & / 0.005454154\} / 110 \times 2.386864288974 \\ & \times 0.907607415992 \times (1 - h_s/110)^{(2.386864288974 - 1)} \\ & \times [1 - (1 - h_s/110)^{2.386864288974}]^{(0.907607415992 - 1)}]^{0.5} \end{aligned}$$

Iterative minimization results in $h_s = 83.785181046$. To determine sawlog volume, use model 6 and

the coefficients from the inside-bark volume ratio table (table S5a) to find the proportion of total stem volume for both the 1-foot stump height and the 7.0-inch top diameter height (R_s):

$$R_1 = [1 - (1 - h_1/H)^{\alpha}]^{\beta}$$

$$R_1 = [1 - (1 - 1/110)^{2.220714200464}]^{0.952218706779} = 0.024198309503$$

$$R_s = [1 - (1 - h_s/H)^{\alpha}]^{\beta}$$

$$R_s = [1 - (1 - 83.785181046/110)^{2.220714200464}]^{0.952218706779} = 0.960553392655$$

where h_1 is stump height (1 foot), h_s is the sawlog height, R_1 is the proportion of volume to 1 foot and R_s is the proportion of volume to the sawlog height.

Then, multiply the ratios by the already estimated total stem wood volume and subtract:

$$Vsaw_{ib} Gross = (R_s \times Vtot_{ib} Gross) - (R_1 \times Vtot_{ib} Gross)$$

$$Vsaw_{ib} Gross = (0.960553392655 \times 88.452275544288) - (0.024198309503 \times 88.452275544288) = 82.822737822255$$

The same procedure can be used to estimate the sawlog outside-bark volume:

$$Vsaw_{ob} Gross = (R_s \times Vtot_{ob} Gross) - (R_1 \times Vtot_{ob} Gross)$$

$$Vsaw_{ob} Gross = (0.960553392655 \times 101.643711776594) - (0.024198309503 \times 101.643711776594) = 95.174606192451$$

Sawlog stem bark volume is then calculated via subtraction:

$$Vsaw_{bk} Gross = Vsaw_{ob} Gross - Vsaw_{ib} Gross$$

$$Vsaw_{bk} Gross = 95.174606192451 - 82.822737822255 = 12.351868370196$$

Stump wood and bark volumes are estimated using the same volume ratio approach:

$$Vstump_{ob} Gross = R_1 \times Vtot_{ob} Gross$$

$$Vstump_{ob} Gross = 0.024198309503 \times 101.643711776594 = 2.459605996608$$

$$Vstump_{ib} Gross = R_1 \times Vtot_{ib} Gross$$

$$Vstump_{ib} Gross = 0.024198309503 \times 88.452275544288 = 2.140395539869$$

$$V_{stump_{bk}Gross} = V_{stump_{ob}Gross} - V_{stump_{ib}Gross}$$

$$V_{stump_{bk}Gross} = 2.459605996608 - 2.140395539869 \\ = 0.319210456739$$

Finally, stem-top volumes are calculated by subtracting the other stem volume subcomponents:

$$V_{top_{ob}Gross} = V_{tot_{ob}Gross} - V_{mer_{ob}Gross} \\ - V_{stump_{ob}Gross}$$

$$V_{top_{ob}Gross} = 101.643711776594 \\ - 98.513884967785 - 2.459605996608 \\ = 0.670220812201$$

$$V_{top_{ib}Gross} = V_{tot_{ib}Gross} - V_{mer_{ib}Gross} \\ - V_{stump_{ib}Gross}$$

$$V_{top_{ib}Gross} = 88.452275544288 - 85.728641209612 \\ - 2.140395539869 = 0.583238794807$$

$$V_{top_{bk}Gross} = V_{top_{ob}Gross} - V_{top_{ib}Gross}$$

$$V_{top_{bk}Gross} = 0.670220812201 - 0.583238794807 \\ = 0.086982017394$$

The same ratio procedure can be used to estimate outside-bark or inside-bark volume between any heights and can be used to estimate many product classes (i.e., sawlog volumes). Additionally, if bark volumes are desired, predict for both outside-bark and inside-bark volumes and then subtract (i.e., $V_{bk} = V_{ob} - V_{ib}$).

Associated sound wood and bark attributes are also needed to account for any rotten/missing cull wood, along with any decay reductions that are specified for dead trees. Notationally, values designated as “Sound” hereafter refer to values occurring after considering any deductions due to cull, broken top, or dead tree density reductions. Although the tree in this example has $CULL = 0$, it is shown how cull would be applied to any inside-bark volumes at this point:

$$V_{tot_{ib}Sound} = V_{tot_{ib}Gross} \times (1 - CULL/100)$$

$$V_{tot_{ib}Sound} = 88.452275544288 \times (1 - 0/100) \\ = 88.452275544288$$

where $CULL$ is the percentage of rotten/missing wood in the main stem below any missing top. For the example tree used here, all sound attributes are equal to their gross counterparts due to the tree being alive with no cull.

An outside-bark volume that includes wood cull (note that bark volume predictions are unaffected by the $CULL$ value) can be determined by adding the appropriate bark volume to the sound wood volume estimates:

$$V_{tot_{ob}Sound} = V_{tot_{ib}Sound} + V_{top_{bk}Sound}$$

$$V_{tot_{ob}Sound} = 88.452275544288 + 13.191436232306 \\ = 101.643711776594$$

Total stem wood volume is converted to total stem wood dry weight in pounds (lb) using the wood density (specific gravity) value from the FIADB REF_SPECIES table, which is 0.45 for $SPCD = 202$. To convert volume to weight, multiply this value by the weight of a cubic foot of water (62.4 lb/ft³):

$$W_{tot_{ib}} = V_{tot_{ib}Gross} \times WDSG \times 62.4$$

$$W_{tot_{ib}} = 88.452275544288 \times 0.45 \times 62.4 \\ = 2483.739897283610$$

It is considered that most cull material will be rotten wood, which would still contribute to the stem weight. As such, it is assumed the density of cull wood is reduced by the proportion for $DECAYCD = 3$ (see table 1; wood density proportion ($DensProp$) is 0.54 for hardwood species and 0.92 for softwood species) as reported by Harmon et al. (2011) to obtain the reduced weight due to cull. In this example, $CULL = 0$, so no reduction in weight is incurred:

$$W_{tot_{ib}red} = V_{tot_{ib}Gross} \times [1 - CULL/100 \\ \times (1 - DensProp)] \times WDSG \times 62.4$$

$$W_{tot_{ib}red} = 88.452275544288 \times [1 - 0/100 \\ \times (1 - 0.92)] \times 0.45 \times 62.4 = 2483.739897283610$$

Next, total stem bark weight can be estimated using the appropriate model form and coefficients. Consulting the stem bark weight coefficient table (table S6a), use model 1 with the appropriate coefficients:

$$W_{tot_{bk}} = a \times D^b \times H^c$$

$$W_{tot_{bk}} = 0.009106538193 \times 20^{1.437894424586} \\ \times 110^{1.336514272981} = 361.782496100100$$

Total branch weight can then be estimated using the appropriate model form and coefficients. Consulting

the branch weight coefficient table (table S7a), use model 1 with the appropriate coefficients:

$$W_{branch} = a \times D^b \times H^c$$

$$W_{branch} = 9.521330809106 \times 20^{1.762316117442} \times 110^{-0.40574259177} = 277.487756904646$$

Reductions to bark and branch weights are only considered for dead trees and trees with broken tops. As neither of these conditions is present in the current example, $W_{tot_bk_red} = W_{tot_bk}$ and $W_{branchred} = W_{branch}$.

Now, total aboveground biomass (AGB) can be estimated using the appropriate equation form and coefficients. The total biomass coefficient table (table S8a) prescribes the use of model 1 with the appropriate coefficients:

$$AGB_{Predicted} = a \times D^b \times H^c$$

$$AGB_{Predicted} = 0.135206506787 \times 20^{1.713527048035} \times 110^{1.047613377046} = 3154.5539926725$$

The next series of steps are designed to ensure consistent estimates between the three independently estimated components (total stem wood weight, total stem bark weight, and branch weight) and the predicted total aboveground biomass ($AGB_{Predicted}$). First, estimate a second total aboveground biomass by summing the three components and then calculate the difference between the two AGB estimates:

$$AGB_{Component_red} = W_{tot_ib_red} + W_{tot_bk_red} + W_{branchred}$$

$$AGB_{Component_red} = 2483.739897283610 + 361.782496100100 + 277.487756904646 = 3123.010150288360$$

A reduction factor is now calculated to modify $AGB_{Predicted}$ to account for any component rot or loss (none in this case):

$$AGB_{Reduce} = AGB_{Component_red} / (W_{tot_ib} + W_{tot_bk} + W_{branch})$$

$$AGB_{Reduce} = 3123.010150288360 / (2483.739897283610 + 361.782496100100 + 277.487756904646) = 1.000000000000$$

$$AGB_{Predicted_red} = AGB_{Predicted} \times AGB_{Reduce}$$

$$AGB_{Predicted_red} = 3154.5539926725 \times 1.000000000000 = 3154.5539926725$$

$$AGB_{Diff} = AGB_{Predicted_red} - AGB_{Component_red}$$

$$AGB_{Diff} = 3154.5539926725 - 3123.0101502883 = 31.543842384153$$

Next, to harmonize the three components with the predicted total aboveground biomass, proportionally distribute AGB_{Diff} across the components. Mathematically, this can be accomplished with the following calculations:

$$Wood_{Harmonized} = AGB_{Predicted_red} \times (W_{tot_ib_red} / AGB_{Component_red})$$

$$Wood_{Harmonized} = 3154.5539926725 \times (2483.7398972836 / 3123.01015028834) = 2508.826815376370$$

$$Bark_{Harmonized} = AGB_{Predicted_red} \times (W_{tot_bk_red} / AGB_{Component_red})$$

$$Bark_{Harmonized} = 3154.5539926725 \times (361.7824961001 / 3123.01015028834) = 365.436666110811$$

$$Branch_{Harmonized} = AGB_{Predicted_red} \times (W_{branchred} / AGB_{Component_red})$$

$$Branch_{Harmonized} = 3154.5539926725 \times (277.487756904647 / 3123.01015028834) = 280.290511185328$$

At this point, all the individual tree components have been harmonized and are additive with the predicted total aboveground biomass estimate. The final biomass component that can be predicted is foliage weight. Consulting the foliage weight coefficient table (table S9) indicates the use of model 2 with the appropriate coefficients:

$$W_{foliage} = a \times k^{(b-b1)} \times D^{b1} \times H^c$$

$$W_{foliage} = 0.477184595914 \times 9^{(2.592670351881 - 1.249237428914)} \times 20^{1.249237428914} \times 110^{-0.325050455055} = 83.634788855934$$

Reductions to foliage weight are only considered for live trees with a broken top. As no broken top is present in the current example, $W_{foliagered} = W_{foliage}$. Foliage biomass is kept separate from total biomass values, which consist of wood, bark, and branch mass.

Finally, calculate a new adjusted wood density using the harmonized total stem wood weight and the predicted total stem wood volume. Careful attention is needed for this calculation because cull is treated differently for volume vs. biomass in the NSVB

framework. The wood volume basis does not include a deduction for cull but does include deductions for missing wood (i.e., broken top). In this example, no cull nor broken top is present such that $V_{tot_ib_Gross}$ and $V_{tot_bk_Gross}$ are representative of the actual existing wood and bark volume, respectively:

$$WDSG_{Adj} = Wood_{Harmonized} / V_{tot_ib_Gross} / 62.4$$

$$WDSG_{Adj} = 2508.826815376370 / 88.452275544288 / 62.4 = 0.454545207473$$

Similarly, an adjusted bark density is calculated using the harmonized total stem bark weight and the predicted total stem bark volume:

$$BKSG_{Adj} = Bark_{Harmonized} / V_{tot_bk_Gross} / 62.4$$

$$BKSG_{Adj} = 365.436666110811 / 13.191436232306 / 62.4 = 0.4439514186$$

The adjusted wood density can convert any stem wood volume subcomponents (e.g., merchantable or sawlog portion of the stem) to weights compatible with the harmonized total stem wood weight. The adjusted bark density can similarly be used to convert any stem bark volume subcomponents to weights compatible with the harmonized total stem bark weight. Merchantable stem wood and bark weights can be determined using the same volume basis (e.g., *Gross*) as above for the adjusted specific gravity calculations:

$$Wmer_{ib} = Vmer_{ib_Gross} \times WDSG_{Adj} \times 62.4$$

$$Wmer_{ib} = 85.728641209612 \times 0.454545207473 \times 62.4 = 2431.57468351127$$

$$Wmer_{bk} = Vmer_{bk_Gross} \times BKSG_{Adj} \times 62.4$$

$$Wmer_{bk} = 12.785243758174 \times 0.4439514186 \times 62.4 = 354.184091263592$$

The merchantable stem outside-bark weight is then calculated via addition:

$$Wmer_{ob} = Wmer_{ib} + Wmer_{bk}$$

$$Wmer_{ob} = 2431.57468351127 + 354.184091263592 = 2785.75877477486$$

$Wmer_{ob}$ is equivalent in definition to the FIADB variable DRYBIO_BOLE (dry biomass in the merchantable bole). Similarly, stump weights are calculated as follows:

$$Wstump_{ib} = Vstump_{ib_Gross} \times WDSG_{Adj} \times 62.4$$

$$Wstump_{ib} = 2.140395539869 \times 0.454545207473 \times 62.4 = 60.709367768006$$

$$Wstump_{bk} = Vstump_{bk_Gross} \times BKSG_{Adj} \times 62.4$$

$$Wstump_{bk} = 0.319210456739 \times 0.4439514186 \times 62.4 = 8.842949550309$$

$$Wstump_{ob} = Wstump_{ib} + Wstump_{bk}$$

$$Wstump_{ob} = 60.709367768006 + 8.842949550309 = 69.552317318315$$

$Wstump_{ob}$ is equivalent in definition to the FIADB variable DRYBIO_STUMP (dry biomass in the tree stump).

The NSVB component analogous to the current FIADB component DRYBIO_TOP (dry biomass in the top and branches of the tree) is the total AGB minus the merchantable stem components and the stump:

$$DRYBIO_TOP = AGB_{Predicted}^{red} - Wmer_{ob} - Wstump_{ob}$$

$$\begin{aligned} DRYBIO_TOP &= 3154.5539926725 \\ &- 2785.75877477486 - 69.552317318315 \\ &= 299.242900579325 \end{aligned}$$

As the sum of the biomass components is equal to $AGB_{Predicted}^{red}$, the carbon content (C) of the stem and branches (but not foliage) is obtained via multiplication by the appropriate C fraction for *SPCD* = 202 (table S10a):

$$C = AGB_{Predicted}^{red} \times CF$$

$$\begin{aligned} C &= 3154.5539926725 \times 0.515595833333 \\ &= 1626.474894645920 \end{aligned}$$

Example 2

Assume a red maple (*Acer rubrum*; *SPCD* = 316) tree with $D = 11.1$ inches, $H = 38$ feet, and $CULL = 3$ percent growing in the Warm Continental Mountains (*DIVISION* = M210). The first step is to predict total stem wood volume using the appropriate equation form and coefficients. Consulting the inside-bark wood volume coefficient table (table S1a), there are no coefficients for the *SPCD*/*DIVISION* combination of 316/M210. Therefore, the species-level coefficients are to be used. Use model 1 with the appropriate coefficients:

$$V_{tot_ib_Gross} = a \times D^b \times H^c$$

$$\begin{aligned} V_{tot_ib_Gross} &= 0.001983918881 \times 11.1^{1.810559393287} \\ &\times 38^{1.129417635145} = 9.427112777611 \end{aligned}$$

Next, total bark volume will be predicted. Consulting the bark volume coefficient table (table S2a), use model 2 with the appropriate coefficients:

$$Vtot_{bk}Gross = a \times k^{(b-b1)} \times D^{b1} \times H^c$$

$$Vtot_{bk}Gross = 0.003743084443$$

$$\times 11^{(2.226890355309 - 1.685993125661)} \times 11.1^{1.685993125661}$$

$$\times 38^{0.275066356213} = 2.155106401987$$

Outside-bark volume is then calculated via addition:

$$Vtot_{ob}Gross = Vtot_{ib}Gross + Vtot_{bk}Gross$$

$$Vtot_{ob}Gross = 9.427112777611 + 2.155106401987$$

$$= 11.582219179599$$

Merchantable and sawlog stem volumes are calculated next using equation 7, which can be minimized to estimate the height to any top diameter. For the height to a 4.0-inch top diameter (h_m), inserting the correct coefficients from tables S3a and S4a results in the following:

$$|4 - \{0.003068676884 \times 11.1^{1.811800477506}$$

$$\times 38^{1.054949234246} / 0.005454154 / 38 \times 2.500241064397$$

$$\times 0.88374141693 \times (1 - h_m/38)^{(2.500241064397 - 1)}$$

$$\times [1 - (1 - h_m/38)^{2.500241064397}]^{(0.88374141693 - 1)}\}^{0.5}|$$

Iterative minimization results in $h_m = 28.047839250135$. To determine merchantable volume, use model 6 and the coefficients from the inside-bark volume ratio table (table S5a) to find the proportion of total stem volume from the 1-foot stump to the 4.0-inch top:

$$R_1 = [1 - (1 - h_1/H)^{\alpha}]^{\beta}$$

$$R_1 = [1 - (1 - 1/38)^{2.533953226865}]^{0.8781223155}$$

$$= 0.091117585499$$

$$R_m = [1 - (1 - h_m/H)^{\alpha}]^{\beta}$$

$$R_m = [1 - (1 - 28.047839250135/38)^{2.533953226865}]^{0.8781223155}$$

$$= 0.970485778632$$

Then, multiply the ratios by the already estimated total inside-bark stem wood volume and subtract the 1-foot stump volume from the 4.0-inch top volume:

$$Vmer_{ib}Gross = (R_m \times Vtot_{ib}Gross) - (R_1 \times Vtot_{ib}Gross)$$

$$Vmer_{ib}Gross = (0.970485778632 \times 9.427112777611)$$

$$- (0.091117585499 \times 9.427112777611)$$

$$= 8.289903129704$$

The same procedure can be used to estimate the merchantable outside-bark volume:

$$Vmer_{ob}Gross = (R_m \times Vtot_{ob}Gross) - (R_1 \times Vtot_{ob}Gross)$$

$$Vmer_{ob}Gross = (0.970485778632 \times 11.582219179599)$$

$$- (0.091117585499 \times 11.582219179599)$$

$$= 10.185035152427$$

Merchantable stem bark volume is then calculated via subtraction:

$$Vmer_{bk}Gross = Vmer_{ob}Gross - Vmer_{ib}Gross$$

$$Vmer_{bk}Gross = 10.185035152427 - 8.289903129704$$

$$= 1.895132022724$$

Calculation of cubic-foot volume in the sawlog portion of the stem (1-foot stump height to 7.0-inch top diameter for softwoods ($SPCD < 300$; $D \geq 9.0$ inches) or 9.0-inch top diameter for hardwoods ($SPCD \geq 300$; $D \geq 11.0$ inches)) proceeds similarly, with sawlog height (h_s) being obtained from the following calculation:

$$|9 - \{0.003068676884 \times 11.1^{1.811800477506}$$

$$\times 38^{1.054949234246} / 0.005454154 / 38 \times 2.500241064397$$

$$\times 0.88374141693 \times (1 - h_s/38)^{(2.500241064397 - 1)}$$

$$\times [1 - (1 - h_s/38)^{2.500241064397}]^{(0.88374141693 - 1)}\}^{0.5}|$$

Iterative minimization results in $h_s = 9.98078332380462$. To determine sawlog volume, use model 6 and the coefficients from the inside-bark volume ratio table (table S5a) to find the proportion of total stem volume for both the 1-foot stump height and the 9.0-inch top diameter height (R_s):

$$R_1 = [1 - (1 - h_1/H)^{\alpha}]^{\beta}$$

$$R_1 = [1 - (1 - 1/38)^{2.533953226865}]^{0.8781223155}$$

$$= 0.091117585499$$

$$R_s = [1 - (1 - h_s/H)^{\alpha}]^{\beta}$$

$$R_s = [1 - (1 - 9.98078332380462/38)^{2.533953226865}]^{0.8781223155}$$

$$= 0.580175217851$$

Then, multiply the ratios by the already estimated total inside-bark stem wood volume and subtract:

$$Vsaw_{ib}Gross = (R_s \times Vtot_{ib}Gross) - (R_1 \times Vtot_{ib}Gross)$$

$$Vsaw_{ib}Gross = (0.580175217851 \times 9.427112777611)$$

$$- (0.091117585499 \times 9.427112777611)$$

$$= 4.610401454934$$

The same procedure can be used to estimate the sawlog outside-bark volume:

$$Vsaw_{ob\ Gross} = (R_s \times Vtot_{ob\ Gross}) - (R_1 \times Vtot_{ob\ Gross})$$

$$Vsaw_{ob\ Gross} = (0.580175217851 \times 11.582219179599) - (0.091117585499 \times 11.582219179599) = 5.664372689357$$

Sawlog stem bark volume is then calculated via subtraction:

$$Vsaw_{bk\ Gross} = Vsaw_{ob\ Gross} - Vsaw_{ib\ Gross}$$

$$Vsaw_{bk\ Gross} = 5.664372689357 - 4.610401454934 = 1.053971234423$$

Stump volumes are estimated using the same volume ratio approach as previously used:

$$Vstump_{ob\ Gross} = R_1 \times Vtot_{ob\ Gross}$$

$$Vstump_{ob\ Gross} = 0.091117585499 \times 11.582219179599 = 1.055343846369$$

$$Vstump_{ib\ Gross} = R_1 \times Vtot_{ib\ Gross}$$

$$Vstump_{ib\ Gross} = 0.091117585499 \times 9.427112777611 = 0.858975754526$$

$$Vstump_{bk\ Gross} = Vstump_{ob\ Gross} - Vstump_{ib\ Gross}$$

$$Vstump_{bk\ Gross} = 1.055343846369 - 0.858975754526 = 0.196368091843$$

Finally, stem-top volumes are calculated by subtracting the other stem volume subcomponents:

$$Vtop_{ob\ Gross} = Vtot_{ob\ Gross} - Vmer_{ob\ Gross} - Vstump_{ob\ Gross}$$

$$Vtop_{ob\ Gross} = 11.582219179599 - 10.185035152427 - 1.055343846369 = 0.341840180802$$

$$Vtop_{ib\ Gross} = Vtot_{ib\ Gross} - Vmer_{ib\ Gross} - Vstump_{ib\ Gross}$$

$$Vtop_{ib\ Gross} = 9.427112777611 - 8.289903129704 - 0.858975754526 = 0.278233893382$$

$$Vtop_{bk\ Gross} = Vtop_{ob\ Gross} - Vtop_{ib\ Gross}$$

$$Vtop_{bk\ Gross} = 0.341840180802 - 0.278233893382 = 0.06360628742$$

Cull is applied to any inside-bark stem volumes at this point to obtain estimates of sound volume:

$$Vtot_{ib\ Sound} = Vtot_{ib\ Gross} \times (1 - CULL/100)$$

$$Vtot_{ib\ Sound} = 9.427112777611 \times (1 - 3/100) = 9.144299394283$$

Because cull deductions only apply to inside-bark wood and no adjustments to bark are needed to account for a broken top or dead tree decay, $Vtot_{bk\ Sound} = Vtot_{bk\ Gross}$. An outside-bark volume that includes cull can be determined by adding the appropriate bark volume to the sound wood volume estimates:

$$Vtot_{ob\ Sound} = Vtot_{ib\ Sound} + Vtot_{bk\ Sound}$$

$$Vtot_{ob\ Sound} = 9.144299394283 + 2.155106401987 = 11.299405796270$$

Distribution of sound volume into stump, merchantable stem, and top components is accomplished using the same ratios as gross volume.

Total stem wood volume is converted to total stem wood dry weight using the correct value from the wood density table (FIADB REF_SPECIES table) in conjunction with the weight of one cubic foot of water (62.4 lb). Also, it is considered that most cull will be rotten wood, which would still contribute to the stem weight. As such, it is assumed the density of cull wood is reduced by the proportion for *DECAYCD* = 3 (see table 1; wood density proportion (*DensProp*) is 0.54 for hardwood species and 0.92 for softwood species) as reported by Harmon et al. (2011) to obtain the reduced weight due to cull:

$$Wtot_{ib} = Vtot_{ib\ Gross} \times WDSG \times 62.4$$

$$Wtot_{ib} = 9.427112777611 \times 0.49 \times 62.4 = 288.243400288234$$

$$Wtot_{ib\ red} = Vtot_{ib\ Gross} \times [1 - CULL/100 \times (1 - DensProp)] \times WDSG \times 62.4$$

$$Wtot_{ib\ red} = 9.427112777611 \times [1 - 3/100 \times (1 - 0.54)] \times 0.49 \times 62.4 = 284.265641364256$$

Total stem bark weight can be estimated by consulting the stem bark weight coefficient table (table S6a), which indicates the use of model 1 with

the appropriate coefficients. For live trees with intact tops, no bark deductions are incurred:

$$Wtot_{bk} = a \times D^b \times H^c$$

$$Wtot_{bk} = 0.061595466174 \times 11.1^{1.818642599217} \times 38^{0.654020672095} = 52.945466015848$$

$$Wtot_{bk,red} = Wtot_{bk} = 52.945466015848$$

The total stem weight considering the cull deduction is calculated as follows:

$$Wtot_{ob,red} = Wtot_{ib,red} + Wtot_{bk,red}$$

$$Wtot_{ob,red} = 284.265641364256 + 52.945466015848 = 337.211107380104$$

Total branch weight can then be estimated by consulting the branch weight coefficient table (table S7a), where the use of model 1 with the appropriate coefficients is indicated. For live trees with intact tops, no branch deductions are incurred:

$$Wbranch = a \times D^b \times H^c$$

$$Wbranch = 0.011144618401 \times 11.1^{3.269520661293} \times 38^{0.421304343724} = 135.001927997271$$

$$Wbranchred = Wbranch = 135.001927997271$$

Total aboveground biomass can be estimated by consulting the total biomass coefficient table (table S8a) that stipulates the use of model 4 with the appropriate coefficients:

$$AGB_{Predicted} = a \times D^b \times H^c \times \exp^{-(b1 \times D)}$$

$$AGB_{Predicted} = 0.31573027567 \times 11.1^{1.853839844372} \times 38^{0.740557378679} \times \exp^{-(0.024745684975 \times 11.1)} = 532.584798820042$$

Next, the three independently estimated components (stem wood weight, stem bark weight, and branch weight) need to be harmonized with the predicted total aboveground biomass. First, estimate an alternative total aboveground biomass by summing the three components:

$$AGB_{Component,red} = Wtot_{ib,red} + Wtot_{bk,red} + Wbranchred$$

$$AGB_{Component,red} = 284.265641364256 + 52.945466015848 + 135.001927997271 = 472.213035377375$$

Subsequently, $AGB_{Predicted}$ needs to be reduced to account for component rot and loss by calculating a reduction factor. For harmonization purposes, determine the difference between the reduced predicted and component-based values:

$$AGB_{Reduce} = AGB_{Component,red} / (Wtot_{ib} + Wtot_{bk} + Wbranch)$$

$$AGB_{Reduce} = 472.213035377375 / (288.243400288234 + 52.945466015848 + 135.001927997271) = 0.991646711840$$

$$AGB_{Predicted,red} = AGB_{Predicted} \times AGB_{Reduce}$$

$$AGB_{Predicted,red} = 532.584798820042 \times 0.991646711840 = 528.135964525863$$

$$AGB_{Diff} = AGB_{Predicted,red} - AGB_{Component,red}$$

$$AGB_{Diff} = 528.135964525863 - 472.213035377375 = 55.922929148488$$

Next, proportionally distribute AGB_{Diff} across the components:

$$Wood_{Harmonized} = AGB_{Predicted,red} \times (Wtot_{ib,red} / AGB_{Component,red})$$

$$Wood_{Harmonized} = 528.135964525863 \times (284.265641364256 / 472.213035377375) = 317.930462388645$$

$$Bark_{Harmonized} = AGB_{Predicted,red} \times (Wtot_{bk,red} / AGB_{Component,red})$$

$$Bark_{Harmonized} = 528.135964525863 \times (52.945466015848 / 472.213035377375) = 59.215656211618$$

$$Branch_{Harmonized} = AGB_{Predicted,red} \times (Wbranchred / AGB_{Component,red})$$

$$Branch_{Harmonized} = 528.135964525863 \times (135.001927997271 / 472.213035377375) = 150.989845925600$$

Foliage weight can be estimated using the foliage weight coefficient table (table S9a), which prescribes the use of model 1 with the appropriate coefficients:

$$Wfoliage = a \times D^b \times H^c$$

$$Wfoliage = 0.850316556558 \times 11.1^{1.998961809584} \times 38^{-0.418446486365} = 22.807960563788$$

Reductions to foliage weight are only considered for live trees having a broken top. As no broken top is present in the current example, $W_{foliagered} = W_{foliage}$.

At this point, calculate a new adjusted wood density using the harmonized total stem wood weight and the predicted total stem wood volume. As noted in the previous example, it is important that the volume basis used here does not include any cull deduction but does account for missing wood and bark. Thus, $V_{tot_ib_Gross}$ and $V_{tot_bk_Gross}$ again provide the appropriate volume bases:

$$WDSG_{Adj} = Wood_{Harmonized} / V_{tot_ib_Gross} / 62.4$$

$$WDSG_{Adj} = 317.930462388645 / 9.427112777611 / 62.4 = 0.540466586276$$

Similarly, calculate an adjusted bark density using the harmonized total stem bark weight and the predicted total stem bark volume:

$$BKSG_{Adj} = Bark_{Harmonized} / V_{tot_bk_Gross} / 62.4$$

$$BKSG_{Adj} = 59.215656211618 / 2.155106401987 / 62.4 = 0.440335033421$$

Merchantable stem wood and bark weights can be determined as follows:

$$Wmer_{ib} = V_{tot_ib_Gross} \times (R_m - R_l) \times WDSG_{Adj} \times 62.4$$

$$Wmer_{ib} = 9.427112777611 \times (0.970485778632 - 0.091117585499) \times 0.540466586276 \times 62.4 = 279.577936252521$$

$$Wmer_{bk} = Vmer_{bk_Gross} \times BKSG_{Adj} \times 62.4$$

$$Wmer_{bk} = 1.895132022724 \times 0.440335033421 \times 62.4 = 52.072364607955$$

Merchantable stem outside-bark weight is then calculated via addition:

$$Wmer_{ob} = Wmer_{ib} + Wmer_{bk}$$

$$Wmer_{ob} = 279.577936252521 + 52.072364607955 = 331.650300860476$$

Similarly, stump weights are calculated as follows:

$$Wstump_{ib} = Vstump_{ib_Gross} \times WDSG_{Adj} \times 62.4$$

$$Wstump_{ib} = 0.858975754526 \times 0.540466586276 \times 62.4 = 28.969056089533$$

$$Wstump_{bk} = Vstump_{bk_Gross} \times BKSG_{Adj} \times 62.4$$

$$Wstump_{bk} = 0.196368091843 \times 0.440335033421 \times 62.4 = 5.395587617753$$

$$Wstump_{ob} = Wstump_{ib} + Wstump_{bk}$$

$$Wstump_{ob} = 28.969056089533 + 5.395587617753 = 34.364643707286$$

The component DRYBIO_TOP (dry biomass in the top and branches of the tree) is determined by:

$$DRYBIO_TOP = AGB_{Predicted}^{red} - Wmer_{ob} - Wstump_{ob}$$

$$DRYBIO_TOP = 528.135964525863 - 331.650300860476 - 34.364643707286 = 162.121019958101$$

The carbon content (C) of the tree is obtained via multiplication by the appropriate C fraction for SPCD = 316 (table S10a):

$$C = AGB_{predicted}^{red} \times CF$$

$$C = 528.135964525863 \times 0.485733333333 = 256.533242502186$$

Example 3

Assume the following measurements were taken for a dead ($DECAYCD = 2$) tanoak (*Notholithocarpus densiflorus*; $SPCD = 631$) tree having $D = 11.3$ inches, $H = 28$ feet, and a broken top (actual height $AH = 21$ feet) with $CULL = 10$ percent growing in the Marine Mountains ($DIVISION = M240$, $PROVINCE = M242$). Note that $PROVINCE = M242$ is a subarea within $DIVISION = M240$ (Cleland et al. 2007, Nowacki et al. 2003), and the more spatially explicit ecoprovince designation facilitates the use of table S11 in the context of a dead tree with a broken top.

The first step is to predict total stem wood volume using the inside-bark wood volume coefficient table (table S1b). There are no coefficients for the $SPCD/DIVISION$ combination of 631/M240 nor any species-level coefficients. Therefore, the appropriate Jenkins group ($JENKINS_SPGRPCD$) coefficients are to be used. Tanoak is in the Other hardwoods group ($JENKINS_SPGRPCD = 8$ as shown in the FIADB REF_SPECIES table). Use model 1 with the appropriate coefficients:

$$V_{tot_ib_Gross} = a \times D^b \times H^c$$

$$V_{tot_ib_Gross} = 0.002340041369 \times 11.3^{1.89458735401} \times 28^{1.035094060155} = 7.283117547652$$

Total bark volume is predicted by consulting the bark volume coefficient table (table S2b), which indicates the use of model 1 with the appropriate coefficients:

$$Vtot_{bk}Gross = a \times D^b \times H^c$$

$$Vtot_{bk}Gross = 0.001879520673 \times 11.3^{1.721074101914} \times 28^{0.825002196089} = 1.907136145131$$

Outside-bark volume is then calculated via addition:

$$Vtot_{ob}Gross = Vtot_{ib}Gross + Vtot_{bk}Gross$$

$$Vtot_{ob}Gross = 7.283117547652 + 1.907136145131 = 9.190253692783$$

Merchantable and sawlog stem volumes are calculated next by minimizing equation 7 to estimate the height to any top diameter. For the merchantable height to a 4.0-inch top (h_m), insert the correct coefficients from tables S3b and S4b to produce the following:

$$\begin{aligned} &|4 - \{0.00334258499 \times 11.3^{1.861924531448} \\ &\times 28^{1.015964521941} / 0.005454154 / 28 \times 2.317280548447 \\ &\times 0.846218848701 \times (1 - h_m / 28)^{(2.317280548447 - 1)} \\ &\times [1 - (1 - h_m / 28)^{2.317280548447}]^{(0.846218848701 - 1)}\}^{0.5}| \end{aligned}$$

Iterative minimization results in $h_m = 21.790361419761$. To determine merchantable volume, use model 6 and the coefficients from the inside-bark volume ratio table (table S5b) to find the proportion of total stem volume to the 1-foot stump and the 4.0-inch top diameter height:

$$R_1 = [1 - (1 - h_1/H)^a]^\beta$$

$$R_1 = [1 - (1 - 1/28)^{2.353772358051}]^{0.831640004254} = 0.124985332188$$

$$R_m = [1 - (1 - h_m/H)^a]^\beta$$

$$R_m = [1 - (1 - 21.790361419761/28)^{2.353772358051}]^{0.831640004254} = 0.975933190572$$

Then, multiply the ratios by the already estimated total inside-bark stem wood volume and subtract:

$$Vmer_{ib}Gross = (R_m \times Vtot_{ib}Gross) - (R_1 \times Vtot_{ib}Gross)$$

$$\begin{aligned} Vmer_{ib}Gross &= (0.975933190572 \times 7.283117547652) \\ &- (0.124985332188 \times 7.283117547652) \\ &= 6.197553279533 \end{aligned}$$

The same procedure can be used to estimate the merchantable outside-bark volume:

$$Vmer_{ob}Gross = (R_m \times Vtot_{ob}Gross) - (R_1 \times Vtot_{ob}Gross)$$

$$\begin{aligned} Vmer_{ob}Gross &= (0.975933190572 \times 9.190253692783) \\ &- (0.124985332188 \times 9.190253692783) \\ &= 7.820426697879 \end{aligned}$$

Merchantable stem bark volume is then calculated via subtraction:

$$Vmer_{bk}Gross = Vmer_{ob}Gross - Vmer_{ib}Gross$$

$$\begin{aligned} Vmer_{bk}Gross &= 7.820426697879 - 6.197553279533 \\ &= 1.622873418346 \end{aligned}$$

Calculation of cubic-foot volume in the sawlog portion of the stem (1-foot stump height to 7.0-inch top diameter for softwoods ($SPCD < 300$; $D \geq 9.0$ inches) or 9.0-inch top diameter for hardwoods ($SPCD \geq 300$; $D \geq 11.0$ inches)) proceeds similarly, with calculation of the sawlog height (h_s) being obtained from minimization the following:

$$\begin{aligned} &|9 - \{0.00334258499 \times 11.3^{1.861924531448} \\ &\times 28^{1.015964521941} / 0.005454154 / 28 \times 2.317280548447 \\ &\times 0.846218848701 \times (1 - h_s / 28)^{(2.317280548447 - 1)} \\ &\times [1 - (1 - h_s / 28)^{2.317280548447}]^{(0.846218848701 - 1)}\}^{0.5}| \end{aligned}$$

Iterative minimization results in $h_s = 8.10427459853$. To determine sawlog volume, use model 6 and the coefficients from the inside-bark volume ratio table (table S5b) to find the proportion of total stem volume for both the 1-foot stump height and the 9.0-inch top diameter height (R_s):

$$R_1 = [1 - (1 - h_1/H)^a]^\beta$$

$$R_1 = [1 - (1 - 1/28)^{2.353772358051}]^{0.831640004254} = 0.124985332188$$

$$R_s = [1 - (1 - h_s/H)^a]^\beta$$

$$\begin{aligned} R_s &= [1 - (1 - 8.10427459853/28)^{2.353772358051}]^{0.831640004254} \\ &= 0.610622756652 \end{aligned}$$

Then, multiply the ratios by the already estimated total inside-bark stem wood volume and subtract:

$$Vsaw_{ib}Gross = (R_s \times Vtot_{ib}Gross) - (R_1 \times Vtot_{ib}Gross)$$

$$\begin{aligned} Vsaw_{ib}Gross &= (0.610622756652 \times 7.283117547652) \\ &- (0.124985332188 \times 7.283117547652) \\ &= 3.536954447910 \end{aligned}$$

The same procedure can be used to estimate the sawlog outside-bark volume:

$$Vsaw_{ob\ Gross} = (R_s \times Vtot_{ob\ Gross}) - (R_1 \times Vtot_{ob\ Gross})$$

$$Vsaw_{ob\ Gross} = (0.610622756652 \times 9.190253692783) - (0.124985332188 \times 9.190253692783) = 4.463131133534$$

Sawlog stem bark volume is then calculated via subtraction:

$$Vsaw_{bk\ Gross} = Vsaw_{ob\ Gross} - Vsaw_{ib\ Gross}$$

$$Vsaw_{bk\ Gross} = 4.463131133534 - 3.536954447910 = 0.926176685624$$

Stump volumes are estimated using the same volume ratio approach as previously used:

$$Vstump_{ob\ Gross} = R_1 \times Vtot_{ob\ Gross}$$

$$Vstump_{ob\ Gross} = 0.124985332188 \times 9.190253692783 = 1.148646910689$$

$$Vstump_{ib\ Gross} = R_1 \times Vtot_{ib\ Gross}$$

$$Vstump_{ib\ Gross} = 0.124985332188 \times 7.283117547652 = 0.910282866061$$

$$Vstump_{bk\ Gross} = Vstump_{ob\ Gross} - Vstump_{ib\ Gross}$$

$$Vstump_{bk\ Gross} = 1.148646910689 - 0.910282866061 = 0.238364044628$$

At this point, calculations are needed to account for the broken top. The broken top at $AH = 21$ feet occurs at a height below the calculated 4.0-inch top diameter height ($h_m = 21.790361419761$); therefore, no stem top wood component is present and the volume of the merchantable stem needs to be reduced. Any cull that might be present is also considered ($CULL = 10$ percent in this example) to obtain sound wood volume. Initially, the volume of the merchantable stem is adjusted by recalculating R_m based on AH :

$$R_m = [1 - (1 - 21/28)^{2.353772358051}]^{0.831640004254} = 0.968066877159$$

$$Vmer_{ib\ Sound} = [(R_m \times Vtot_{ib\ Gross}) - (R_1 \times Vtot_{ib\ Gross})] \times (1 - CULL/100)$$

$$Vmer_{ib\ Sound} = [(0.968066877159 \times 7.283117547652) - (0.124985332188 \times 7.283117547652)] \times (1 - 10/100) = 5.526235794852$$

Similarly estimate the remaining merchantable component bark volume:

$$Vmer_{bk\ Sound} = (R_m \times Vtot_{bk\ Gross}) - (R_1 \times Vtot_{bk\ Gross})$$

$$Vmer_{bk\ Sound} = (0.968066877159 \times 1.907136145131) - (0.124985332188 \times 1.907136145131) = 1.607871287707$$

Merchantable stem sound volume outside bark arises via addition:

$$Vmer_{ob\ Sound} = Vmer_{ib\ Sound} + Vmer_{bk\ Sound}$$

$$Vmer_{ob\ Sound} = 5.526235794852 + 1.607871287707 = 7.134107082559$$

Calculations for stump wood volumes are unaffected by the broken top, but any cull present affects the amount of sound stump wood:

$$Vstump_{ib\ Sound} = Vstump_{ib\ Gross} \times (1 - CULL/100)$$

$$Vstump_{ib\ Sound} = 0.910282866061 \times (1 - 10/100) = 0.819254579455$$

Because bark is unaffected by wood cull, it is not included in the following calculation:

$$Vstump_{ob\ Sound} = Vstump_{ib\ Sound} + Vstump_{bk\ Gross}$$

$$Vstump_{ob\ Sound} = 0.819254579455 + 0.238364044628 = 1.057618624083$$

Now the total sound wood inside- and outside-bark volumes can be obtained, in this case, by summing the stem components present (no top wood):

$$Vtot_{ob\ Sound} = Vmer_{ob\ Sound} + Vstump_{ob\ Sound}$$

$$Vtot_{ob\ Sound} = 7.134107082559 + 1.057618624083 = 8.191725706642$$

$$Vtot_{ib\ Sound} = Vmer_{ib\ Sound} + Vstump_{ib\ Sound}$$

$$Vtot_{ib\ Sound} = 5.526235794852 + 0.819254579455 = 6.345490374317$$

$$Vtot_{bk\ Sound} = Vtot_{ob\ Sound} - Vtot_{ib\ Sound}$$

$$Vtot_{bk\ Sound} = 8.191725706642 - 6.345490374317 = 1.846235332335$$

Stem-top volumes are calculated by subtracting the other stem volume subcomponents. Due to the broken top height being below the height to a

4.0-inch top diameter, the stem-top wood and bark volumes are zero:

$$V_{top_{ob}Sound} = V_{tot_{ob}Sound} - V_{mer_{ob}Sound} - V_{stump_{ob}Sound}$$

$$V_{top_{ob}Sound} = 8.191725706642 - 7.134107082559 - 1.057618624083 = 0.000000000000$$

$$V_{top_{ib}Sound} = V_{tot_{ib}Sound} - V_{mer_{ib}Sound} - V_{stump_{ib}Sound}$$

$$V_{top_{ib}Sound} = 6.345490374317 - 5.526235794852 - 0.819254579455 = 0.000000000000$$

$$V_{top_{bk}} = V_{top_{ob}Sound} - V_{top_{ib}Sound}$$

$$V_{top_{bk}} = 0.000000000000 - 0.000000000000 = 0.000000000000$$

Total stem wood volume is next converted to total stem wood dry weight (lb) using the correct *WDSG* value from the FIADB REF_SPECIES table and the water weight conversion factor (62.4 lb/ft³):

$$W_{tot_{ib}} = V_{tot_{ib}Gross} \times WDSG \times 62.4$$

$$W_{tot_{ib}} = 7.283117547652 \times 0.58 \times 62.4 = 263.590590284621$$

A second calculation accounts for the broken top and the dead tree density reduction (table 1) associated with *DECAYCD* = 2 for this tree. While the inside-bark weight includes the weight loss for wood cull (*CULL*) in live trees, cull weight is not included for dead trees as it is considered to be already accounted for by the density reduction:

$$W_{tot_{ib}red} = V_{tot_{ib}Sound} / (1 - CULL/100) \times WDSG \times DensProp \times 62.4$$

$$W_{tot_{ib}red} = 6.345490374317 / (1 - 10/100) \times 0.58 \times 0.8 \times 62.4 = 204.13865566837$$

Total stem bark weight can be estimated by consulting the stem bark weight coefficient table (table S6b), which indicates the use of model 1 with the appropriate coefficients. Also, calculate the value for the proportion of the stem remaining (via R_m in this case) while incorporating a density reduction factor for dead trees and the remaining bark proportion (*BarkProp*) (table 1):

$$W_{tot_{bk}} = a \times D^b \times H^c$$

$$W_{tot_{bk}} = (0.06020544773 \times 11.3^{1.933727566198} \times 28^{0.590397069325}) = 46.816664266025$$

$$W_{tot_{bk}red} = W_{tot_{bk}} \times R_m \times DensProp \times BarkProp$$

$$W_{tot_{bk}red} = 46.816664266025 \times 0.968066877159 \times 0.8 \times 0.8 = 29.005863664008$$

Consulting the branch weight coefficient table (table S7b), use model 5 with the appropriate coefficients and *WDSG* value to estimate total branch weight. Subsequently, also use table 1 to account for the remaining dead tree branch proportion (*BranchProp*), dead tree wood density reduction (*DensProp*), and branches remaining due to the broken top (*BranchRem*). The latter adjustment requires consulting the crown ratio table (table S11) to assume the proportion of the stem having branch wood, which indicates the expected crown ratio calculated from live trees by hardwood vs. softwood species classification and *PROVINCE*.

$$W_{branch} = a \times D^b \times H^c \times WDSG$$

$$W_{branch} = 0.798604849948 \times 11.3^{2.969162133333} \times 28^{0.301902411279} \times 0.58 = 226.788002348975$$

$$BranchRem = [AH - H \times (1 - CR)] / (H \times CR)$$

$$BranchRem = [21 - 28 \times (1 - 0.378)] / (28 \times 0.378) = 0.338624338624$$

$$W_{branchred} = W_{branch} \times DensProp \times BranchProp \times BranchRem$$

$$W_{branchred} = 226.788002348975 \times 0.8 \times 0.5 \times 0.338624338624 = 30.718374921312$$

Total aboveground biomass can be estimated by consulting the total biomass coefficient table (table S8b), which specifies the use of model 5 with the appropriate coefficients. Again, as Jenkins group coefficients are being used, multiplication by specific gravity (*WDSG*) is required:

$$AGB_{Predicted} = a \times D^b \times H^c \times WDSG$$

$$AGB_{Predicted} = 0.433906440864 \times 11.3^{2.115626101921} \times 28^{0.735074517922} \times 0.58 = 492.621457718427$$

Next, the three independently estimated components (stem wood weight, stem bark weight, and branch weight) need to be harmonized with the predicted total aboveground biomass. First, estimate

a reduced total aboveground biomass based on the reduced component weights:

$$AGB_{Component}^{red} = W_{tot_{ib}}^{red} + W_{tot_{bk}}^{red} + W_{branch}^{red}$$

$$\begin{aligned} AGB_{Component}^{red} &= 204.13865566837 \\ &+ 29.005863664008 + 30.718374921312 \\ &= 263.862894253690 \end{aligned}$$

Subsequently, $AGB_{Predicted}$ needs to be reduced to account for component rot and loss by calculating a reduction factor:

$$AGB_{Reduce} = AGB_{Component}^{red} / (W_{tot_{ib}} + W_{tot_{bk}} + W_{branch})$$

$$\begin{aligned} AGB_{Reduce} &= 263.862894253690 / (263.590590284621 \\ &+ 46.816664266025 + 226.788002348975) \\ &= 0.491186195084 \end{aligned}$$

$$AGB_{Predicted}^{red} = AGB_{Predicted} \times AGB_{Reduce}$$

$$\begin{aligned} AGB_{Predicted}^{red} &= 492.621457718427 \\ &\times 0.491186195084 = 241.968859433448 \end{aligned}$$

$$AGB_{Diff} = AGB_{Predicted}^{red} - AGB_{Component}^{red}$$

$$\begin{aligned} AGB_{Diff} &= 241.968859433448 - 263.862894253690 \\ &= -21.894034820242 \end{aligned}$$

Next, proportionally distribute AGB_{Diff} across the components:

$$\frac{Wood_{Harmonized}}{AGB_{Component}^{red}} = \frac{AGB_{Predicted}^{red} \times (W_{tot_{ib}}^{red})}{AGB_{Component}^{red}}$$

$$\begin{aligned} \frac{Wood_{Harmonized}}{AGB_{Component}^{red}} &= 241.968859433448 \times (204.13865566837 / 263.862894253690) \\ &= 187.200242072923 \end{aligned}$$

$$\frac{Bark_{Harmonized}}{AGB_{Component}^{red}} = \frac{AGB_{Predicted}^{red} \times (W_{tot_{bk}}^{red})}{AGB_{Component}^{red}}$$

$$\begin{aligned} \frac{Bark_{Harmonized}}{AGB_{Component}^{red}} &= 241.968859433448 \times (29.005863664008 / 263.862894253690) \\ &= 26.599100898644 \end{aligned}$$

$$\frac{Branch_{Harmonized}}{AGB_{Component}^{red}} = \frac{AGB_{Predicted}^{red} \times (W_{branch}^{red})}{AGB_{Component}^{red}}$$

$$\begin{aligned} \frac{Branch_{Harmonized}}{AGB_{Component}^{red}} &= 241.968859433448 \times (30.718374921312 / 263.862894253690) \\ &= 28.169516461881 \end{aligned}$$

In the case of dead trees, foliage weight is assumed to be zero:

$$W_{foliage} = 0$$

Finally, calculate a new adjusted wood density using the harmonized total stem wood weight and the total sound inside-bark stem wood volume. Although $V_{tot_{ib}}^{Gross}$ and $V_{tot_{bk}}^{Gross}$ provided the correct bases in previous examples, their use here is inappropriate as reductions incurred by the broken top are not accounted for. Also, any reductions due to $CULL > 0$ need to be excluded. Thus, $V_{tot_{ib}}^{Sound}$ with $CULL$ deductions removed and $V_{tot_{bk}}^{Sound}$ are the appropriate volumes to use. For this example, $V_{tot_{ib}}^{Sound} / (1 - CULL/100) = 6.345490374317 / (1 - 10/100) = 7.050544860341$. The calculations proceed as follows:

$$WDSG_{Adj} = \frac{Wood_{Harmonized}}{(1 - CULL/100)} / V_{tot_{ib}}^{Sound} / 62.4$$

$$\begin{aligned} WDSG_{Adj} &= 187.200242072923 / 7.050544860341 / 62.4 \\ &= 0.425499580359 \end{aligned}$$

Similarly, calculate an adjusted bark density using the harmonized total stem bark weight and the predicted total stem bark volume:

$$BKSG_{Adj} = \frac{Bark_{Harmonized}}{V_{tot_{bk}}^{Sound}} / 62.4$$

$$\begin{aligned} BKSG_{Adj} &= 26.599100898644 / 1.846235332335 / 62.4 \\ &= 0.230884782206 \end{aligned}$$

Merchantable stem wood and bark weights can be determined as follows:

$$\begin{aligned} W_{mer_{ib}} &= (V_{tot_{ib}}^{Sound} / (1 - CULL/100) \\ &- V_{stump_{ib}}^{Sound} - V_{top_{ib}}^{Sound}) \times WDSG_{Adj} \times 62.4 \end{aligned}$$

$$\begin{aligned} W_{mer_{ib}} &= (7.050544860341 - 0.910282866061 \\ &- 0.000000000000) \times 0.425499580359 \times 62.4 \\ &= 163.031163476092 \end{aligned}$$

$$\begin{aligned} W_{mer_{bk}} &= (V_{tot_{bk}}^{Sound} - V_{stump_{bk}}^{Sound} \\ &- V_{top_{bk}}^{Sound}) \times BKSG_{Adj} \times 62.4 \end{aligned}$$

$$\begin{aligned} W_{mer_{bk}} &= (1.846235332335 - 0.238364044628 \\ &- 0.000000000000) \times 0.230884782206 \times 62.4 \\ &= 23.164939953637 \end{aligned}$$

Merchantable stem outside-bark weight is then calculated via addition:

$$W_{mer_{ob}} = W_{mer_{ib}} + W_{mer_{bk}}$$

$$\begin{aligned} W_{mer_{ob}} &= 163.031163476092 + 23.164939953637 \\ &= 186.196103429729 \end{aligned}$$

Similarly, stump weights are calculated:

$$Wstump_{ib} = Vstump_{ib} Sound \times WDSG_{Adj} \times 62.4$$

$$Wstump_{ib} = 0.910282866061 \times 0.425499580359 \times 62.4 = 24.169078597057$$

$$Wstump_{bk} = Vstump_{bk} Sound \times BKSG_{Adj} \times 62.4$$

$$Wstump_{bk} = 0.238364044628 \times 0.230884782206 \times 62.4 = 3.434160945052$$

$$Wstump_{ob} = Wstump_{ib} + Wstump_{bk}$$

$$Wstump_{ob} = 24.169078597057 + 3.434160945052 = 27.603239542109$$

The component DRYBIO_TOP (dry biomass in the top and limbs of the tree) is calculated as follows:

$$DRYBIO_TOP = AGB_{Predicted}^{red} - Wmer_{ob} - Wstump_{ob}$$

$$DRYBIO_TOP = 241.968859433448 - 186.196103429729 - 27.603239542109 = 28.169516461610$$

The carbon content (C) of the dead tree is obtained via multiplication by the appropriate C fraction for a hardwood species (tanoak, $SPCD = 631$) with $DECAYCD = 2$ (table S10a):

$$C = AGB_{predicted}^{red} \times CF$$

$$C = 241.968859433448 \times 0.473000000000 = 114.451270512021$$

Example 4

Assume the following measurements were taken for a live white oak (*Quercus alba*; $SPCD = 802$) tree having $D = 18.1$ inches, $H = 65$ feet, a broken top (actual height (AH) = 59 feet), $CULL = 2$ percent, and a crown ratio of 30 percent ($CR = 30$) growing in the Hot Continental Mountains ($DIVISION = M220$).

The first step is to predict total inside-bark stem wood volume by consulting the inside-bark wood volume coefficient table (table S1a). There are coefficients given for the $SPCD/DIVISION$ combination of 802/M220 along with the specification to use model 1:

$$Vtot_{ib} Gross = a \times D^b \times H^c$$

$$Vtot_{ib} Gross = 0.002062931814 \times 18.1^{1.852527628718} \times 65^{1.09312644716} = 42.277832913225$$

Total bark volume is accomplished by consulting the bark volume coefficient table (table S2a), which indicates the use of model 2 with the appropriate coefficients:

$$Vtot_{bk} Gross = a \times k^{(b-b1)} \times D^{b1} \times H^c$$

$$Vtot_{bk} Gross = 0.002020025979 \times 11^{(1.957775262905 - 1.618455676343)} \times 18.1^{1.618455676343} \times 65^{0.677400740385} = 8.361568823386$$

Total outside-bark volume is then calculated via addition:

$$Vtot_{ob} Gross = Vtot_{ib} Gross + Vtot_{bk} Gross$$

$$Vtot_{ob} Gross = 42.277832913225 + 8.361568823386 = 50.639401736611$$

Merchantable and sawlog stem volumes are calculated using equation 7 that can be minimized to estimate the height to any top diameter. For the height to a 4.0-inch top diameter (h_m), inserting the correct coefficients from tables S3a and S4a produces the following:

$$\left| 4 - \left\{ 0.003504073654 \times 18.1^{1.821357964958} \times 65^{1.031766698583} / 0.005454154 / 65 \times 2.413673220682 \times 0.851093936311 \times (1 - h_m/65)^{(2.413673220682 - 1)} \times [1 - (1 - h_m/65)^{2.413673220682}]^{(0.851093936311 - 1)} \right\}^{0.5} \right|$$

Iterative minimization results in $h_m = 56.72042843$. The broken top actual height of 59 feet is greater than the predicted h_m for an intact top, so the merchantable top height is unaffected (see example 3 for $AH < h_m$). To determine merchantable volume, use model 6 and the coefficients from the inside-bark volume ratio table (table S5a) to find the proportion of total stem volume for both the 1-foot stump height and the 4.0-inch top diameter height:

$$R_1 = [1 - (1 - h_1/H)^\alpha]^\beta$$

$$R_1 = [1 - (1 - 1/65)^{2.466800456074}]^{0.842271677308} = 0.062976290396$$

$$R_m = [1 - (1 - h_m/H)^\alpha]^\beta$$

$$R_m = [1 - (1 - 56.72042843/65)^{2.466800456074}]^{0.842271677308} = 0.994774693648$$

where h_1 is stump height (1 foot), h_m is the merchantable height, R_1 is the proportion of volume to 1 foot, and R_m is the proportion of volume to the merchantable height.

Then, multiply the ratios by the already estimated total stem wood volume and subtract:

$$Vmer_{ib} Gross = (R_m \times Vtot_{ib} Gross) - (R_1 \times Vtot_{ib} Gross)$$

$$Vmer_{ib} Gross = (0.994774693648 \times 42.277832913225) - (0.062976290396 \times 42.277832913225)$$

$$= 39.394417201498$$

The same procedure can be used to estimate the merchantable outside-bark volume:

$$Vmer_{ob} Gross = (R_m \times Vtot_{ob} Gross) - (R_1 \times Vtot_{ob} Gross)$$

$$Vmer_{ob} Gross = (0.994774693648 \times 50.639401736611) - (0.062976290396 \times 50.639401736611)$$

$$= 47.185713679811$$

Merchantable stem bark volume is then calculated via subtraction:

$$Vmer_{bk} Gross = Vmer_{ob} Gross - Vmer_{ib} Gross$$

$$Vmer_{bk} Gross = 47.185713679811 - 39.394417201498$$

$$= 7.791296478313$$

Calculating cubic-foot volume in the sawlog portion of the stem (1-foot stump height to 7.0-inch top diameter for softwoods ($SPCD < 300$) and 9.0-inch top diameter for hardwoods ($SPCD \geq 300$)) proceeds similarly, with the sawlog height (h_s) being obtained from the following:

$$|9 - \{0.003504073654 \times 18.1^{1.821357964958} \times 65^{1.031766698583} / 0.005454154/65 \times 2.413673220682 \times 0.851093936311 \times (1 - h_s/65)^{(2.413673220682 - 1)} \times [1 - (1 - h_s/65)^{2.413673220682}]^{(0.851093936311 - 1)}\}^{0.5}|$$

Iterative minimization results in $h_s = 39.214128405$. The broken top actual height of 59 feet is greater than the predicted h_s for an intact top, so the sawlog top height is unaffected. To determine merchantable volume, use model 6 and the coefficients from the inside-bark volume ratio table (table S5a) to find the proportion of total stem volume for both the 1-foot stump height and the 9.0-inch top diameter height (R_s):

$$R_1 = [1 - (1 - h_1/H)^\alpha]^\beta$$

$$R_1 = [1 - (1 - 1/65)^{2.466800456074}]^{0.842271677308}$$

$$= 0.062976290396$$

$$R_s = [1 - (1 - h_s/H)^\alpha]^\beta$$

$$R_s = [1 - (1 - 39.214128405/65)^{2.466800456074}]^{0.842271677308}$$

$$= 0.913186793241$$

where h_1 is stump height (1 foot), h_s is the merchantable height, R_1 is the proportion of volume to 1 foot, and R_s is the proportion of volume to the sawlog height.

Then, multiply the ratios by the already estimated total stem wood volume and subtract:

$$Vsaw_{ib} Gross = (R_s \times Vtot_{ib} Gross) - (R_1 \times Vtot_{ib} Gross)$$

$$Vsaw_{ib} Gross = (0.913186793241 \times 42.277832913225) - (0.062976290396 \times 42.277832913225)$$

$$= 35.945057580350$$

The same procedure can be used to estimate the sawlog outside-bark volume:

$$Vsaw_{ob} Gross = (R_s \times Vtot_{ob} Gross) - (R_1 \times Vtot_{ob} Gross)$$

$$Vsaw_{ob} Gross = (0.913186793241 \times 50.639401736611) - (0.062976290396 \times 50.639401736611)$$

$$= 43.054151214254$$

Sawlog stem bark volume is then calculated via subtraction:

$$Vsaw_{bk} Gross = Vsaw_{ob} Gross - Vsaw_{ib} Gross$$

$$Vsaw_{bk} Gross = 43.054151214254 - 35.945057580350$$

$$= 7.109093633904$$

Stump volumes are estimated using the same volume ratio approach as used previously:

$$Vstump_{ob} Gross = R_1 \times Vtot_{ob} Gross$$

$$Vstump_{ob} Gross = 0.062976290396 \times 50.639401736611 = 3.189081669245$$

$$Vstump_{ib} Gross = R_1 \times Vtot_{ib} Gross$$

$$Vstump_{ib} Gross = 0.062976290396 \times 42.277832913225 = 2.662501082857$$

$$Vstump_{bk} Gross = Vstump_{ob} Gross - Vstump_{ib} Gross$$

$$Vstump_{bk} Gross = 3.189081669245 - 2.662501082857$$

$$= 0.526580586388$$

Typically, stem-top volumes are calculated by subtracting the other stem volume subcomponents from the total stem volume:

$$Vtop_{ob} Gross = Vtot_{ob} Gross - Vmer_{ob} Gross - Vstump_{ob} Gross$$

$$V_{top_{ob}Gross} = 50.639401736611 - 47.185713679811 - 3.189081669245 = 0.264606387555$$

$$V_{top_{ib}Gross} = V_{tot_{ib}Gross} - V_{mer_{ib}Gross} - V_{stump_{ib}Gross}$$

$$V_{top_{ib}Gross} = 42.277832913225 - 39.394417201498 - 2.662501082857 = 0.220914628870$$

$$V_{top_{bk}Gross} = V_{top_{ob}Gross} - V_{top_{ib}Gross}$$

$$V_{top_{bk}Gross} = 0.264606387555 - 0.220914628870 = 0.043691758685$$

In this case, the stem-top volume must account for the broken top height ($AH = 59$). Thus, determination of the missing top volume requires a ratio calculation to obtain the proportion of remaining stem volume R_b :

$$R_b = [1 - (1 - AH/H)^{\alpha}]^{\beta}$$

$$R_b = [1 - (1 - 59/65)^{2.466800456074}]^{0.842271677308} = 0.997639540140$$

Thus, the missing volume amount is calculated as follows:

$$V_{miss_{ob}Gross} = V_{tot_{ob}Gross} \times (1 - R_b)$$

$$V_{miss_{ob}Gross} = 50.639401736611 \times (1 - 0.997639540140) = 0.119532275134$$

$$V_{miss_{ib}Gross} = V_{tot_{ib}Gross} \times (1 - R_b)$$

$$V_{miss_{ib}Gross} = 42.277832913225 \times (1 - 0.997639540140) = 0.099795127559$$

$$V_{miss_{bk}Gross} = V_{miss_{ob}Gross} - V_{miss_{ib}Gross}$$

$$V_{miss_{bk}Gross} = 0.119532275134 - 0.099795127559 = 0.019737147575$$

Volumes of the remaining top wood (including the cull deduction) and bark are now defined as follows:

$$V_{top_{ib}Sound} = (V_{tot_{ib}Gross} - V_{mer_{ib}Gross} - V_{stump_{ib}Gross} - V_{miss_{ib}Gross}) \times (1 - CULL/100)$$

$$V_{top_{ib}Sound} = (42.277832913225 - 39.394415319923 - 2.662501082857 - 0.099795127559) \times (1 - 2/100) = 0.118698955228$$

$$V_{top_{ob}Sound} = V_{top_{ib}Sound} + V_{tot_{bk}Gross} \times (1 - R_m) - V_{miss_{bk}Gross}$$

$$V_{top_{ob}Sound} = 0.118698955228 + 8.361568823386 \times (1 - 0.994774693648) - 0.019737147575 = 0.142653566339$$

$$V_{top_{bk}Sound} = V_{top_{ob}Sound} - V_{top_{ib}Sound}$$

$$V_{top_{bk}Sound} = 0.142653566339 - 0.118698955228 = 0.023954611111$$

As shown above, $AH = 59$ occurs at a height above the 4.0-inch top diameter; therefore, sound volumes for the stump and merchantable stem only require deduction of cull:

$$V_{mer_{ib}Sound} = V_{mer_{ib}Gross} \times (1 - CULL/100)$$

$$V_{mer_{ib}Sound} = 39.394417201498 \times (1 - 2/100) = 38.606528857468$$

$$V_{stump_{ib}Sound} = V_{stump_{ib}Gross} \times (1 - CULL/100)$$

$$V_{stump_{ib}Sound} = 2.662501082857 \times (1 - 2/100) = 2.609251061200$$

Sound stem wood volume needed to account for the broken top and cull can be calculated as follows:

$$V_{tot_{ib}Sound} = (V_{mer_{ib}Sound} + V_{stump_{ib}Sound} + V_{top_{ib}Sound})$$

$$V_{tot_{ib}Sound} = (38.606528857468 + 2.609251061200 + 0.118698955228) = 41.334478873896$$

Other sound stem components are also calculated:

$$V_{tot_{ob}Sound} = V_{tot_{ib}Sound} + V_{tot_{bk}Gross} - V_{miss_{bk}Gross}$$

$$V_{tot_{ob}Sound} = 41.334478873896 + 8.361568823386 - 0.019737147575 = 49.676310549707$$

$$V_{tot_{bk}Sound} = V_{tot_{ob}Sound} - V_{tot_{ib}Sound}$$

$$V_{tot_{bk}Sound} = 49.676310549707 - 41.334478873896 = 8.341831675811$$

Total stem wood volume is next converted to total stem wood dry weight using the wood density value from the FIADB REF_SPECIES table. It is considered that some cull will be rotten wood, which would still contribute to the stem weight. As such, it is assumed the density of cull wood is reduced by the proportion for $DECAYCD = 3$ (see table 1; wood density proportion ($DensProp$) is 0.54 for hardwood species, 0.92 for softwood species) as reported by

Harmon et al. (2011) to obtain the reduced weight due to cull. The weight is also reduced to account for missing top wood:

$$Wtot_{ib} = Vtot_{ib}Gross \times WDSG \times 62.4$$

$$Wtot_{ib} = 42.277832913225 \times 0.60 \times 62.4 \\ = 1582.882064271140$$

$$Wtot_{ib}red = (Vtot_{ib}Gross - Vmiss_{ib}Gross) \\ \times [1 - CULL/100 \times (1 - DensProp)] \times WDSG \times 62.4$$

$$Wtot_{ib}red = (42.277832913225 - 0.099795127559) \\ \times [1 - 2/100 \times (1 - 0.54)] \times 0.60 \times 62.4 \\ = 1564.617593936140$$

Next, total stem bark weight can be estimated by consulting the stem bark weight coefficient table (table S6a), which specifies to use model 2 with the appropriate coefficients. Also, calculate the value for the proportion of the stem remaining (via R_b in this case):

$$Wtot_{bk} = a \times k^{(b-b1)} \times D^{b1} \times H^c$$

$$Wtot_{bk} = 0.013653815808 \times 11^{(2.255437355705 - 1.777569692133)} \\ \times 18.1^{1.777569692133} \times 65^{0.830992810735} = 237.154413924445$$

$$Wtot_{bk}red = (a \times k^{(b-b1)} \times D^{b1} \times H^c) \times R_b$$

$$Wtot_{bk}red = (0.013653815808 \times 11^{(2.255437355705 - 1.777569692133)} \\ \times 18.1^{1.777569692133} \times 65^{0.830992810735}) \\ \times 0.997639540140 = 236.594620449755$$

Consulting the branch weight coefficient table (table S7a), use model 1 with the appropriate coefficients to estimate total branch weight. Additionally, account for the branches remaining due to the broken top ($BranchRem$). The latter adjustment requires use of the observed crown ratio ($CR = 30$ percent) based on AH to standardize the CR value to H (CR_H) and then assess the proportion of the branch wood still intact:

$$Wbranch = a \times D^b \times H^c$$

$$Wbranch = 0.003795934624 \times 18.1^{2.337549205679} \\ \times 65^{1.30586951288} = 770.251512414918$$

$$CR_H = [H - AH \times (1 - CR)]/H$$

$$CR_H = [65 - 59 \times (1 - .30)]/65 = 0.364615384615$$

$$BranchRem = [(AH - H \times (1 - CR_H))/(H \times CR_H)]$$

$$BranchRem = [59 - 65 \times (1 - 0.364615384615)]/(65 \\ \times 0.364615384615) = 0.746835443038$$

$$Wbranchred = a \times D^b \times H^c \times BranchRem$$

$$Wbranchred = 0.003795934624 \times 18.1^{2.337549205679} \\ \times 65^{1.30586951288} \times 0.746835443038 \\ = 575.250923828242$$

Now, total aboveground biomass can be estimated by consulting the total biomass coefficient table (table S8a), which indicates the use of model 2 with the appropriate coefficients:

$$AGB_{Predicted} = a \times k^{(b-b1)} \times D^{b1} \times H^c$$

$$AGB_{Predicted} = 0.024470323124 \times 11^{(1.93799905037 - 1.886819489967)} \\ \times 18.1^{1.886819489967} \times 65^{1.403264431619} \\ = 2285.319903933610$$

Next, the three independently estimated components (stem wood weight, stem bark weight, and branch weight) need to be harmonized with the predicted total aboveground biomass. First, estimate a second total aboveground biomass by summing the three components:

$$AGB_{Component}red = Wtot_{ib}red + Wtot_{bk}red + Wbranchred$$

$$AGB_{Component}red = 1564.617593936140 \\ + 236.594620449755 + 575.250923828242 \\ = 2376.463138214140$$

Subsequently, $AGB_{Predicted}$ needs to be reduced to account for component rot and loss by calculating a reduction factor:

$$AGB_{Reduce} = AGB_{Component}red / (Wtot_{ib} + Wtot_{bk} + Wbranch)$$

$$AGB_{Reduce} = 2376.463138214140 / (1582.882064271140 \\ + 237.154413924445 + 770.251512414918) \\ = 0.917451320791$$

$$AGB_{Predicted}red = AGB_{Predicted} \times AGB_{Reduce}$$

$$AGB_{Predicted}red = 2285.319903933610 \\ \times 0.917451320791 = 2096.669764293850$$

$$AGB_{Diff} = AGB_{Predicted}red - AGB_{Component}red$$

$$AGB_{Diff} = 2096.669764293850 - 2376.463138214140 \\ = -279.793373920290$$

Next, proportionally distribute AGB_{Diff} across the components:

$$Wood_{Harmonized} = AGB_{Predicted}^{red} \times (Wtot_{ib}^{red} / AGB_{Component}^{red})$$

$$Wood_{Harmonized} = 2096.669764293850 \times (1564.6175939 / 36140 / 2376.463138214140) = 1380.407021315430$$

$$Bark_{Harmonized} = AGB_{Predicted}^{red} \times (Wtot_{bk}^{red} / AGB_{Component}^{red})$$

$$Bark_{Harmonized} = 2096.669764293850 \times (236.59462044 / 9755 / 2376.463138214140) = 208.739104392067$$

$$Branch_{Harmonized} = AGB_{Predicted}^{red} \times (Wbranch^{red} / AGB_{Component}^{red})$$

$$Branch_{Harmonized} = 2096.669764293850 \times (575.250923 / 828242 / 2376.463138214140) = 507.523638586351$$

At this point, all the individual tree components have been harmonized and are additive with the predicted total aboveground biomass estimate. The final biomass component that may be predicted is foliage weight. Foliage weight can be estimated by consulting the foliage weight coefficient table (table S9a), which stipulates the use of model 1 with the appropriate coefficients:

$$W_{foliage} = a \times D^b \times H^c$$

$$W_{foliage} = 0.03832401169 \times 18.1^{1.740655717258} \times 65^{0.500290321354} = 47.823281355886$$

As with branches, the weight of foliage needs to be reduced to account for remaining portion after the broken top loss:

$$FoliageRem = [AH - H \times (1 - CR_H)] / (H \times CR_H)$$

$$FoliageRem = [59 - 65 \times (1 - 0.364615384615)] / (65 \times 0.364615384615) = 0.746835443038$$

$$W_{foliagered} = W_{foliage} \times FoliageRem$$

$$W_{foliagered} = 47.823281355886 \times 0.746835443038 = 35.716121518954$$

New adjusted wood and bark densities are calculated using the harmonized total stem weights and the appropriate volume bases. As in previous examples, the wood and bark volume bases need to account for missing material due to a broken top but exclude any deductions for $CULL > 0$. Therefore,

the correct values are obtained by subtraction as $V_{tot_{ib}^{Gross}} - V_{miss_{ib}^{Gross}}$ and $V_{tot_{ob}^{Gross}} - V_{tot_{ib}^{Gross}} - V_{miss_{bk}^{Gross}}$ for wood and bark volume bases, respectively:

$$WDSG_{Adj} = Wood_{Harmonized} / (V_{tot_{ib}^{Gross}} - V_{miss_{ib}^{Gross}}) / 62.4$$

$$WDSG_{Adj} = 1380.407021315430 / (42.277832913225 - 0.099795127559) / 62.4 = 0.524488775540$$

Similarly, calculate an adjusted bark density using the harmonized total stem bark weight and the predicted total stem bark volume:

$$BKSG_{Adj} = Bark_{Harmonized} / (V_{tot_{ob}^{Gross}} - V_{tot_{ib}^{Gross}} - V_{miss_{bk}^{Gross}}) / 62.4$$

$$BKSG_{Adj} = 208.739104392067 / (50.639401736611 - 42.277832913225 - 0.019737147575) / 62.4 = 0.401012401713$$

Because the broken top does not affect the merchantable volume and cull is excluded, merchantable stem wood and bark weights can be determined as follows:

$$W_{mer_{ib}} = V_{mer_{ib}^{Gross}} \times WDSG_{Adj} \times 62.4$$

$$W_{mer_{ib}} = 39.394417201498 \times 0.524488775540 \times 62.4 = 1289.304409606240$$

$$W_{mer_{bk}} = V_{mer_{bk}^{Gross}} \times BKSG_{Adj} \times 62.4$$

$$W_{mer_{bk}} = 7.791296478313 \times 0.401012401713 \times 62.4 = 194.962966425323$$

Merchantable stem outside-bark weight is then calculated via addition:

$$W_{mer_{ob}} = W_{mer_{ib}} + W_{mer_{bk}}$$

$$W_{mer_{ob}} = 1289.304409606240 + 194.962966425323 = 1484.267376031560$$

Similarly, stump weights are calculated as follows:

$$W_{stump_{ib}} = V_{stump_{ib}^{Gross}} \times WDSG_{Adj} \times 62.4$$

$$W_{stump_{ib}} = 2.662501082857 \times 0.524488775540 \times 62.4 = 87.138600608067$$

$$W_{stump_{bk}} = V_{stump_{bk}^{Gross}} \times BKSG_{Adj} \times 62.4$$

$$W_{stump_{bk}} = 0.526580586388 \times 0.401012401713 \times 62.4 = 13.176717568116$$

$$Wstump_{ob} = Wstump_{ib} + Wstump_{bk}$$

$$Wstump_{ob} = 87.138600608067 + 13.176717568116 \\ = 100.315318176183$$

The component DRYBIO_TOP (dry biomass in the top and branches of the tree) is calculated using the following equation:

$$DRYBIO_TOP = AGB_{Predicted}^{red} - Wmer_{ob} - Wstump_{ob}$$

$$DRYBIO_TOP = 2096.669764293850 \\ - 1484.267376031560 - 100.315318176183 \\ = 512.087070086107$$

The carbon content (C) of the tree is obtained via multiplication by the appropriate C fraction for *SPCD* = 802 (table S10a):

$$C = AGB_{predicted}^{red} \times CF$$

$$C = 2096.669764293850 \times 0.495700000000 \\ = 1039.319202160460$$

The previous examples use trees with $D \geq 5.0$ inches, which implies a merchantable portion of the stem exists. It is assumed no merchantable volume is present for sapling-sized trees ($1.0 \leq D < 5.0$); however, total stem wood and bark volume components are present. Prediction of biomass (and subsequently carbon) for saplings proceeds in the same manner as for larger trees, with stem and branch components being harmonized with $AGB_{Predicted}$ and foliage biomass being obtained directly from the model. Readers desiring to implement the NSVB

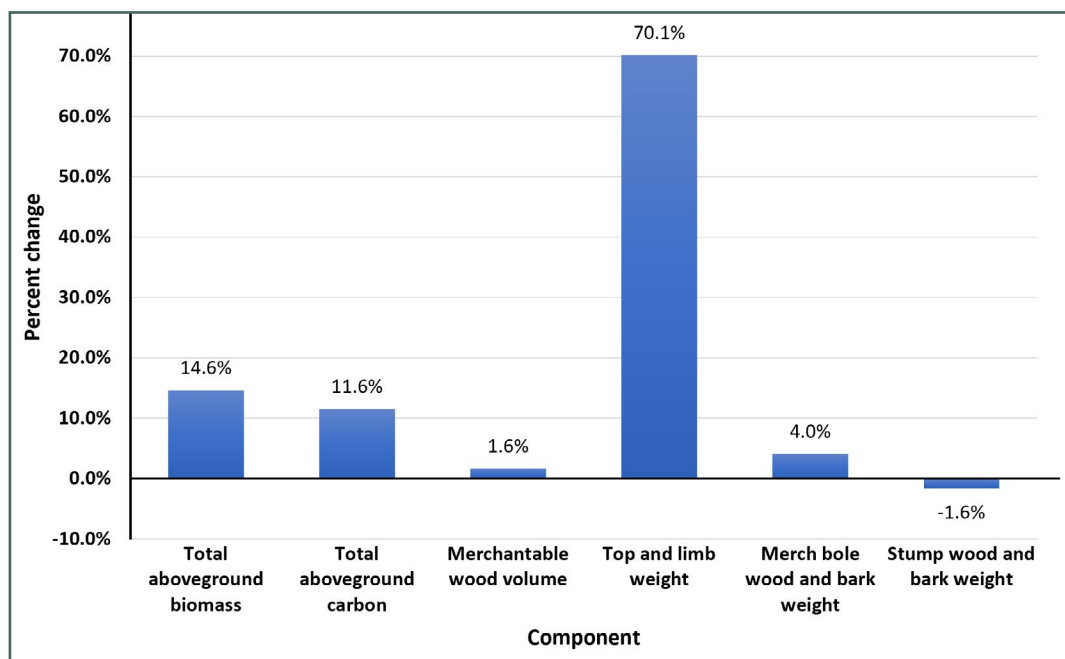
modeling system for their own applications can find resources via the Forest Service National Volume Estimator Library (NVEL): <https://www.fs.usda.gov/forestmanagement/products/measurement/volume/nvel/index.php>.

COMPARISONS WITH CURRENT METHODS

It is useful to examine the results in the context of current FIA tree volume models, the component ratio method (CRM) for biomass (Woodall et al. 2011), and the subsequent carbon values. Due to the nearly limitless number of potential comparisons, only broad-scale differences are illustrated within this publication; however, readers interested in making more customized evaluations are invited to access data tables where the previous and current values of volume and biomass components for individual trees are stored (<https://usfs-public.box.com/s/8xzlkg8epthml2l5idkd5laxs0uy5tbz>).

At the national scale, there were only minor differences in merchantable wood cubic-foot volume (1.6 percent), merchantable wood and bark weight (4.0 percent), and stump wood and bark weight (-1.6 percent). There was a large difference in weights of top/limbs (70.1 percent), which translates into increased tree aboveground biomass of 14.6 percent nationally. The change in biomass basis and implementation of new carbon fractions resulted in a national-scale change for carbon content of 11.6 percent (fig. 2).

Figure 2.—National-scale differences in volume, biomass, and carbon by component between national-scale volume and biomass (NSVB) and regionally implemented volume models/component ratio method (CRM).



Because the CRM is based on volume models implemented within FIA regions, another point of reference is at the regional level, where increases in tree aboveground biomass ranged from 528 to 1,676 million tons across all four regions (fig. 3a). Corresponding percentage increases were 15.7 percent, 7.2 percent, 20.0 percent, and 17.4 percent for Southern, Pacific Northwest, Rocky Mountain, and Northern regions, respectively. Merchantable wood volume increased in the Northern (19,380 million cubic feet; 5.1 percent) and Southern (13,708 million cubic feet; 3.2 percent) regions. In contrast, volume decreased in the Rocky Mountain (-4,918 million cubic feet; -2.4 percent) and Pacific Northwest (-5,679 cubic feet; -1.4 percent) regions (fig. 3b). At this broad spatial scale, these outcomes arise from many sources such as model prediction differences and the influence of relative tree species frequency.

A more detailed examination of biomass component contributions to the overall increases revealed that, in most cases, increases in biomass for the top/limbs component were a large driver of change in aboveground tree biomass for both hardwood and softwood species (fig. 4). It is particularly apparent when both stump and merchantable bole biomass changes are negative or only slightly positive, such that little overall change is observed unless the top/limbs are a primary contributor to the increase. The primary exceptions to this paradigm were for hardwood species in the Southern region and softwood species in the Northern region, where nontrivial increases in both stump and merchantable bole biomass reduced the proportional contribution of the top/limbs to total aboveground biomass. Although various factors may have influenced the systematic underprediction of top/limbs biomass using CRM, one likely cause is that top/limbs biomass is not directly modeled but instead is determined from the difference between total aboveground biomass and the sum of the other tree biomass components (see equation 9 in Woodall et al. 2011).

Within regions, State-level biomass and volume changes depend on various factors, including species composition, tree size class distributions, and differences in the volume and biomass model predictions. For biomass differences, the largest

increases (>25 percent) were present in Oklahoma, Indiana, Illinois, Missouri, and Michigan (fig. 5a). The CRM-based biomass estimates in these States substantially underpredicted values compared to the data used in the NSVB study. Changes in other States were generally positive, except for North Dakota and Washington, where slight decreases were present. The largest volume increases mimicked the biomass increases, most notably in Indiana, Illinois, Missouri, and Michigan (fig. 5b), due to the regional volume models tending to underpredict volume relative to NSVB models. Generally, 23 of the 48 conterminous U.S. States exhibited slight to moderate reductions in volume. Biomass differences for portions of the State of Alaska (fig. 6a) indicated increases in biomass of about 10 percent for coastal areas and 40 percent for interior areas. For volume differences (fig. 6b), a slight decrease in volume was noted in the coastal region of Alaska, whereas interior volume increases were >5 percent.

Comparisons with CRM aboveground biomass (AGB) predictions showed increases in AGB from NSVB models for most species, primarily due to the underestimation of the top/limbs component by CRM (table 2). The top 10 (by cubic-foot volume) eastern species (Southern and Northern FIA regions) all exhibited positive increases ranging from approximately 0.6 percent for loblolly pine to 27.9 percent for quaking aspen (*Populus tremuloides*). Results for the top 10 (by cubic-foot volume) western species (Rocky Mountain and Pacific Northwest regions) were more variable, ranging from about -6.5 percent for western hemlock (*Tsuga heterophylla*) to >25 percent for both subalpine fir (*Abies lasiocarpa*) and white fir (*A. concolor*). Differences between NSVB and regionally implemented volume models/CRM predictions exhibited increases due to NSVB of nearly 0.5 percent (sweetgum) to 10.5 percent (shortleaf pine) for the 10 most common eastern species. In contrast, changes in volume of the 10 primary western species were more mixed with differences ranging from -8.2 percent (Engelmann spruce) to 6.5 percent (white fir). The differences in volume and biomass shown in table 2 underscore the premise that changes between current FIA methods and the NSVB framework depend upon various factors, including species or species assemblages.

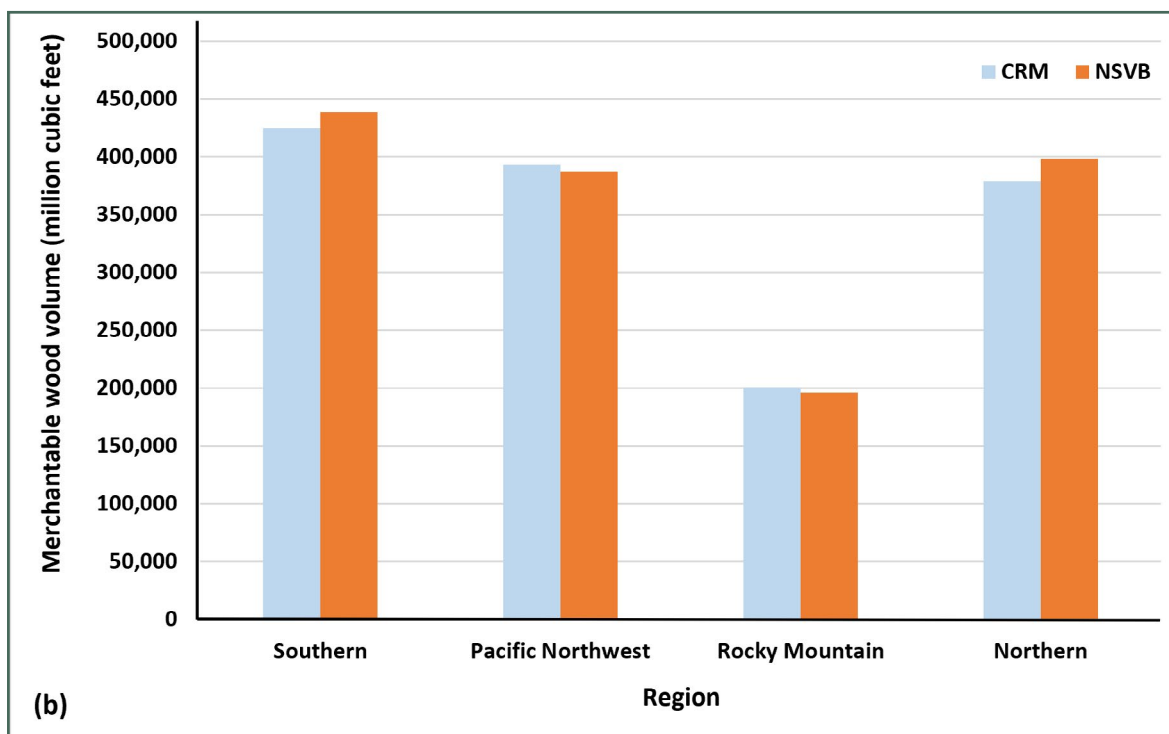
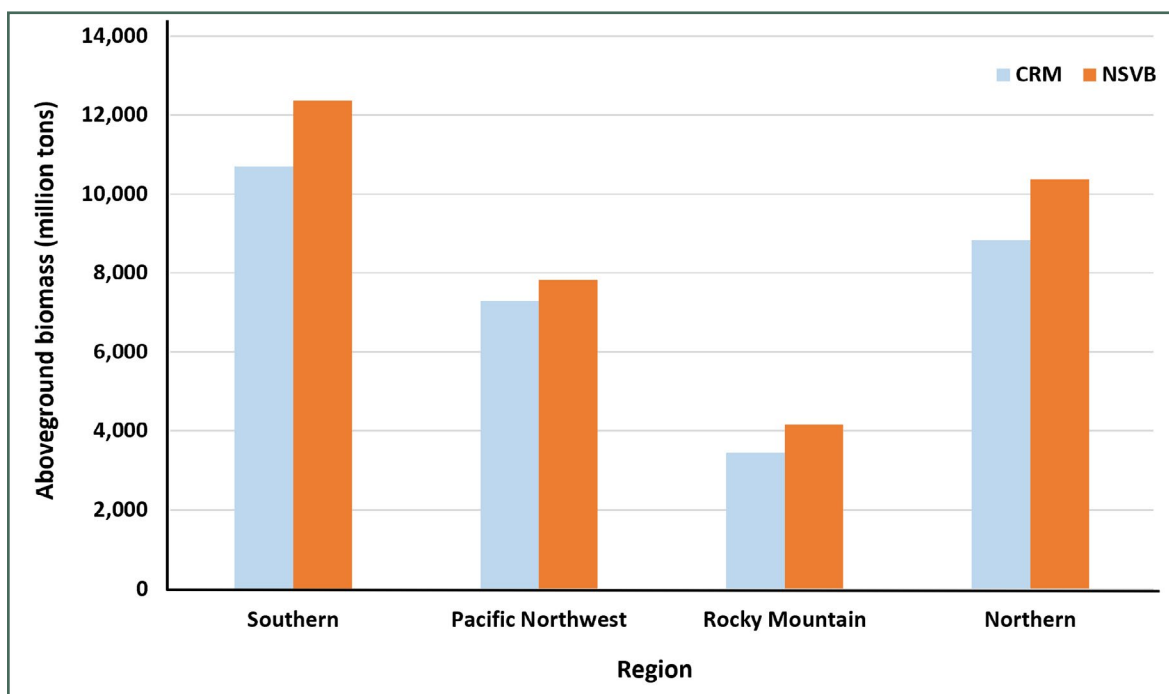


Figure 3.—Differences in (a) aboveground biomass and (b) merchantable wood volume between national-scale volume and biomass (NSVB) and regionally implemented volume models/component ratio method (CRM) by Forest Inventory and Analysis (FIA) region.

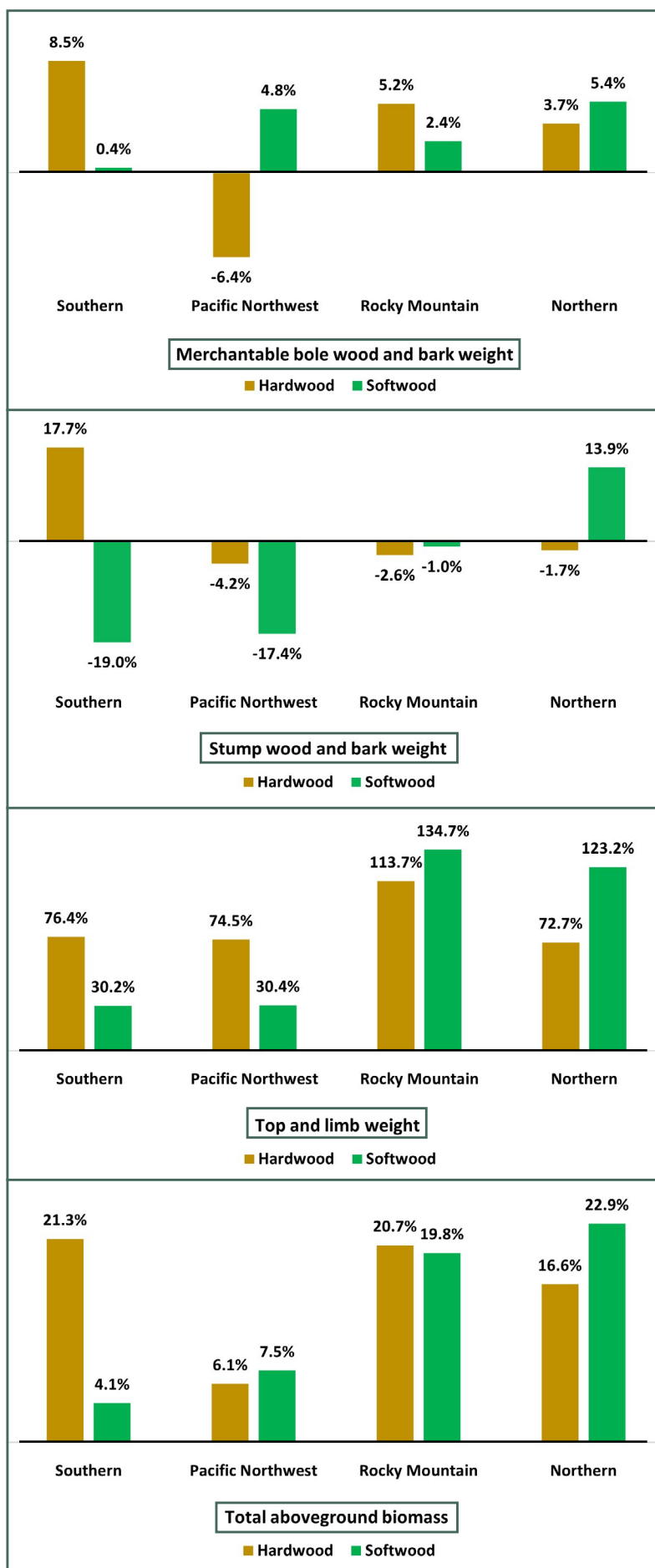


Figure 4.—Percent change in biomass between national-scale volume and biomass (NSVB) and regionally implemented volume models/ component ratio method (CRM) by component, Forest Inventory and Analysis (FIA) region, and hardwood or softwood species classification.

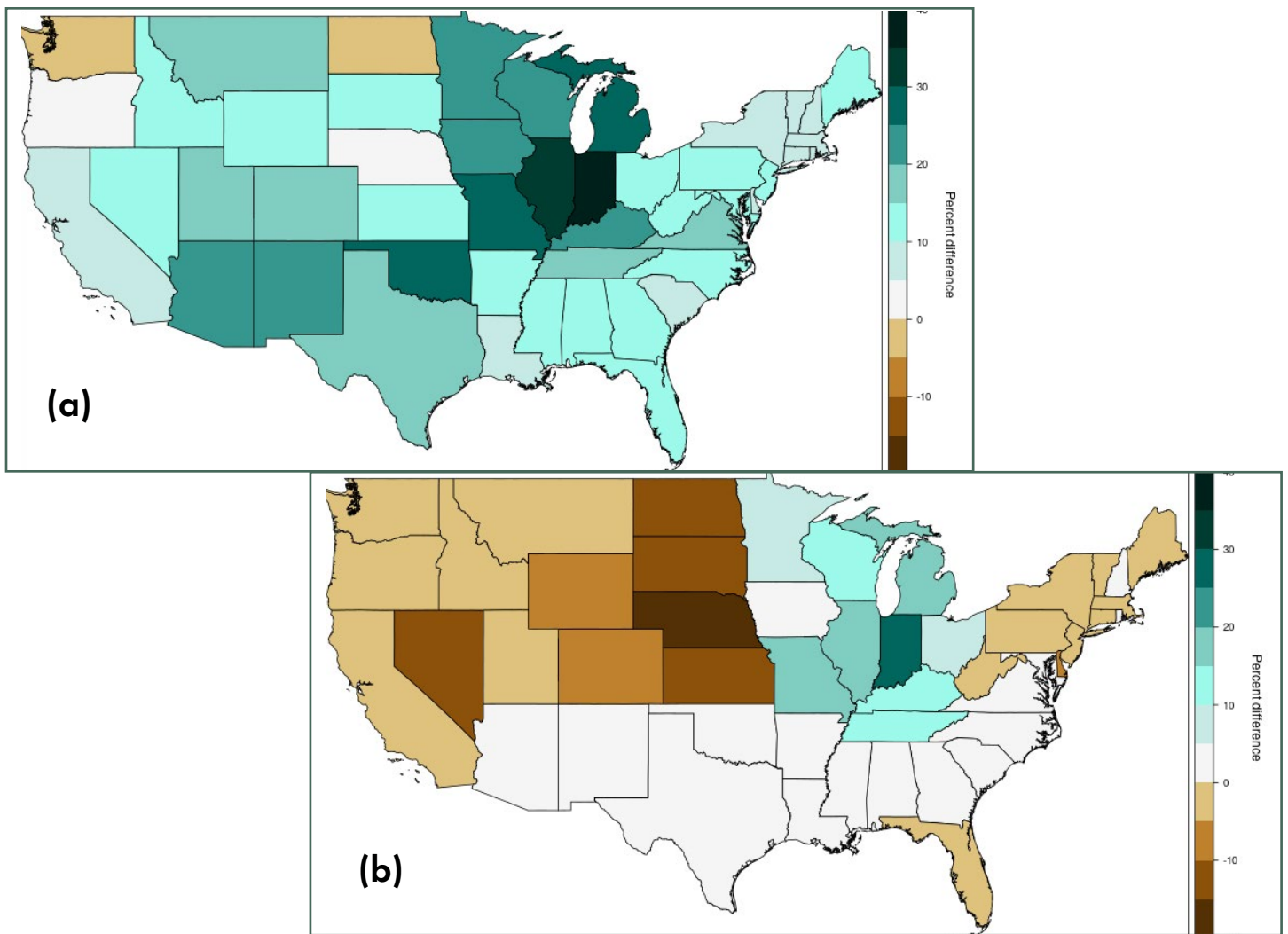


Figure 5.— Percent difference in (a) aboveground biomass and (b) merchantable volume between national-scale volume and biomass (NSVB) and regionally implemented volume models/component ratio method (CRM) for the 48 conterminous U.S. States.

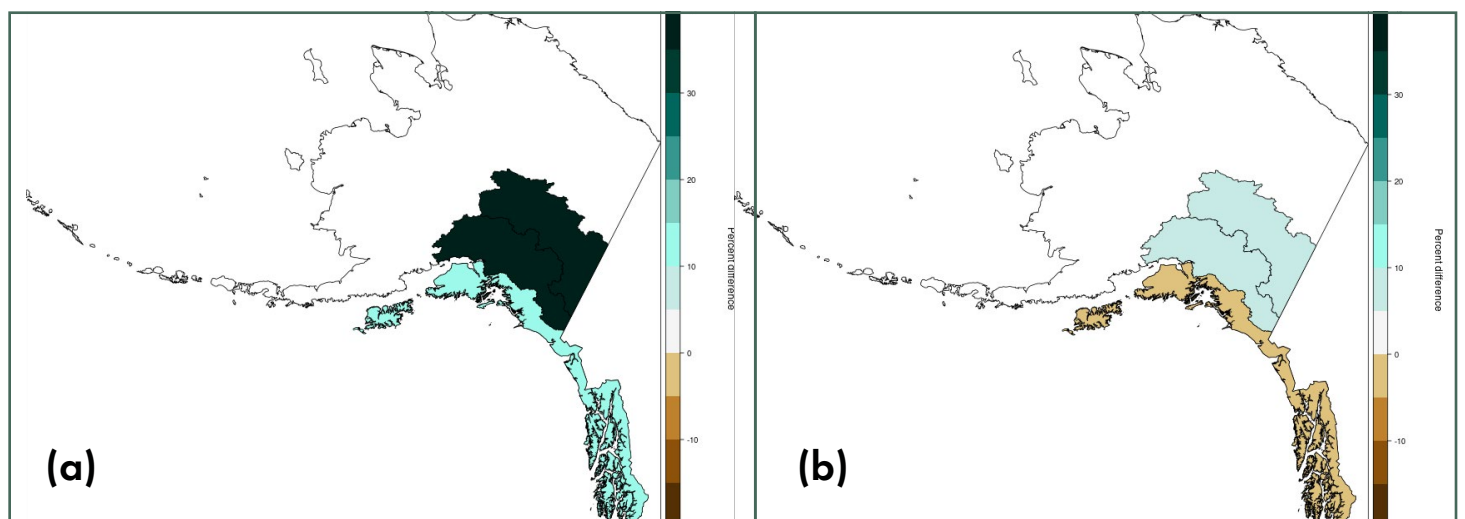


Figure 6.—Percent difference in (a) aboveground biomass and (b) merchantable volume between national-scale volume and biomass (NSVB) and regionally implemented volume models/component ratio method (CRM) for coastal Alaska and portions of interior Alaska completed to date.

Table 2.—Percent change in aboveground biomass and merchantable volume for the 10 most common species (by cubic-foot volume) in the Eastern (Southern and Northern Forest Inventory and Analysis (FIA) regions) and Western (Pacific Northwest and Rocky Mountain FIA regions) United States.

Region	Species common name	Species scientific name	Aboveground biomass (percent change)	Merchantable volume (percent change)
Eastern	Loblolly pine	<i>Pinus taeda</i>	0.59	4.51
Eastern	Red maple	<i>Acer rubrum</i>	20.11	1.30
Eastern	White oak	<i>Quercus alba</i>	24.07	10.27
Eastern	Sugar maple	<i>Acer saccharum</i>	16.22	8.89
Eastern	Sweetgum	<i>Liquidambar styraciflua</i>	5.83	0.45
Eastern	Northern red oak	<i>Quercus rubra</i>	16.04	4.79
Eastern	Yellow-poplar	<i>Liriodendron tulipifera</i>	10.81	3.80
Eastern	Quaking aspen	<i>Populus tremuloides</i>	27.89	5.69
Eastern	Shortleaf pine	<i>Pinus echinata</i>	14.27	10.50
Eastern	Eastern white pine	<i>Pinus strobus</i>	17.47	7.52
Western	Douglas-fir	<i>Pseudotsuga menziesii</i>	0.74	-0.95
Western	Lodgepole pine	<i>Pinus contorta</i>	18.90	-4.67
Western	Ponderosa pine	<i>Pinus ponderosa</i>	18.63	2.70
Western	Subalpine fir	<i>Abies lasiocarpa</i>	27.72	-7.68
Western	Western hemlock	<i>Tsuga heterophylla</i>	-6.47	-1.60
Western	Engelmann spruce	<i>Picea engelmannii</i>	12.83	-8.20
Western	White fir	<i>Abies concolor</i>	29.06	6.45
Western	Grand fir	<i>Abies grandis</i>	19.15	-0.20
Western	Red alder	<i>Alnus rubra</i>	8.12	-3.54
Western	Western redcedar	<i>Thuja plicata</i>	12.98	0.74

DISCUSSION

The NSVB modeling framework presents several potential advantages for the FIA Program and data users. First, tree volume predictions are greatly simplified because only five model specifications are used nationally, and the appropriate form and coefficients can be easily found for any species and ecoregion (*SPCD/DIVISION*) combination. Currently, FIA uses numerous model forms from a wide range of studies, largely depending on broad generalizations of species and location parameters. Second, NSVB eliminates administrative boundaries in favor of more sensible ecological definitions of spatial differences (fig. 1). With some exceptions, current FIA volume model applications are based on State or regional boundaries (Woodall et al. 2011) that often have no relevance to environmental gradients that may influence tree size, form, and growth. Third, the models are based on actual tree measurements instead of pseudo-data that underlie the biomass calculations in the current CRM implementation. Using raw empirical data also allows for accurate quantification of model uncertainty (as indicated in tables S12–S20) so that users can assess the reliability of the predictions. Fourth, the new models provide consistent behavior for all trees measured by FIA ($D \geq 1.0$ inch). In contrast, the CRM uses an ad hoc adjustment factor for saplings to help smooth predictions for trees crossing the $D = 5.0$ -inch threshold. Fifth, conversions from biomass to carbon content use species-specific carbon fractions, compared to a rudimentary 0.5 multiplier used for all trees in the CRM. In summary, taking a holistic national-scale approach resulted in substantial improvements to the tree volume, biomass, and carbon models compared to those currently used by the FIA Program.

While considerable effort was expended to develop a robust prediction framework, several challenges still remain to be addressed. Perhaps the most obvious is the inability to provide adequate coverage of all species occurring on FIA plots nationally. The two main contributing factors are land/tree accessibility and the time/cost necessary to locate specific trees that fill information gaps in spatial

distribution, species, and size (Frank et al. 2019). Regarding the former, a considerable amount of forest land is simply inaccessible due to private ownership or other constraints such as remote location or challenging topographical gradient. Even in accessible areas, it is often difficult to obtain permission to destructively sample large-sized trees that tend to have substantial economic or intrinsic value. More generally, locating uncommon trees often requires a substantial time and cost commitment due to rarity on the landscape. This requires tradeoffs in project execution to balance efficiency against the perceived knowledge gain of rare tree inclusion.

Other potential near-term refinements to the NSVB framework could include: (1) expansion to a broader range of species, e.g., woodland species (see FIADB REF_SPECIES table); (2) incorporation of nonlinear reductions in branches and foliage for broken top trees; (3) more advanced methods of weight deductions for rotten cull wood; and (4) improvements in wood density decay reductions and bark/branch weight loss reductions for dead trees (table 1). This research also serves as a foundation for prospective long-term advances in tree volume, biomass, and carbon prediction where enhancements that further explore ecological differences, provide alternative model formulations, and account for changing environmental conditions may be possible. Realization of these types of improvements depends on numerous factors, particularly the availability of requisite data at appropriate spatial and temporal scales.

CONCLUSIONS

The work presented herein provides transparent and fully documented methods for national-scale prediction of tree volume, biomass, and carbon attributes. Highlights of the new model framework include the following:

- Consistent modeling results for all trees having a diameter at breast height ≥ 1.0 inch.
- Considerable increases in analytical flexibility attained by using the entire tree stem as the basis and the ability to determine attribute values for any desired portion of the stem.
- Explicit separation of stem bark and wood attributes.
- Abandonment of the 0.5 carbon fraction for all species through formulation of more appropriate species-level carbon values.

The models were developed using the most comprehensive database ever assembled for the United States across a wide range of species, tree characteristics, and spatial domains. In this sense, the study results are the best available science to date.

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APPENDIX: SUPPLEMENTAL DATA FILES

The following tables (in CSV format) with statistics and data values used in the national-scale volume and biomass (NSVB) modeling framework for predicting tree volume, biomass, and carbon content across the United States are available at <https://doi.org/10.2737/WO-GTR-104-Supp1>.

Table S1a.—Coefficients for predicting total stem inside-bark wood cubic-foot volume based on FIA species code (SPCD).

Table S1b.—Coefficients for predicting total stem inside-bark wood cubic-foot volume based on Jenkins species group (JENKINS_SPGRPCD).

Table S2a.—Coefficients for predicting total stem bark cubic-foot volume based on FIA species code (SPCD).

Table S2b.—Coefficients for predicting total stem bark cubic-foot volume based on Jenkins species group (JENKINS_SPGRPCD).

Table S3a.—Coefficients for predicting total stem outside-bark cubic-foot volume based on FIA species code (SPCD).

Table S3b.—Coefficients for predicting total stem outside-bark cubic-foot volume based on Jenkins species group (JENKINS_SPGRPCD).

Table S4a.—Coefficients for predicting outside-bark volume ratio based on FIA species code (SPCD).

Table S4b.—Coefficients for predicting outside-bark volume ratio based on Jenkins species group (JENKINS_SPGRPCD).

Table S5a.—Coefficients for predicting inside-bark volume ratio based on FIA species code (SPCD).

Table S5b.—Coefficients for predicting inside-bark volume ratio based on Jenkins species group (JENKINS_SPGRPCD).

Table S6a.—Coefficients for predicting total stem bark biomass based on FIA species code (SPCD).

Table S6b.—Coefficients for predicting total stem bark biomass based on Jenkins species group (JENKINS_SPGRPCD).

Table S7a.—Coefficients for predicting total branch biomass based on FIA species code (SPCD).

Table S7b.—Coefficients for predicting total branch biomass based on Jenkins species group (JENKINS_SPGRPCD).

Table S8a.—Coefficients for predicting total tree biomass based on FIA species code (SPCD).

Table S8b.—Coefficients for predicting total tree biomass based on Jenkins species group (JENKINS_SPGRPCD).

Table S9a.—Coefficients for predicting total foliage biomass based on FIA species code (SPCD).

Table S9b.—Coefficients for predicting total foliage biomass based on Jenkins species group (JENKINS_SPGRPCD).

Table S10a.—Biomass percent carbon fraction for live trees based on FIA species code (SPCD).

Table S10b.—Biomass percent carbon fraction for dead trees based on hardwood vs. softwood classification and FIA decay code (*DECAYCD*).

Table S11.—Mean crown ratio (CR) percentage by ecoprovince and hardwood vs. softwood species classification.

Table S12.—Model fit statistics for volume and biomass components.

Table S13.—Model fit statistics for volume and biomass components by FIA species code (SPCD).

Table S14.—Model fit statistics for volume and biomass components by current FIA volume model region.

Table S15.—Model fit statistics for volume and biomass components by FIA species code (SPCD) and current FIA volume model region.

Table S16.—Model fit statistics for volume and biomass components by State.

Table S17.—Model fit statistics for volume and biomass components by FIA species code (SPCD) and State.

Table S18.—Model fit statistics for volume and biomass components by ecodivision.

Table S19.—Model fit statistics for volume and biomass components by FIA species code (SPCD) and ecodivision.

Table S20.—Model fit statistics for volume and biomass components by tree diameter class.

