



Terminal value: A crucial and yet often forgotten element in timber harvest scheduling and timberland valuation

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ABSTRACT

A forest investment's returns are generated from three sources: the land's gain in value, the timber's growth in size and product class improvement, and the timber price change. Land appreciation is rapidly leading to an inverse relationship with tenure. This phenomenon has turned what was once an academic exercise of land appraisal into a practical one that incorporates the asset's terminal value. We found that failing to account for the terminal value can lead to sizable differences in forest value, although those differences diminish with increasing planning time horizons. The findings can be of use to those who conduct appraisal work for larger timberland owners because (1) land held under short holding times has offered an increasingly large share of timber supply, (2) classical models fail to capture the complexity of management decisions under this regime, and (3) "short term" holders face a terminal value risk, which we evaluate through scenario analysis.

1. Introduction

Timberland asset are popular alternative investments among institutional and independent investors because of their desirable characteristics such as (i) hedging against inflation (Baral and Mei, 2023; Wan et al., 2013), (ii) low correlation with other financial assets (Cascio and Clutter, 2008; Redmon and Cabbage, 1988), and (iii) high risk-adjusted returns (Restrepo et al., 2020; Wan et al., 2015). In addition, over the last few decades, timberland assets have become potential carbon and Environmental, Social, and Governance (ESG) investments, aligning with the growing emphasis on sustainable and responsible investing (The Economist, 2023).

To achieve the expected returns of timberland investments in a portfolio, forestland managers must regularly evaluate and perform harvest scheduling through a complex process involving multiple interactions between biological and economic variables. This process combines multiple management regimes, operational costs, and timber markets to maximize the forest value subject to regional constraints such as physical (e.g., timber production), economic (e.g., demand of timber), and political (environmental regulation) (e.g., Diaz-Balteiro et al., 2009;

González-González et al., 2022; Jumppanen et al., 2003). A dubious evaluation of the harvest schedule can misallocate resources and mislead investors and landowners. Given the large scale of the forest operations (e.g., the top ten private landowners in the US and Canada own 10.6 million acres – FORISK, 2021), even marginal deviation from the optimal can lead to millions of dollars in financial losses.

While various aspects of forestland value and optimal harvest decisions have been extensively addressed in the literature on harvest scheduling modeling (e.g., Chang and Zhang, 2023; Yu et al., 2023; Singleton and Straka, 2010; Straka, 2007), one area that has received relatively less attention is the value of a timberland asset upon liquidation, or the terminal value (Clutter et al., 1983). The terminal value is a crucial parameter in the harvesting schedule because it defines the ecological structure (e.g., age distribution or species distributions) and the financial value at the terminal period of the planning horizon (McTague and Oppenheimer, 2015). Therefore, as in McIntosh & Zhang (2024), we suggest that defining the terminal value with a solid microeconomic foundation, such as in Faustmann Formula, is essential to guarantee the sustainable use of forest resources.

This paper aims to investigate the impact of the terminal value on the

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final forest value through a harvest schedule model; we simulated the decision process of a profit maximizer landowner in a pine plantation under different planning horizon lengths. Our results reveal that the terminal value's impact on the forest value varied from -48.3% to -1.4% , depending on the planning horizon and constraints.

2. Literature review

Studies on forest valuation are far from new. Notably, Faustmann (1849) laid the groundwork for this field by demonstrating the theory and analytical application of the Land Expectation Value in forest management; his work has led to hundreds of publications addressing the forest optimal rotation problem (Chang, 2001; McIntosh & Zhang, 2024; Newman, 2002) and have shaped the microeconomic principles of timberland value (Zhang, 2021). These studies have not only contributed to our understanding of sustainable forest management but have also paved the way for its implementation; however, most of their focus was on unconstrained optimization at the stand level, which is not necessarily applicable to forest-level management.

At the forest level, the forest value has been estimated through well-established and defined mathematical optimization techniques (Bell, 1977; Curtis, 1962; Nautiyal and Pearce, 1967; Singleton and Straka, 2010; Kaya et al., 2016). At the end of the planning horizon, these models normally add a terminal value to simulate a perpetual management regime (Clutter et al., 1983). The application of the terminal value follows the concept that the harvest schedule converges to the optimal rotation age in the long run (Heaps, 2015; Mitra and Wan, 1986; Tahvonen and Viitala, 2006). Previous literature has indicated the need to use terminal value in discrete time control problems, like harvest scheduling (Standiford and Howitt, 1992), because it renders the problem atemporal (Brooks et al., 1974; Wear and Parks, 1994; McTague and Oppenheimer, 2015).

Although the concept of terminal value is not new, it is unclear how different assumptions about its value would impact the final forest value and the harvest schedule. Even methods to calculate the terminal value are not well established. For instance, Clutter et al. (1983) proposed two solutions to add at the end of the planning period: (i) assuming a hypothetical liquidation with an expected sale value, which can be subjective and vary based on individual expectations, and (ii) calculating the forest value based on capital budget approach, thereby establishing a link between forest management and economic theory. In this paper, we employ the second approach proposed by Clutter et al. (1983) and compare it to other alternative values like a positive and negative deviation of 10% (to mimic some variation of the actual value) and not using terminal value at all.

Our study contributes to the existing literature in several ways: (1) We develop a straightforward harvest schedule model that can be replicated in various mathematical optimization software platforms; (2) We provide a detailed step-by-step guide for incorporating the terminal value at the terminal period; and (3) We analytically and numerically examine the impact of the terminal value on forest value. This research not only serves as a valuable resource for landowners and investors engaged in forest valuation but also contributes to advancing the understanding of the complexities inherent in this critical aspect of sustainable forest management.

3. Microeconomic foundation

To simplify the theoretical analysis, we focus on the variable of interest (the area of the unharvested inventory at the terminal period) and generalize the other variables and constraints. A profit maximizer landowner would have the following utility function:

$$\begin{aligned} \max_{A_{i,t}, H_{i,t}, R_i} & \underbrace{\sum_{t=0}^{T-1} \sum_i f_{i,t}(A_{i,t}, H_{i,t}, R_i, p_k, q_{i,k}, C_t) / (1 + \delta)^t}_{(A)} \\ & + \underbrace{\left[\sum_k \sum_i p_k q_{i,k} H_{i,t=T} / (1 + \delta)^{t=T} \right] - C_{t=T} / (1 + \delta)^{t=T}}_{(B)} \\ & + \underbrace{\sum_i F_i R_i / (1 + \delta)^{t=T}}_{(C)} \end{aligned} \quad (1)$$

Subject to

$$A_{i,t=T} - H_{i,t=T} = R_i \quad \forall i \in [0, I] \quad (2)$$

$$R_i \geq 0 \quad \forall i \in [0, I] \quad (3)$$

$$g(A_{i,t}, H_{i,t}, R_i) \geq 0 \quad \forall i \in [0, I] \text{ and } t \in [0, T] \quad (4)$$

where i is the forest age ranging from zero (bare land) to I , t is the period from the present (zero) to the terminal period T . Notice that the maximum age I is exogenous and is not necessarily equal to the optimal rotation age. The p_k are the timber prices per ton by product k , $q_{i,k}$ is the volume of timber per acre at age i per product k , C_t is the cost (\$) at period t , $f_{i,t}(\cdot)$ is a function that represents the generalization of all previous activities. $A_{i,t}$ is the area (acres) at age i and period t , $H_{i,t}$ is the area (acres) of age i harvested in period t , F_i is the forest value (\$) per acre of each age i , R_i is the residual area (acres) at terminal period T . The residual area is the difference between the area available ($A_{i,t=T}$) and the area harvested ($H_{i,t=T}$) at period T as in Eq. (2). We broke down the profit (Eq. (1)) into two sections: Section (A) represents the NPV of all operations in a harvest schedule when t is less than the terminal period T . Section (B) shows the profit at the terminal period T represented just by the timber market ($p_k q_{i,k} H_{i,t=T}$) and a periodic cost ($C_{t=T}$). Last, section (C) shows the terminal value, or the forest value (F_i) for each age i multiplied by the area residual (R_i) at the terminal period T , and discounted to the present by the factor $(1 + \delta)^{t=T}$. Eq. (3) guarantees the residual inventory area is non-negative. Eq. (4) represents all other possible constraints in a harvest schedule model. The rationale that $f_{i,t}(A_{i,t}, H_{i,t}, R_i, p_k q_{i,k}, C_t)$ and $g(A_{i,t}, H_{i,t}, R_i)$ are also a function of R_i across the entire planning horizon is discussed in Appendix.

Substituting the harvested area by ($H_{i,t=T} = A_{i,t=T} - R_i$) from Eq. (2) in Eq. (1), the Lagrangian formulation ($\Omega(A_{i,t}, H_{i,t}, R_i, p_k, q_{i,k}, C_t, \lambda)$) of Eqs. (1), (3) and (4) is:

$$\begin{aligned} \Omega(A_{i,t}, H_{i,t}, R_i, p_k, q_{i,k}, C_t, \lambda) = & \sum_{t=0}^{T-1} \sum_i f_{i,t}(A_{i,t}, H_{i,t}, R_i, p_k, q_{i,k}, C_t) / (1 + \delta)^t \\ & + \sum_k \sum_i p_k q_{i,k} (A_{i,t=T} - R_i) / (1 + \delta)^{t=T} \\ & - C_{t=T} / (1 + \delta)^{t=T} \\ & + \sum_i F_i R_i / (1 + \delta)^{t=T} \\ & + \sum_i \lambda_i^a (R_i) + \lambda g(A_{i,t}, H_{i,t}, R_i) \end{aligned} \quad (5)$$

where the Lagrange multiplier λ_i^a is the shadow price of imposing $R_i = 0$ for each age i , or the "cost" of not adding one unit of the terminal value to a harvest schedule.

We derive the value of λ_i^a by taking the first-order condition of Eq. (1) with respect to R_i :

$$\lambda_i^a = \underbrace{\sum_{t=T-1}^I \sum_i \frac{\partial}{\partial R_i} f_{it}(A_{i,t}, H_{i,t}, R_i, p_k, q_{i,k}, C_i)}_{(A)} - \underbrace{\sum_k \sum_i p_k q_{i,k}}_{(B)} \bigg/ (1 + \delta)^{t=T} + \underbrace{F_i / (1 + \delta)^{t=T}}_{(C)} + \underbrace{\frac{\partial}{\partial R_i} \lambda g(A_{i,t}, H_{i,t}, R_i)}_{(D)} \quad (6)$$

Notice that the impact of constraint $R_i = 0$ in Eq. (6) depends on the magnitude of the marginal effects from previous returns (Section A), the present value of the marginal revenue at $t = T$ (Section B), the terminal value of the forest (Section C), the shadow prices (λ), and the marginal effect of R_i on the other constraints ($\frac{\partial}{\partial R_i} g(A_{i,t}, H_{i,t}, R_i)$) (Section D).

4. Methods

We start our analysis by defining the estimation of the forest and terminal values following Clutter et al. (1983). The forest value estimated here is based on a Discounted Cash Flow (DCF) analysis. This method of estimating the forest value depends on three rules that are directly related to the current age class of a forest and its optimal rotation age. These rules are (1) bare land value, (2) immature timber (younger than the optimal rotation age), and (3) mature timber (at or older than the optimal rotation).

4.1. Bare land value

The bare land value is the Land Expectation Value (LEV) (Faustmann, 1849):

$$L_{i^*} = \frac{\sum_k p_k q_{i^*,k} - \sum_{i=0}^{i^*} C_i (1 + \delta)^{i^* - i}}{(1 + \delta)^{i^*} - 1} \quad (7)$$

where, i^* is the age when the rotation is optimal (maximum LEV), L_{i^*} is the LEV for rotation age i^* and, $p_k q_{i^*,k}$ is the revenue at age i^* , and $\sum_{i=0}^{i^*} C_i (1 + \delta)^{i^* - i}$ is the cumulative cost from age zero to i^* , and δ is the discount rate. L_{i^*} represents the value of the forest when it is bare land, or $F_{i=0}$.

4.2. Immature timber ($0 < i < i^*$)

The value of a stand younger than the optimal rotation i^* is based on the potential income in future years and the Land Expectation Value (L_{i^*}). Mathematically, we can estimate it as in:

$$F_i = \frac{\sum_k p_k q_{i,k} - \sum_{i=0}^{i^*} C_i (1 + \delta)^{i^* - i} + L_{i^*}}{(1 + \delta)^{(i^* - i)}} \quad (8)$$

where F_i is the Forest Value per acre at age i . Eq. (8) estimates the total value of forestland; L_{i^*} shows that the optimal silvicultural treatment will repeat in perpetuity. Notice that the value of the land and timber is computed jointly; to extract only the timber's value, we subtract L_{i^*} from F_i .

4.3. Mature timber ($i \geq i^*$)

The value of forestland that is equal to or older than the optimal rotation i^* is straightforward:

$$F_i = \sum_k p_k q_{i,k} - C_i + L_{i^*} \quad (9)$$

Eq. (9) is simply calculating the current profits from timber

production ($\sum_k p_k q_{i,k} - C_i$) plus the LEV at optimal rotation i^* .

4.4. Terminal value

Once we estimate the forest value F_i , we calculate the terminal value as:

$$Z_i = \frac{F_i}{(1 + \delta)^{t=T}} \quad (10)$$

where Z_i equals the terminal value per acre of a forest at age i , T is the terminal period. F_i is the Forest Value calculated from Eqs. (7) and (8) or (9).

4.5. Harvest schedule

Eqs. (7) to (9) optimize the forest value per unit of area at stand level and don't account for possible constraints faced by landowners such as timber demand, operational capacity, etc. We now build a harvest schedule model to simulate how the terminal value can affect the returns of a profit-maximizing landowner at the forest level. The model is described in Eqs. (11) to (19); all the notations were already introduced except the annual costs (ac_t) and the initial forest area by age i (a_i).

$$\begin{aligned} & \max_{A_{i,t}, H_{i,t}, R_i} \sum_k \sum_i \sum_t \frac{p_k q_{i,k} H_{i,t}}{(1 + \delta)^t} \\ & - \sum_i \sum_t A_{i,t} (C_i + ac_t) \bigg/ (1 + \delta)^t \\ & + \sum_i \frac{F_i R_i}{(1 + \delta)^{t=T}} \end{aligned} \quad (11)$$

Subjected to:

$$A_{i,t} \geq H_{i,t} \quad \forall i \in [0, I] \text{ and } t \in [0, T] \quad (12)$$

$$A_{i,t=0} = a_i \quad \forall i \in [0, I] \quad (13)$$

$$A_{i=0,t} = \sum_i H_{i,t-1} \quad \forall t \in (0, T] \quad (14)$$

$$A_{i,t} = A_{i-1,t-1} - H_{i-1,t-1} \quad \forall i \in (0, I) \text{ and } t \in (0, T] \quad (15)$$

$$A_{i=L,t} = A_{i=L-1,t-1} - H_{i=L-1,t-1} + A_{i=L,t-1} - H_{i=L,t-1} \quad \forall t \in (0, T] \quad (16)$$

$$A_{i,t=T} - H_{i,t=T} = R_i \quad \forall i \in [0, I] \quad (17)$$

$$H_{i,t} = 0 \quad \forall i \in [0, merch) \text{ and } t \in [0, T] \quad (18)$$

$$A_{i,t} \geq 0; H_{i,t} \geq 0 \quad \forall i \in [0, I] \text{ and } t \in [0, T] \quad (19)$$

$$R_i \geq 0 \quad \forall i \in [0, I] \quad (20)$$

where Eq. (11) is the objective function, in which the first term is the present value of the revenues (timber volume multiplied by prices), the second term is the present value of costs, and the last term is the terminal value at $t = T$. Eq. (12) guarantees the area harvested is less than or equal to the area stock. Eq. (13) sets the initial forest condition at $t = 0$; Eqs. (14), (15), and (16) are the transition functions between age classes; Eq. (14) ensures the area harvested at any age i in period $t - 1$ is planted at period t . Eq. (15) impose that the area not harvested in $t - 1$ and age $i - 1$ moves to age i at period t ; similarly, Eq. (16) guarantees the area stock accumulates at the oldest age (I). Eq. (17) calculates the area after harvesting (area stock minus area harvested) at period T . Eq. (18) restrict the harvest area to merchantable ages (*merch* - here ten years old). Eqs. (19) and (20) are the non-negative constraints.

The initial harvest schedule model is limited only by the current and

projected timber inventory; however, in practice, there are many other constraints linked to timber supply, including, but not limited to, operational capacity and local environmental regulations. Given the large number of possible constraints, we added a wood flow constraint by imposing that the volume stock of timber at period t should fluctuate $\pm 15\%$ around the volume stock from period $t - 1$.

$$S_t \leq S_{t-1} \times (1 + 0.15) \quad \forall t \in (0, T] \quad (21)$$

$$S_t \geq S_{t-1} \times (1 - 0.15) \quad \forall t \in (0, T] \quad (22)$$

where, $S_t = \sum_k \sum_i q_{ik} A_{it}$. These constraints will force some harvesting regulations and a more realistic timber supply.

5. Data

5.1. Yield curves, prices and costs, and initial forest inventory

All yield curves were produced by PTAEDA (Burkhart, 2008); we simulated a typical pine plantation in the U.S. South with 8×8 feet spacing - 681 trees per acre (TPA) - and a Site Index of 75 ft (Fig. 1). There are three merchantable products - pulpwood (P.W.), chip and saw (CNS), and sawtimber (S.T.) - and one non-merchantable (biomass).¹

All costs except seedlings were sourced from Costs & Trends of Southern Forestry Practices 2020 (Maggard and Barlow, 2019). Due to unusual inflationary effects over the prior two years (2021 and 2022), we used a Consumer Price Index (CPI) adjustment to update Maggard and Barlow to 2022.² Seedling costs were 7.5 cents per seedling, and they were sourced from published state nursery costs from Georgia (Georgia Forestry Commission, 2022) and Arkansas (Arkansas Department of Agriculture, 2023). Establishment costs included chemical treatment, planting costs, and seedling costs of \$234.66 for $8' \times 8'$ spacing arrangements (Table 1). Annual management expenses were \$9.55 per/acre, adjusted for inflation from a 2017 estimate (Forest2-Market, 2018). Timber prices were taken from Timber Mart South (2022) and represent Southside averages for 2022. For simplicity, both costs and prices were held constant over time, and a 4% real discount rate was applied.³

To mimic a realistic forest structure, we assume the initial forest conditions follow the same age class distribution as the state of Mississippi according to the USDA Forest Inventory Analysis (FIA) (See Fig. 2. (Stanke et al., 2020)). We selected only pine (*Pinus taeda*) plantations owned by private entities. All results are provided on a per-acre basis.

5.2. Scenarios

We combined five planning horizons (5, 15, 30, 60, and 100 years) and four assumptions about terminal value based on Eqs. (7) to (9): (i) No Terminal Value, (ii) -10% the Terminal Value, (iii) Terminal Value, and (iv) $+10\%$ the Terminal Value. In combination, there are 20 scenarios for restrictions assumption, unconstrained and constrained by biological restriction.

6. Results

We estimated the effect of the terminal value on the forest value and harvest schedule dynamics by calculating (1) the forest and terminal value at stand levels, (2) the forest inventory dynamics at the forest level, (3) age class distribution at the terminal period and (4) the total

effect of the terminal value in the forest value. For results in (1), we used Eqs. (7) to (10), and for variables (2), (3), and (4) we used the Equations in the harvest schedule (HS1) to (HS10), including scenarios with (E1 - A) and (E1 - B).

6.1. Forest value and terminal value at stand level

Fig. 3 illustrates the forest and terminal values per acre for each age class (from zero to 80 years old) and terminal period ($T = 5, T = 15, T = 30, T = 60$, and $T = 100$) at a 4% discount rate using Eqs. (7) to (10). These values are interpreted as follows: a 40-year-old pine plantation with an 8×8 ft. spacing is worth \$3378 per acre under projected costs, timber price, and yield assumption. If a land-owner possessed the same 40-year-old forest 30 years from now, its terminal value would be $\frac{\$3,378}{(1+0.04)^{30}} = \1041 per acre. The same logic follows a bare land worth \$592.25 per acre today, and its terminal value is $\$486 \left(\frac{\$592}{(1+0.04)^5} \right)$ per acre 5 years from now. Fig. 3 shows these values for every age and terminal period.

As anticipated, the forest value exhibited diminishing marginal returns as the forest matured. The abrupt growth in value between age zero and one is attributed to the transition from bare land to a planted forest, while the subsequent decreasing rate of value growth after reaching the peak is linked to slower biological growth and discount rates. The Land Expectation Value (LEV) of an 8×8 ft pine plantation was \$592.25 at a 4% discount rate; this value is like the ones practiced in other studies (Cubbage et al., 2020), validating our assumptions.

Two results demonstrate the impact of the planning horizon on the terminal value. First, as the terminal period (T) increases, the terminal value decreases due to the discount factor, causing the curves to shift downward. For example, the difference between bare land values at $T = 5$ (\$486.79 per acre) and $T = 100$ (\$11.72 per acre) is \$475.07 per acre. Second, as the terminal period (T) increases, the difference between the forest value across age classes decreases. In the extreme case of $T = 100$, the difference between the highest value (\$69.01 per acre) and the minimum value (\$11.72 per acre) is \$57.29 per acre, while when $T = 5$, the highest and lowest value, \$2864.80 per acre and \$486.80 per acre respectively, differs \$2378 per acre. These results at the stand level provide essential information to forest managers. However, they have neither temporal nor spatial interaction with other stands like those in forest-level management. In the following sub-topics, we show the results at forest-level harvest schedule from Eqs. (11) to (19) in the unconstrained model, HS1 to HS9, and E1-A and E1-B.

6.2. Forest inventory dynamics

The presence of the terminal value in forest planning impacted the harvest decision substantially. Fig. 4A and B show, respectively, the share of the timberland growing stock with respect to the initial inventory ($\frac{\text{Inventory at } t}{\text{Inventory at } t=0}$) per year and at the terminal year. While Fig. 5A and B display the volume harvested relative to the inventory ($\frac{\text{Volume harvested at } t}{\text{Inventory at } t}$) per year and average, respectively.

To investigate the influence of the terminal value on forest dynamics, a direct comparison between the unconstrained and wood flow control models is challenging due to the inherent constraints. Despite these differences, both models exhibit parallel trends characterized by volume accumulation until reaching the optimal rotation age, succeeded by clearcutting (Fig. 4A and B). Notably, the constrained model displays more frequent harvest activities across planning horizons, a strategic approach aimed at maximizing net present value while holding wood flow restrictions (Fig. 5A and B).

The impact of the terminal value becomes evident under the not-constrained and constrained models. In scenarios with No TV and the unconstrained model, forest inventory accumulates until the terminal period, culminating in the final clearcut to optimize harvested volume

¹ DBH by product: Biomass: $< 6"$, Pulpwood: $6-9"$, Chip-n-saw: $10-13"$, Sawtimber: $\geq 14"$

² The Consumer Price Index for All Urban Consumers: All Items in U.S. City Average, Index 1982-1984 = 100, Monthly, Seasonally Adjusted

³ 4% discount rate is a common practice in the US South (Cubbage et al., 2020)

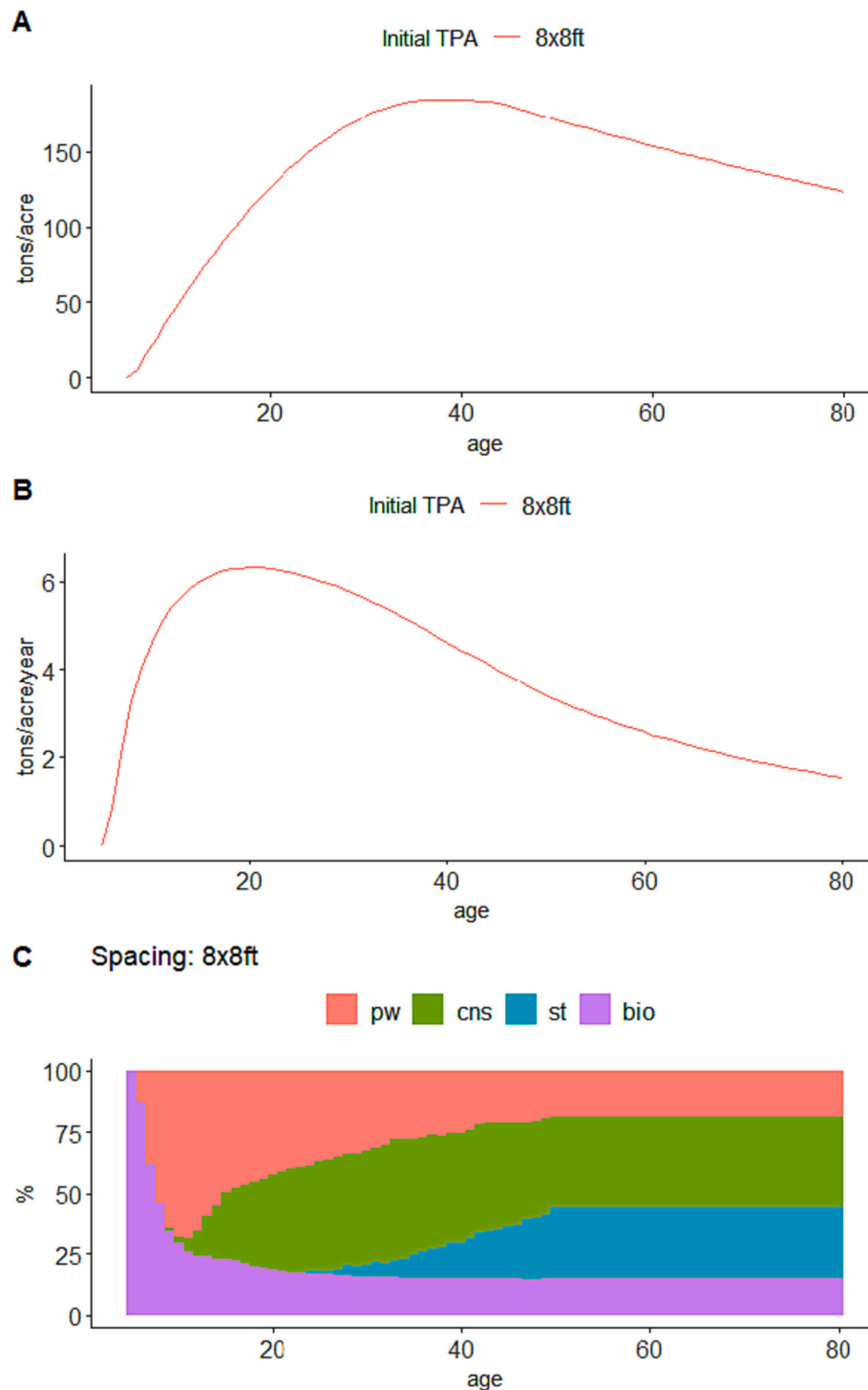


Fig. 1. (A) Yield curves based on initial trees per acre, TPA: 8x8ft.

(B) Mean Annual Increment by initial TPA and age. (C) Product shares for the yield curve with 8x8ft initial space. PW = Pulpwood, CNS = Chip and Saw, ST = Sawtimber, Bio = biomass. Mean Annual Increment were maximized at 20.

and profitability (Fig. 4A). In the most extreme case, in the scenario $T = 15$, the forest volume accumulates to a remarkable 550% greater than the initial inventory, while at $T = 100$, the difference was around 300%. The only exception occurs at $T = 5$, where the inventory is not fully harvested at the terminal period due to insufficient time for non-merchantable forests to mature into merchantable timber (Fig. 4B).

Comparing scenarios with and without terminal value (No TV), results indicate less frequent harvesting without terminal value. For instance, in the scenario $T = 60$ and No TV, only 21% of the inventory is

harvested at $t = 44$, with a full clearcut occurring 16 years later at $t = 60$. Conversely, in the same-T scenario with TV added in the terminal year, a consistent annual harvest of at least 7% occurs between $t = 48$ and $t = 60$, and no clearcut at $t = 60$. On the other hand, the intensity of harvesting (average share harvested – Fig. 5B) was not subtle at longer periods. Without constraint, only at $T = 30$ and shorter, there was a difference between the average harvested among the scenarios with and without TV. We observed a similar trend with a constraint with a starker difference in periods shorter than 30 but still a fair difference in longer

Table 1
Input prices and costs applied to the harvest schedule model.

Product	Timber Prices (\$/ton) ¹
Pulpwood	10
Chip and Sawn	19
Sawtimber	27

Costs	\$/acre
Annual Management ²	\$9.55
Chemical treatment ³	\$97.30
Planting ³	\$79.62
Seedlings (7.5 per) – 8x8ft ^{4,5}	\$ 57.74

Note: Timber Prices are rounded to the nearest whole number to mask specific prices. Sources: ¹TimberMart South - Timber Price Report, ²Forest2Market (2018), ³Maggard and Barlow (2019), ⁴Arkansas Department of Agriculture (2023), ⁵Georgia Forestry Commission (2022).

periods ($T = 60$ and $T = 100$).

A consistent result across all scenarios with TV and unconstrained models is that no harvesting activities occur in the terminal period. Intriguingly, the higher the terminal value (+10% TV), the earlier the harvest activity starts in the final years of the planning horizon. For instance, in scenarios $T = 100$ and + 10% TV, harvesting initiates at $t = 88$ and concludes at $t = 98$, clearing 95% of the inventory. In contrast, for -10% TV under the same planning horizon, harvesting begins at $t = 92$, with 4% harvested, concluding at $t = 98$ with 100% of the inventory harvested.

A distinct trend emerged in our analysis of wood flow constraint, particularly evident in the scenario with no terminal value (No TV) and a five-year planning horizon ($T = 5$). This specific scenario exhibited a noteworthy pattern characterized by an aggressive harvesting strategy at the initial year in the planning horizon ($t = 0$), removing 81% of the initial inventory. However, this intensive harvesting led to a significant constraint on the inventory's recovery capacity throughout the planning period, with the inventory only reaching 40% of its initial value by $t = 5$.

In contrast, among the constrained models, scenarios involving TV or variations of $\pm 10\%$ in TV demonstrated a more nuanced picture. In $T = 5$, harvesting percentages at $t = 0$ were notably lower compared to the

No TV scenario: 75% for TV, 76% for -10% TV, and 71% for +10% TV. Interestingly, unlike the scenario with no terminal value, these scenarios showcased a more favorable recovery trajectory for inventory, reaching 80% of the initial inventory by the conclusion of the five-year planning horizon. On the other hand, in the scenario $T = 100$, harvesting at $t = 0$ was the same across every scenario with 60%. The total inventory did not recover to >100% of the initial inventory at the terminal period. Still, it was higher in scenarios with terminal value, 30%, 27%, and 32% the initial inventory for TV, -10% TV and + 10 TV, respectively, and 20% in the scenario with no TV.

This analysis underscores the influence of terminal value on harvesting dynamics and subsequent inventory recovery, with scenarios incorporating TV exhibiting more balanced outcomes compared to the stark reductions observed in the absence of terminal value.

6.3. Age class composition at the terminal period

The previous section examines the variations in inventory across various scenarios, yet it falls short of providing a comprehensive assessment of the forest's structure at the terminal period. In this section, we investigate the influence of the terminal value on age class composition during the terminal period across different planning horizons. To facilitate interpretation, we categorized forest age classes into ten groups (Class 1: 0 – bare land, Class 2: 1 to 5 years old, Class 3: 6 to 10 years old, and so forth, up to Class 10: ≥ 40 years). Table 2 shows the proportion of the area within each age class at the terminal period.

Our findings present distinct trends in age class distribution at the terminal period. In unconstrained models, harvesting activities at the terminal period solely emerged in scenarios lacking the terminal value for all planning horizons, except when $T = 5$. In the case of $T = 5$, featuring a shorter planning horizon, the transition of non-merchantable forests into merchantable ones resulted in 78.96% classified as bareland (Class 1), with 12.72% and 8.31% in Class 2 and 3, respectively. In other scenarios without the terminal value and unconstrained model, the entire age class composition was 100% bareland. The constrained model exhibited an equivalent trend, with differences between scenarios minimized due to imposed wood flow constraints. For instance, the bare land area did not exceed 5% of the total age class composition in any scenarios featuring the terminal value. The lower the terminal value, the higher the concentration of area in the bareland category; the highest

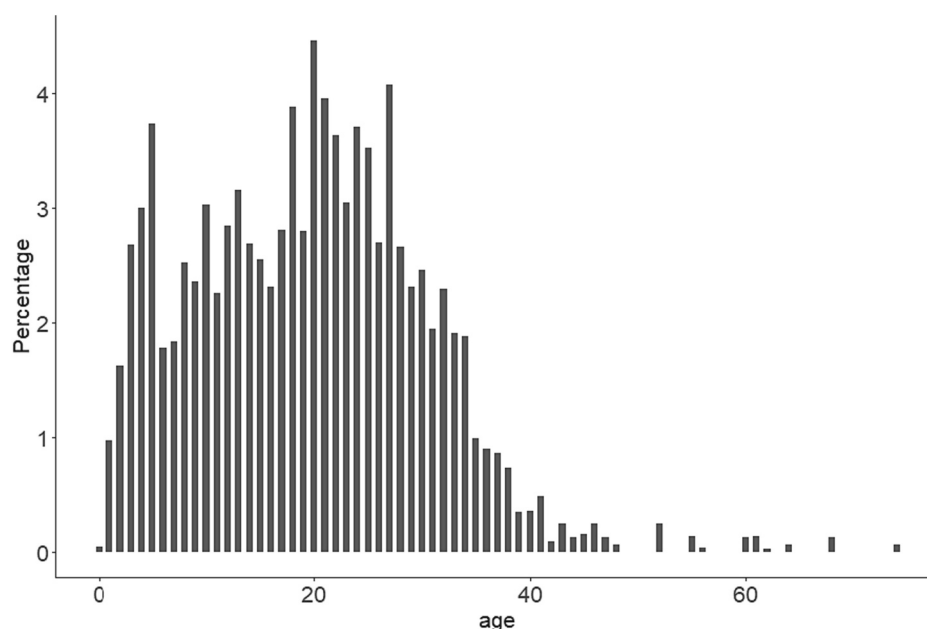


Fig. 2. Age class distribution of the state of Mississippi (pine plantation in private ownership).

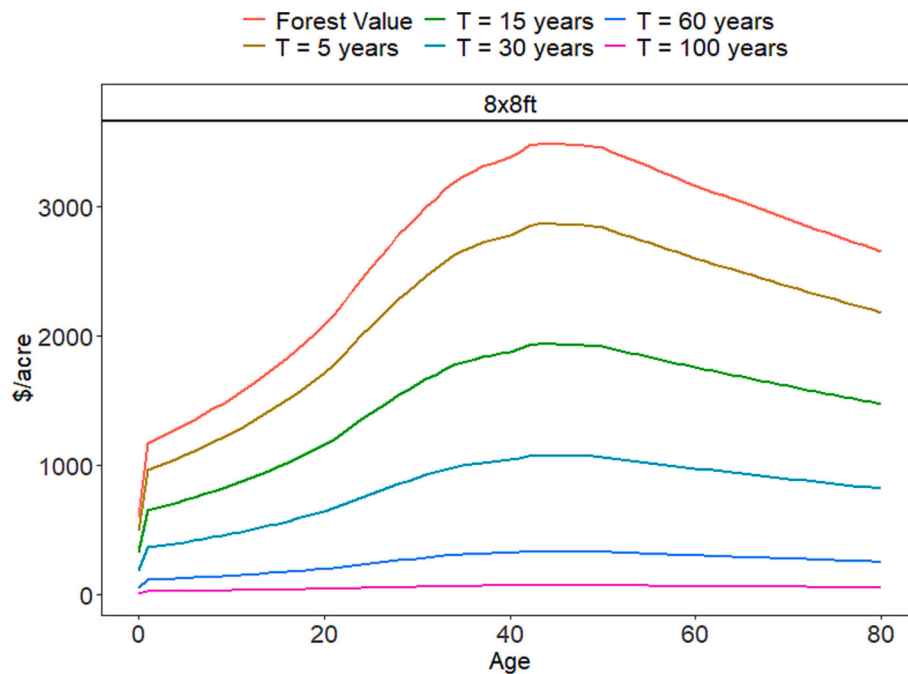


Fig. 3. Forest value (\$/acre) by age class, and terminal value (\$/acre) by age class and terminal period.

Both forest and terminal values were calculated at a 4% discounted rate. Eqs. (1) to (3) estimated the forest value, while terminal value was calculated using Eqs. (1) to (3) and discounted to the present as in Eq. (5).

values were observed at -10% TV and $T = 30$ and -10% TV and $T = 100$, where 1.90% and 1.94% were classified as bareland at the terminal period, respectively. On the other hand, in the scenarios with $+10\%$ TV, there was virtually no area allocated to bare land, only 0.10% when $T = 5$, 0.16% when $T = 15$, and 0.49% when $T = 60$.

The introduction of the terminal value significantly impacted the age class distribution that remained unharvested during the terminal period. In unconstrained models, a 10% increase in the terminal value resulted in a higher proportion of stands in age class 5: from 5.28% to 8.13% at $T = 5$, from 1.02% to 2.64% at $T = 15$, and from zero to 2.36% at $T = 30$. Similarly, the scenarios of wood flow control exhibited a parallel trend. Without the terminal value, the oldest age class consistently remained below 5. Conversely, with the introduction of the terminal value, at $T = 30$ and $T = 100$, a portion of the forest transitioned to age class 6 (3.11% and 0.13%, respectively). This share notably increased to 7.46% and 5.01%, with a positive 10% adjustment in the terminal value at $T = 30$ and $T = 100$, respectively.

These findings underscore the complex relationship between terminal values and the resulting age class distribution at the terminal period. The observed concentration in older age classes highlights the financial incentives associated with retaining stands until the end of the planning horizon (or initiating harvest activities before that period), particularly in scenarios with augmented terminal values.

6.4. Forest value

Table 3 displays the forest value per acre derived from the harvest schedule model. Generally, omitting the terminal value consistently results in a lower forest value, diminishing differences as the terminal period extends from 5 to 100 years. In addition, shorter terminal periods yielded higher forest values than more extended terminal periods in every scenario with terminal value and model constraints. Conversely, forest value without terminal value increases as the terminal period increases.

Although long planning horizons in forest management mitigate the overall loss of forest value due to the terminal value, in practice, the large scale of forest operations substantially reduces profits for

timberland investors. For instance, forest planning with a horizon of five years ($T = 5$) and an unconstrained scenario, failing to consider the terminal value, could cost investors $-\$1277.84$ per acre. This difference reduces to $-\$27.65$ when $T = 100$. In the constrained model, the disparity in forest value per acre between adding and not adding the terminal value remains substantial, from $-\$1056.49$ per acre at $T = 5$ to $-\$25.38$ per acre at $T = 100$.

The sensitivity analysis on the terminal value reveals that a 10% deviation from the actual terminal value does not necessarily result in a symmetrical increase or decrease in forest value. Specifically, in the case of unconstrained management, a positive increase of 10% in the terminal value has a much greater effect than a negative 10%. For instance, when considering forest management with a 60-year planning horizon, a terminal value that is 10% lower results in a deviation of only 0.9% from the true forest value. Conversely, a terminal value that is 10% higher leads to overestimating the forest value by 1.3%. On the other hand, when the forest valuation is constrained, the 10% variation presents symmetric outcomes for all scenarios.

This difference is even more pronounced in shorter planning horizons. In the scenario $T = 5$ and unconstrained, a negative reduction of 10% in the terminal value would lead to a -5.9% reduction in forest value, while a 10% increase in the terminal value would lead to a 9.6% increase in the forest value.

7. Discussion and conclusions

We investigated the effect of the terminal value on the timber harvest schedule and the final forest value through analytical and numeric approaches. We extended the work from Clutter et al. (1983), which used the principle of Faustmann formula to estimate the terminal value. Our results show that, even with very long planning horizons, not accounting for the terminal value can have a sizable impact on the final timberland property value.

This paper is an educational tool for those engaging in discounted cash flow modeling for large forest estates comprising multiple stands. Our study emphasized the wide range of outcomes that arise when different planning horizons are selected, leading to values that diverge

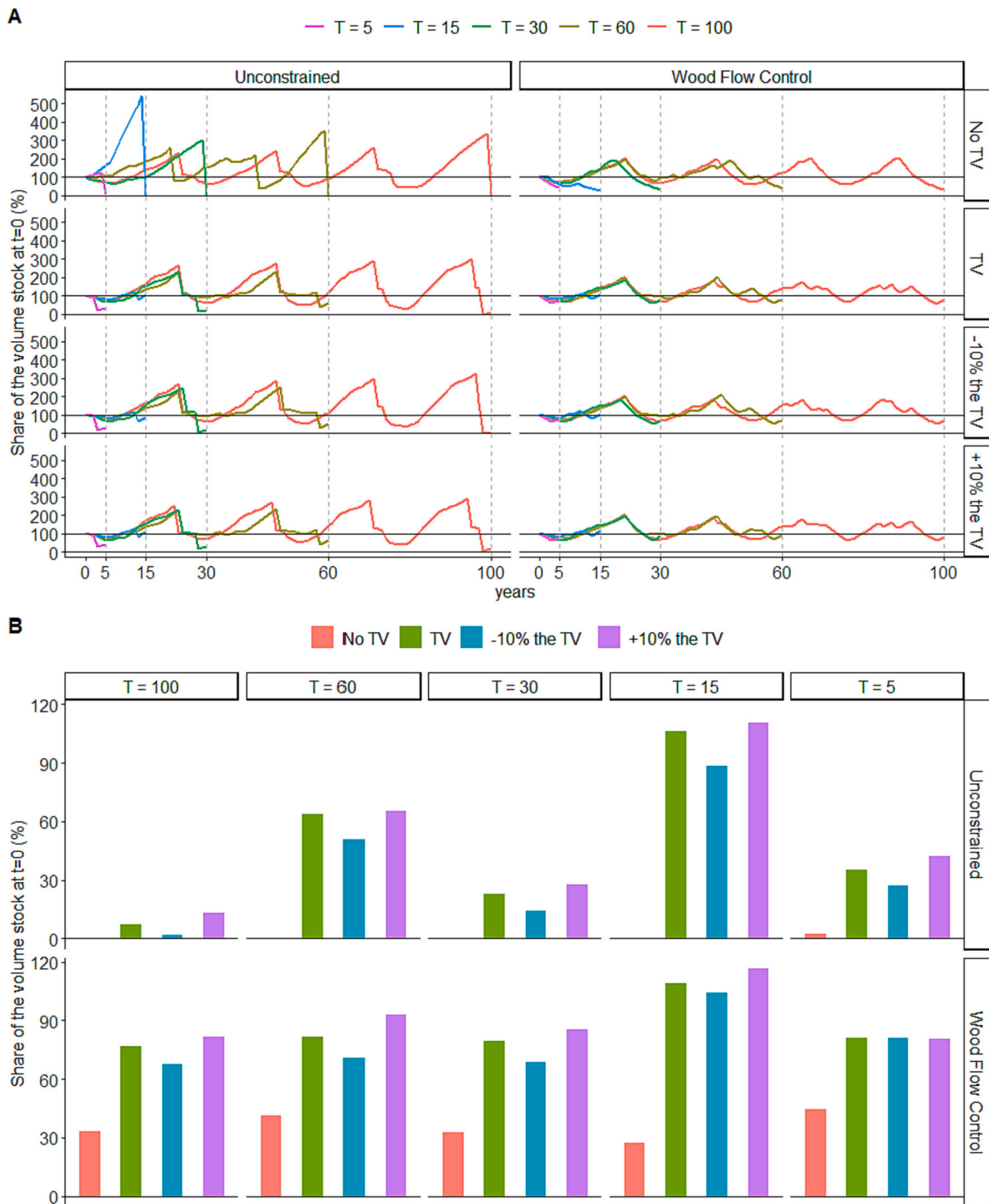


Fig. 4. (A) Timber inventory share ($\frac{\text{Inventory at } t}{\text{Inventory at } t=0}$) by year and terminal period.

(B) Timber inventory share ($\frac{\text{Inventory at } t}{\text{Inventory at } t=0}$) at the terminal period. Both graphs are by terminal period, constraint, and terminal value assumptions. No TV, TV, -10% the TV, +10% the TV represent the assumption of no terminal value, with terminal value, with -10% the terminal value, and with +10% the terminal value, respectively.

from traditional Faustmann outcomes. Although the terminal value can add more uncertainty to the forest value (given the assumption to calculate it), it represents a continuation of the harvest schedule through perpetuity and provides solid economic principles. Not having that, the forest is undervalued, and harvest schedule decisions could be sub-optimal and ultimately lead to a non-sustainable use of forest resources. We hope our approach enlightens the scientific and mainly practitioner community to use the terminal value to manage forests for both timber

production and multiple uses.

Our results reinforce the convergence to the optimal rotation and a regulated forest when maximizing net present values without any constraint, like findings from Heaps (2015), Mitra and Wan (1986), and Tahvonen and Viitala (2006). In addition, we complemented their studies by adding wood flow control, which works as a proxy for other possible constraints when modeling forest-level management.

In fast-growth plantations, not using the terminal value properly

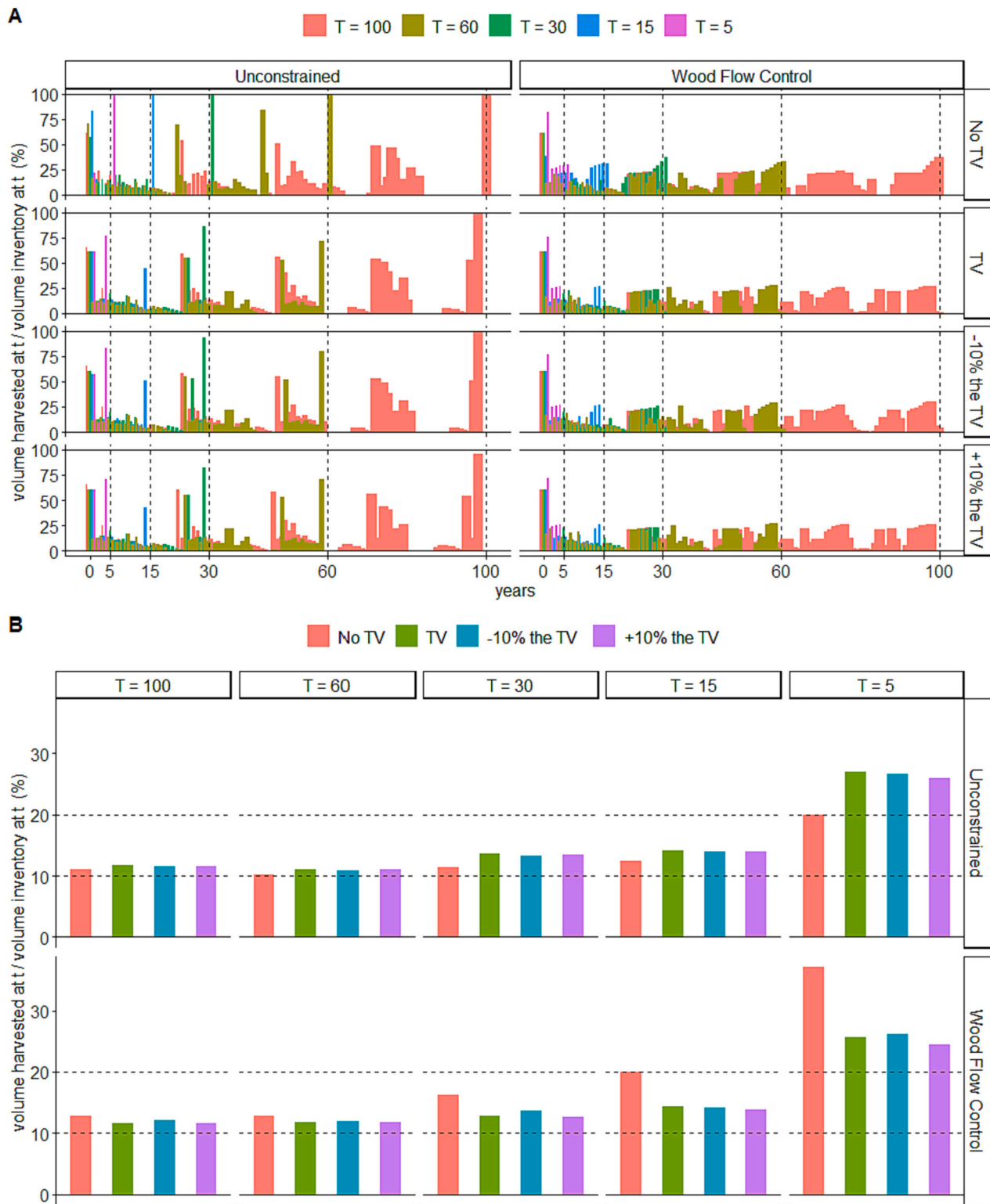


Fig. 5. (A) Relative harvest volume by year.

(B) Average relative harvest volume. Both graphs are by terminal period, constraint, and terminal value assumptions. No TV, TV, -10% the TV, +10% the TV represent the assumption of no terminal value, with terminal value, with -10% the terminal value, and with +10% the terminal value, respectively.

might generate financial losses even more critically than in slow-growth forests. A poor definition of terminal value could lead to forest managers making poor harvesting decisions and perhaps cause marginal divestment of timberland assets. Undervaluation of forest assets might also have more profound implications at large scales; in the U.S. South, for instance, investors look for opportunities to profit from the timber

market; however, the expansion of urbanization pressures managers to convert timberland to better-use alternatives. An undervalued asset would exacerbate this process, creating problematic ecological consequences such as defragmentation, land use change, and carbon release.

The financial outcomes are of primary interest for multiple investors; perhaps just as significant are the effects it implies for overall forest

Table 2

Area distribution in percentage by age classes and scenarios at the terminal year T.

Scenarios/Age Classes	1	2	3	4	5	6	7	8	9	10
Unconstrained										
No TV and T = 5	78.96	12.72	8.31	–	–	–	–	–	–	–
TV and T = 5	–	74.18	8.31	12.22	5.28	–	–	–	–	–
–10% and T = 5	–	76.43	8.31	12.22	3.03	–	–	–	–	–
+10% and T = 5	–	71.33	8.31	12.22	8.13	–	–	–	–	–
No TV and T = 15	100.00	–	–	–	–	–	–	–	–	–
TV and T = 15	–	30.80	14.23	53.95	1.02	–	–	–	–	–
–10% and T = 15	–	34.45	14.34	51.16	0.05	–	–	–	–	–
+10% and T = 15	–	29.18	14.23	53.95	2.64	–	–	–	–	–
No TV and T = 30	100.00	–	–	–	–	–	–	–	–	–
TV and T = 30	–	42.70	44.44	12.86	–	–	–	–	–	–
–10% and T = 30	–	84.34	8.31	7.34	–	–	–	–	–	–
+10% and T = 30	–	40.34	44.44	12.86	2.36	–	–	–	–	–
No TV and T = 60	100.00	–	–	–	–	–	–	–	–	–
TV and T = 60	–	39.31	17.90	42.79	–	–	–	–	–	–
–10% and T = 60	–	42.12	18.72	39.17	–	–	–	–	–	–
+10% and T = 60	–	38.34	17.90	43.77	–	–	–	–	–	–
No TV and T = 100	100.00	–	–	–	–	–	–	–	–	–
TV and T = 100	–	88.00	12.00	–	–	–	–	–	–	–
–10% and T = 100	–	92.71	7.29	–	–	–	–	–	–	–
+10% and T = 100	–	86.17	8.31	5.51	–	–	–	–	–	–
Wood Flow Control										
No TV and T = 5	8.80	71.88	8.31	11.01	–	–	–	–	–	–
TV and T = 5	0.24	67.74	8.31	12.22	11.48	–	–	–	–	–
–10% and T = 5	0.35	68.60	8.31	12.22	10.51	–	–	–	–	–
+10% and T = 5	0.10	65.40	8.31	12.22	13.96	–	–	–	–	–
No TV and T = 15	23.33	34.45	18.86	23.35	–	–	–	–	–	–
TV and T = 15	0.28	28.90	14.23	53.95	2.64	–	–	–	–	–
–10% and T = 15	0.25	33.23	11.54	53.95	1.02	–	–	–	–	–
+10% and T = 15	0.16	26.33	14.23	53.95	5.32	–	–	–	–	–
No TV and T = 30	15.72	24.51	51.41	8.35	–	–	–	–	–	–
TV and T = 30	0.42	29.06	37.24	17.16	13.02	3.11	–	–	–	–
–10% and T = 30	1.90	27.39	42.18	17.16	11.37	–	–	–	–	–
+10% and T = 30	–	30.33	34.92	17.16	10.13	7.46	–	–	–	–
No TV and T = 60	19.15	32.77	27.97	20.12	–	–	–	–	–	–
TV and T = 60	0.57	36.15	13.94	49.34	–	–	–	–	–	–
–10% and T = 60	1.47	34.68	17.63	46.23	–	–	–	–	–	–
+10% and T = 60	0.49	34.55	9.49	50.43	5.05	–	–	–	–	–
No TV and T = 100	14.96	23.92	54.17	6.95	–	–	–	–	–	–
TV and T = 100	0.66	32.67	21.28	36.59	8.67	0.13	–	–	–	–
–10% and T = 100	1.94	31.61	24.08	41.55	0.81	–	–	–	–	–
+10% and T = 100	–	32.95	24.52	31.80	5.72	5.01	–	–	–	–

Note: Age classes were aggregated by 5 years classes (class 1: 0 - bareland, class 2: 1 to 5 years old, ...class 9: 40 above).

Table 3

Forest Value in \$/acre for an 8×8ft loblolly pine forest plantation with and without constraints across multiple terminal periods and terminal value scenarios.

Gross Value in \$/acre for an 8000 foot long pine forest plantation with and without constraints across multiple terminal periods and terminal value scenarios.										
Terminal Period	Terminal Value	No Terminal Value			–10% the Terminal Value			+10% the Terminal Value		
		Value	Diff	Diff %	Value	Diff	Diff %	Value	Diff	Diff %
8×8ft - Unconstrained (per acre)										
5	\$2647	\$1370	-\$1277	–48.3%	\$2491	-\$155	–5.9%	\$2903	\$255	9.6%
15	\$2434	\$1613	-\$821	–33.7%	\$2319	-\$115	–4.7%	\$2580	\$145	6.0%
30	\$2251	\$1803	-\$447	–19.9%	\$2194	-\$56	–2.5%	\$2349	\$98	4.4%
60	\$2086	\$1951	-\$134	–6.5%	\$2067	-\$18	–0.9%	\$2112	\$26	1.3%
100	\$2028	\$2001	-\$27	–1.4%	\$2025	-\$3	–0.2%	\$2035	\$6	0.3%
8×8ft – Wood Flow Control (per acre)										
5	\$2224	\$1167	-\$1056	–47.5%	\$2110	-\$113.46	–5.1%	\$2338	\$114	5.1%
15	\$2133	\$1397	-\$735	–34.5%	\$2050	-\$82.98	–3.9%	\$2216	\$83	3.9%
30	\$2085	\$1690	-\$394	–18.9%	\$2041	-\$44.53	–2.1%	\$2131	\$45	2.2%
60	\$2035	\$1912	-\$122	–6.0%	\$2021	-\$13.44	–0.6%	\$2048	\$13	0.6%
100	\$2017	\$1992	-\$25	–1.2%	\$2014	-\$2.82	–0.1%	\$2020	\$2	0.1%

composition. The majority of annual timber harvests come from Corporate (46%) and Family (42%) forestlands (Butler et al., 2022). The implication for forest age class harvesting is for those with short planning horizons to engage in bimodal harvest activity- harvesting in both younger and older age classes- compared to those with longer horizons and area constraints. Institutional ownership growth in the U.S. South

would not grow over mature timber regardless of subsequent ownership classification and negotiated sale price for the forest property.

Although not computationally efficient, running a harvest schedule model for twice the typical planning horizon (100 years as opposed to 50 years) would reduce the impact of the terminal value on the final forest value even further. However, in practice, forest-level management

is subjected to constraints, such as harvest adjacency relationships, timber flow, and habitat, so sub-optimal strata-level decisions are selected to conform to the restrictions.

In a harvest schedule and strategic planning, the terminal period is between 30 and 50 years from the starting period in the U.S. South (Tanger et al., 2022) or 100 to 150 years in locations with slower biological growth (Andersson and Eriksson, 2007; Martell et al., 1998). Therefore, cash flows are often no more than marginally impacted by discounting the terminal value to the present from these long periods. In addition, when maximizing Net Present Value (NPV) within such a time frame, the silvicultural prescriptions tend to converge to the optimal rotation age or treatment, becoming a steady state choice (Mittra and Wan, 1986; Uusivuori and Kuuluvainen, 2005).

We recognize a series of limitations in our approach: (1) modeling real data from an actual property with multiple stands subjected to local constraints could provide more realistic results instead of a simulated forest and constraint, (2) we could also add hardwood management to check the impact of slow-growth and uneven age forest, and (3) we used just timber product, which could provide periodic returns, many land-owners have annual return from activities like hunting. Further exploration in these areas is encouraged to better inform the potential consequences on forest returns and ecosystem services.

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Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the author(s) used ChatGPT and

Appendix A

We use the following equation at the terminal period T as in Eq. (15).

$$A_{i,t=T} = A_{i-1,t=T-1} - H_{i-1,t=T-1} \quad \forall i \in [0, I] \quad (A1)$$

We know that $A_{i,T} = R_i + H_{i,T}$, after a few arithmetic substitutions Eq. (A1) becomes:

$$A_{i-1,T-1} = H_{i-1,T-1} + R_i + H_{i,T} \quad \forall i \in [0, I] \quad (A2)$$

We also know that $A_{i-1,T-1} = A_{i-2,T-2} - H_{i-2,T-2}$, from Eq. (A1). Therefore Eq. (A2) could be represented as:

$$A_{i-2,T-2} = H_{i-1,T-1} + R_i + H_{i,T} + H_{i-2,T-2} \quad \forall i \in [0, I] \quad (A3)$$

Generalizing Eq. (A3), we have:

$$A_{i-n',T-n''} = R_i + H_{i,T} + \sum_{n'}^{n'} \sum_{n''}^{n''} H_{i-n',T-n''} \quad \forall i \in [\text{merch}, I], \quad (A4)$$

$$n' \in [0, I], \quad n'' \in [0, I], \quad \text{and } t \in [0, T]$$

where *merch* is the age at which timber is merchantable age.

So, the terminal value and the harvest area at $t = T$ will impact the area inventory as the n' value changes from *merchantable age* to the oldest forest age, and n'' from the initial year to T .

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Grammarly in order to correct grammar and spelling. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the publication's content.

Credit authorship contribution statement

Bruno Kanieski da Silva: Writing – review & editing, Writing – original draft, Validation, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Fatemeh Rezaei:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization. **Shaun Tanger:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization. **Jesse Henderson:** Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Conceptualization. **Eric McConnell:** Writing – review & editing, Formal analysis, Conceptualization. **Changyou Sun:** Writing – review & editing, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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