AN INTEGRATED SYSTEM APPROACH TO MODEL TREE AND STAND GROWTH

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ABSTRACT

An integrated system contains a unified mathematical structure for modeling tree and stand growth that can be applied at any level of resolution. This paper outlines the recent progress that has made this concept a reality: 1) the integrated system is based on a whole-stand growth model; 2) given a tree list, an individual-tree growth model can be derived from the stand model; 3) a diameter distribution (e.g., Weibull) can be partitioned into individual trees, which can be grown according to the tree growth model from 2) and 4) a stand table can be converted into a tree list, which can be used to derive a tree growth model by use of the technique from 2). This integrated system can provide growth predictions for the current stand. The inputs can be a plot summary, a tree list, a stand table, or a diameter distribution. The outputs are constrained to have numerical consistency across all levels of resolution.

INTRODUCTION

Contemporary forest management relies on the application of growth and yield models to forecast future stand growth for various scenarios. These models are classified based on the level of detail (or resolution) of their outputs into four broad categories (Burkhart and Tomé 2012). Whole-stand models are relatively simple models that provide information for the entire stand. The predicted stand attributes can be stand survival (Stankova 2016), basal area per unit area (Naing 2020), or both (Zhang and others 2022). Size-class models deal with diameter classes; these models can be stand-table projection models that project a stand table (containing the number of trees in each diameter class) into the future (Allen and others 2011), or diameter-distribution models that use a probability density function (PDF) to model the frequency of tree diameters (Carretero and Alvarez 2013). Individual-tree models deliver detailed information for each tree, in terms of tree survival (Karlsson and Norell 2005), tree diameter growth (Subedi and Sharma 2011), or both (Sun and others 2019). The user sometimes has difficulty in deciding between a model that provides reliable estimates and another that gives sufficient details. Daniels and Burkhart (1988) promoted the idea of creating an integrated system that can provide consistent growth and yield estimates at various levels of resolution, because it is based on a unified mathematical structure.

The concept of an integrated system has recently become a reality. This paper presents such a system, one that can handle inputs in a variety of formats and still give consistent results.

INPUT: STAND SUMMARY

A stand-level growth model that projects current stand attributes into the future is the heart of the integrated system. Either an existing model is used or a new model is constructed for this purpose. The stand growth model predicts number of trees per hectare ($\hat{N}_{2i}$) and quadratic mean diameter ($\hat{Q}_{2i}$) for plot i at time 2 (future) from stand summary at time 1 (present) in terms of age (A_{1i}) dominant height (H_{1i}), number of trees per hectare (N_{1i}), and quadratic mean diameter (Q_{1i}). The following example is a stand growth model from Cao (2022a):

$$\hat{N}_{2i} = N_{1i} - exp(a_0 + a_1 RS_{1i} + a_2 H_{1i} + a_3 N_{1i}/A_{1i} + a_4/A_{1i}) \quad (1)$$

$$\hat{Q}_{2i} = Q_{1i} + exp(a_5 + a_6 N_{1i}/A_{1i} + a_7/A_{1i}) \quad (2)$$

$$\hat{B}_{2i} = K\,\hat{N}_{2i}\,\hat{Q}_{2i}^2 \quad (3)$$

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where

$\hat{B}_{2i}$ = predicted stand basal area (m^2/ha) at the end of a 5-year growth period

A_{1i} = stand age in years for plot *i* at the beginning of the growth period

N_{1i} = number of trees per hectare at age A_{1i}

H_{1i} = dominant height (m) at age A_{1i}

$RS_{1i} = \frac{\sqrt{10000/N_{1i}}}{H_{1i}}$ = relative spacing at age A_{1i}

$K = \pi/40{,}000$

INPUT: TREE LIST

The earliest attempts to link tree models to stand models was the disaggregation approach (Ritchie and Hann 1997), in which information obtained from the tree model is used to disaggregate stand growth (predicted by a whole-stand model) among trees in the tree list, which is a list of tree diameters in a plot. Cao (2021) presented an integrated system in which the tree model contains identical terms as does the stand model, with an additional term to describe individual-tree variations. Recently, Cao (2019) showed how one can derive a tree survival model from any existing stand survival model; the level of accuracy and precision depended on the stand model performance and on whether or not tree-level survival data were available.

In the case when tree-level survival data are not available, Cao (2022a) developed a two-step procedure to derive a tree-level growth model from a stand-level model by use of a tree list. In the first step, an individual-tree survival equation can be derived from the stand survival equation as follows:

$$p_{ij} = 1 - exp\left[\alpha_1\left(d_{1ij} - 0.95\ Dmin_{1i}\right)\right] \tag{4}$$

where

p_{ij} = survival probability of tree *j* in plot *i*

d_{1ij} = diameter at breast height (d.b.h., cm) of tree *j* in plot *i* at age A_{1i}

$Dmin_{1i}$ = minimum d.b.h. of trees in plot *i*

α_1 is the solution of

$$\hat{N}_{2i} = \sum_{j=1}^{n_{1i}} p_{ij}/s \tag{5}$$

where

p_{ij} = survival probability of tree *j* in plot *i* at age A_{1i}

s = plot size in hectares

In the second step, the predicted d.b.h. at time 2 ($\hat{d}_{2ij}$) is computed from the d.b.h. at time 1 (d_{1ij}) as follows:

$$\hat{d}_{2ij} = d_{1ij}exp\left(\alpha_2 d_{1ij}\right) \tag{6}$$

where

α_2 is solved such that

$$\hat{B}_{2i} = K\sum_{j=1}^{n_{1i}} p_{ij}\hat{d}_{2ij}^2/s \tag{7}$$

The derived tree model was evaluated against other individual-tree models in terms of their abilities to predict tree diameter and survival. The derived tree model performed better than either the unadjusted or disaggregated tree model. The sum of the ranks (lower is better) was 12.91 for the derived tree model, 15.83 for the unadjusted tree model, and 17.28 for the disaggregated tree model (Cao 2022a). Note that both the derived and disaggregated models were constrained by output from the stand-level model.

INPUT: DIAMETER DISTRIBUTION

In this scenario, a PDF is used to describe the diameter distribution of a forest stand. Instead of the commonly used 2-cm diameter classes, 0.2-cm diameter classes are used; each could be considered as a single tree. The diameter of this tree is the midpoint of the diameter class; the number of trees per hectare that this tree represents is obtained by integrating the PDF for each 0.2-cm class. Cao (2022b) used this technique to generate a tree list. Now that the input is a tree list, an individual-tree model can be derived from the stand model by use of the technique described in the previous section.

This method was evaluated against the methods of 1) parameter recovery from $\hat{\hat{D}}_2$ and $\hat{Q}_2$, which are predicted from stand variables, 2) parameter recovery $\hat{\hat{D}}_2$ from $\hat{Q}_2$ and, which are projected from D_1 and Q_1, and 3) growing the PDF from time 1 to time 2. Results from Cao (2022b) indicated that the new method performed better than the first two methods, yielding lower error indices (Reynolds and others 1988) and relative differences between observed and predicted number of trees in each diameter class. The third method requires many steps involving mathematical integration. The new method is analogous to solving the numerical integration problem by use of the trapezoid integration approach, and as a result, its performance was almost identical to that of the third method.

Bailey (1980) observed that if the diameter distributions at both ages A_1 and A_2 belong to the same family of distributions (e.g., Weibull), then the diameter random variable Y at age A_2 is implied to be transformed from variable X at age A_1. The attempt to link a diameter-distribution model to an individual-tree model was later expanded by Cao (1997) and Qin and others (2007). The new method described in this section treats the diameter variable X as individual trees, each with its own expansion factor (number of trees per hectare) depending on its diameter. This process 1) converts a diameter-distribution model at age A_1 into an individual-tree model, 2) grows the trees to age A_2, and 3) converts them back to a diameter-distribution model. The method provides a linkage across the spectrum of growth and yield models: individual-tree, diameter-distribution, and whole-stand models.

INPUT: STAND TABLE

In a ground-breaking paper, Nepal and Somers (1992) suggested the use of a truncated Weibull distribution to approximate the number of trees in each diameter class. After evaluating the uniform, Weibull, and S_B distributions, Cao and Coble (2014) concluded that the truncated Weibull distribution was most appropriate for modeling stand table data.

Cao (2023) proposed a procedure for handling stand-table input as follows:

- Fit a complete Weibull distribution to the stand table by use of the cumulative distribution function regression (Cao and Coble 2014).
- Establish a truncate Weibull distribution for each diameter class.
- Generate a tree list from 0.2-cm diameter classes (Cao 2022b).
- Derive an individual-tree growth model from the stand growth model (Cao 2022a).
- Convert the new tree list to the future stand table.

The new method was found to perform better than the existing stand-table projection models (Cao 2007, Cao and Baldwin 1999, Nepal and Somers 1992) in terms of two error indices (Reynolds and others 1988).

CONCLUSION

This paper outlines the recent progress that has turned into reality the concept of an integrated system, as envisioned by Daniels and Burkhart (1988). The heart of this integrated system is a whole-stand growth model. If the input is a tree list, then an individual-tree growth model can be derived from the stand growth model. If the input is either a diameter distribution or a stand table, a tree list can be generated, and then used for the derivation of the tree growth model. The advantage of this system is that outputs are constrained to have numerical consistency across all levels of resolution. Even though the system was developed from loblolly pine (*Pinus taeda*) plantations, the same approach could be applied to multispecies stands with some modifications.

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