A STAND-LEVEL GROWTH AND YIELD MODEL SYSTEM FOR SHORTLEAF PINE PLANTATIONS IN THE WESTERN HALF OF THE SOUTHEASTERN UNITED STATES: AN INITIAL ATTEMPT

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ABSTRACT

A system of growth and yield equations is presented for shortleaf pine (*Pinus echinata*) plantations using data from across the western half of the Southeastern United States. Data used in model fitting were obtained from Arkansas, Mississippi, Oklahoma, and Tennessee (n = 168 plot measurements). Planting densities ranged from 194 to 1,742 seedlings per acre, and measurement ages ranged from 1 to 30 years. The majority of the measurement ages were ≤16 years of age (n = 152). A mix of cutover and old-field sites were modeled. Predictions of dominant height, trees per acre, basal area per acre, and total volume per acre were obtained directly. Quadratic mean diameter was mathematically derived from the number of trees and basal area per acre estimates. Hopefully, as interest grows, a more complete dataset can become available resulting in an improved model system as compared to this initial modeling effort.

INTRODUCTION

Shortleaf pine (*Pinus echinata*) is native to much of the western half of the Southeastern United States, as well as southern Missouri (Guldin 2019, Little 1971). According to the Shortleaf Pine Initiative (https://shortleafpine.org/) and Guldin (2019), shortleaf was often found in open woodlands in both pine-dominated and mixed pine-oak (*Quercus* spp.) forests, allowing for a diverse assortment of native wildlife to flourish. Over the last 30 to 40 years, this extensive shortleaf pine ecosystem has lost >50 percent of its former acreage (Clabo and Clatterbuck 2015, Guldin 2019). Changes in timber management practices, altered fire regimes, massive southern pine beetle (*Dendroctonus frontalis*) outbreaks in poorly managed stands, disease, and land-use changes have contributed to this rapid decline.

Coordinated efforts by groups like the Shortleaf Pine Initiative to restore the shortleaf pine ecosystem are beginning. Although overstory woody vegetation is a major component of the restoration effort, other components of these ecosystems such as understory vegetation, biological diversity, wildlife habitat, and recreation are also highly desired. The restoration process of this ecosystem will likely begin mostly through the use of plantations (Guldin 2019).

Managers need tools to examine how different management options will likely impact financial returns. Although a more open woodland setting may be the ultimate goal by the majority of agencies, timber production will help to offset the initial conversion and regeneration costs. Additionally, partners wanting to restore or manage for components of the ecosystem such as wildlife or water quality will benefit from gaining an idea of how these stands will develop.

According to a 2022 estimate by the Forest Inventory and Analysis Unit of the U.S. Department of Agriculture, Forest Service (https://apps.fs.usda.gov/fiadb-api/evalidator), there are approximately 255,000 acres of shortleaf pine plantations, and northwestern Arkansas and southeastern Oklahoma have considerable amounts of shortleaf-dominated, naturally regenerated stands (table 1). Shortleaf pine

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Table 1—Acreage of shortleaf pine forest types in 2022 as defined by the Forest Inventory and Analysis program of the U.S. Department of Agriculture, Forest Service

	Natural		Planted		
State	Shortleaf	Shortleaf/oak	Shortleaf	Shortleaf/oak	Total
	~~ *acres* ~~~				
Alabama	111,573	114,843	3,103	0	229,519
Arkansas	1,162,452	757,307	87,132	9,319	2,016,210
Louisiana	37,010	28,850	0	0	65,860
Mississippi	164,269	169,436	10,426	8,569	352,700
Missouri	153,297	303,267	85,864	9,563	551,991
Oklahoma	509,206	454,735	26,640	0	990,581
Tennessee	50,087	133,148	8,964	0	192,199
Texas	252,732	127,357	5,425	0	385,514
Total	2,440,626	2,088,943	227,554	27,451	
Total	4,529,569		255,005		4,784,574

The Forest Inventory and Analysis program has two forest type classifications with shortleaf pine as a dominant species: shortleaf pine (FORTYPCD = 162) and shortleaf pine/oak (FORTYPCD = 404).

stands are also relatively abundant in Mississippi, Missouri, and Texas. A small amount of planting has occurred in the southern parts of Illinois and Indiana.

Projections from this current modeling effort will be compared with the only known comprehensive stand-level model system for shortleaf pine plantations. Smalley and Bailey (1974) presented a system of equations to predict yields of unthinned old-field plantations in the highlands of Alabama, Georgia, and Tennessee. Site index was predicted using an equation presented in Smalley and Bower (1971), while survival rates were predicted as a function of the average height of dominants and codominants (i.e., site quality), trees planted, and age. All (n = 104) but one plantation had planting densities exceeding 500 seedlings per acre with the majority being 1,000 seedlings per acre or greater. All equations presented in Smalley and Bailey (1974) were estimated using cross-sectional data obtained within temporarily established plots, or measurements across a range of ages obtained at the same time. Thus, projections are not based on repeated plot measures.

Smalley and Bailey (1974) predicted basal area development through the use of the survival equation and a three-parameter Weibull parameter-prediction equation system. For their per acre yield estimates, initially, the number of trees within a particular diameter class was estimated based on the stand-level survival equation and parameter estimates of the three-parameter Weibull distribution using the parameter prediction approach. Following this, an equation was used to estimate average height in each diameter class, and then individual tree volumes were predicted using an equation presented by Smalley and Bower (1968). These individual tree volumes were then multiplied by the estimated number of trees within each diameter class. Volumes across all diameter classes were then summed to produce a per acre estimate.

The objective of this paper is to present a system of equations using data from across the western half of the Southeastern United States to determine the site index if desired, predict dominant height, trees per acre, basal area per acre, and total volume per acre of shortleaf pine plantations. Quadratic mean diameter can be mathematically derived from the trees per acre and basal area per acre estimates and estimates of tons per acre can be obtained by using common conversion factors between volume and weight.

METHODS

Data

A variety of sources were used in model development (table 2, fig. 1). The first data source was a shortleaf pine spacing study established in February of 1955 on the University of Arkansas, Southwest Research and Extension Center (then named the Fruit and Truck Branch Experiment Station) near Hope, AR. Four planting densities were established of an Arkansas seed source (most likely 1-0 bareroot seedlings): 1,742 (5 feet by 5 feet), 1,210 (6 feet by 6 feet), 908 (6 feet by 8 feet), and 681 (8 feet by 8 feet) seedlings per acre. Three replications were established of each planting density. Total tree heights were recorded annually following the first through fifth growing seasons on every live tree, and then again in the seventh growing season. Diameters at breast height (d.b.h., 4.5 feet above ground level) were recorded at ages 3, 5, and 7 years. Total cubic foot volumes were estimated using an equation developed by Lenhart and others (1987).

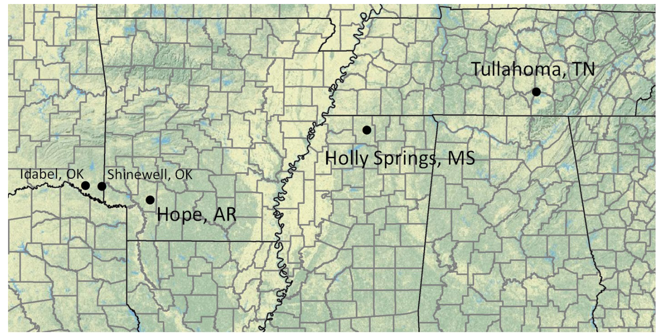


Figure 1—Locations of the five general study sites used within this study.

A second data source was from old-field plantations established on Mississippi State University's Agricultural and Forestry Experiment Station, North Mississippi Branch Experiment Station at Holly Springs, MS. This study was reported earlier by Kushla (2009).

Table 2—Number of observations, measurement ages, and summary statistics

Characteristic	Statistic	Study and State 1 Arkansas (n = 72)	2 Mississippi (n = 4)	3 Tennessee (n = 32)	4 Oklahoma (n = 28)	5 Oklahoma (n = 32)	All (n = 168)
Age (years)	**Range**	1 to 5, 7	16	5, 10, 22, 30	1 to 5, 9, 10	1 to 5, 8, 9, 10	1 to 5, 7, 8, 9, 10, 16, 22, 30
Dominant height (feet)	**Mean**	9	36	42	11	13	17
	SD	5	2	28	10	10	19
	Min	2	34	6	1	1	1
	Max	21	39	73	28	31	73
Trees per acre	**Mean**	1,068	500	423	377	318	674
	SD	375	28	301	33	101	445
	Min	626	460	139	326	198	139
	Max	1,742	520	1,139	415	442	1,742
Basal area (square feet per acre)	**Mean**	26	148	83	20	16	38
	SD	29	7	55	24	19	46
	Min	0	140	13	0	0	0
	Max	89	155	221	56	61	221
Volume (cubic feet per acre)	**Mean**	228	2,673	2,242	356	254	813
	SD	229	229	1,842	305	258	1,281
	Min	3	2,437	34	45	6	3
	Max	712	2,911	6,731	791	843	6,731

SD = standard deviation; min = minimum; max = maximum.

Subsoil tillage treatments were performed prior to planting in December 2004 using a 20-inch ripping shank pulled by a tractor on 10-foot planting centers. Bareroot 1-0 seedlings were then planted on the retired farmland in March 2005 at a nominal density of 622 trees per acre (10 feet by 7 feet spacing). Herbaceous weed control (HWC), consisting of a combination of herbicide application and mowing, was conducted during the first year. Herbicide was applied in April of 2005 and row centers were mowed three times during the growing season on treated areas.

Four 1/20th-acre circular plots were established, and total tree height and d.b.h. were measured in late December of 2021. For modeling purposes, the plantation was considered to be 16 years old (trees were 17 years old from seed). Total cubic foot volumes were estimated using equations presented by Smalley and Bower (1968).

A third data source was published information from a cutover spacing study established southeast of Tullahoma, TN, at the Highland Rim Forestry Experiment Station (Schubert and others 2004). In 1965, the native forest on the site was harvested, cull trees injected with 2,4-D and the area mist-blown to kill the herbaceous vegetation (Schubert and others 2004). In the late winter of 1966, 1-0 seedlings (most likely bareroot) from the Tennessee Valley Authority's Clinton, TN, nursery were planted. Four planting densities were established: 1,210 (6 feet by 6 feet), 538 (9 feet by 9 feet), 303 (12 feet by 12 feet), and 194 (15 feet by 15 feet) seedlings per acre. A total of four blocks were established, but due to extremely poor survival on two blocks, only observations from the other two blocks were used in model development. Percent survival and average d.b.h. were available for ages 5 and 10 (Omiyale 1976), 22 (Schubert 2001), and 30 (Schubert and others 2004).

No site index or dominant height values were reported. However, arithmetic mean height was presented by block for ages 5, 10, 22, and 30. Thus, to estimate dominant height for each of the two blocks at a particular age, first, across all of the three other sources of data described herein, the average ratio between dominant height (the average height of the tallest 50 percent of all surviving trees) and arithmetic mean height was calculated, a value of 1.1664. Second, this average value was then multiplied by the reported arithmetic mean heights to produce estimates of dominant heights for each of the two blocks and for each measurement age.

Reported volumes were to a 4-inch diameter outside bark. Hence, to be consistent with the other studies where volume is total cubic foot volume (or to the tip of the tree), a volume-basal area ratio (VBAR) approach was used. First, the VBAR on a per acre basis for this study was estimated from Smalley and Bower (1968: Table 2):

$$VBAR = 0.5020\ Ht_{Arith} - 5.9771 / QMD^2 \qquad (1)$$

where

Ht_{Arith} is arithmetic mean height (feet)

QMD is quadratic mean diameter (inches)

For the purpose of this study, Ht_{Arith} and QMD replace individual tree height and d.b.h. in Smalley and Bower's equation. Second, the reported basal areas per acre were multiplied by the predicted VBARs to estimate total cubic foot volume per acre.

A fourth source was cutover plantations from two sites in southeastern Oklahoma: Idabel and Shinewell (Dipesh and others 2015). The study was a generalized randomized complete block design; i.e., a randomized complete block study was installed at each site. No mechanical site preparation was conducted, but there was some level of chemical site preparation. At the Shinewell site (study site 4 in table 2) 1-0 bareroot seedlings were planted in late February 2002 and at Idabel 1-0 (study site 5 in table 2) bareroot seedlings were planted early April 2002. Improved seedlings were from the Western Gulf Tree Improvement Program grown at the Oklahoma Forestry Services nursery in Goldsby, OK, from seed collected at the Oklahoma Forestry Services seed orchard in Idabel, OK. Each site had four blocks. Trees were planted at a nominal spacing of 8 feet by 10 feet (544 trees per acre). Each measurement plot ranged from 0.1236 to 0.1730 acres, depending on the available space, and contained 69–90 trees and an additional strip of buffer trees on each side.

Height was measured annually following the first through fifth growing seasons on every live tree at both sites. At the Idabel site, d.b.h. was measured after the third, fourth, and fifth growing seasons but at the Shinewell site only after the fourth and fifth growing seasons. Additionally, at the Idabel site, height and d.b.h. measurements were conducted at ages 8, 9, and 10. At the Shinewell site, height and d.b.h. measurements resumed at ages 9 and 10. Total cubic foot volumes were predicted using an equation presented in Dipesh and others (2015).

Stand-Level Model Development

Equations were selected based on both biological and statistical considerations. Parameter estimates were examined for consistency with biological theory (e.g., with age, basal area per acre or volume per acre should increase). Additionally, predicted values of stand development were examined to determine whether a particular equation or set of equations produced reasonable estimates across a range of ages and values of the regressors. After accounting for biologically meaningful variables and parameter estimates, the function with the highest adjusted R^2 was selected. Residuals were examined for trends. Proc Model (SAS Institute, Inc. 1989) and the Gauss-Newton algorithm were used to estimate parameters of all equations.

Model Fitting

Dominant height equation

A simple anamorphic guide curve approach (Clutter and others 1983) was selected using equation (2):

$$Ht_D = e^{\beta_0 + \beta_1 A^{-1}} \quad (2)$$

where

Ht_D = dominant height (feet), defined as the average height of the tallest 50 percent of surviving shortleaf pine trees (other species were not selected)

A = plot age (plantation age from time of planting, not from seed)

β_0, β_1= parameters to be estimated

A total of 168 observations were used in model development. After fitting (adjusted R^2 = 0.9430) and algebraic manipulation, equation (2) became:

$$SI = Ht_D e^{(25^{-1} - A^{-1})^{-12.5133}} \quad (3)$$

where

SI = dominant height at a reference base age of 25 years and others as previously defined.

Equation (4) was used to predict dominant height:

$$Ht_D = SI e^{(25^{-1} - A^{-1})^{12.5133}} \quad (4)$$

When using equation (3), and based on the oldest plot age, the average site indices at the Mississippi site (n = 4 plots), Idabel, OK (study site 5, n = 4 plots), Shinewell, OK (study site 4, n = 4 plots), and the Tennessee site (n = 8 plots) were 48.2, 58.6, 57.5, and 64.1 feet (base age 25), respectively. Due to the young maximum age of only 7 years at the Arkansas site, site index was not calculated. Figure 2 shows dominant height across time for six site indices (base age 25).

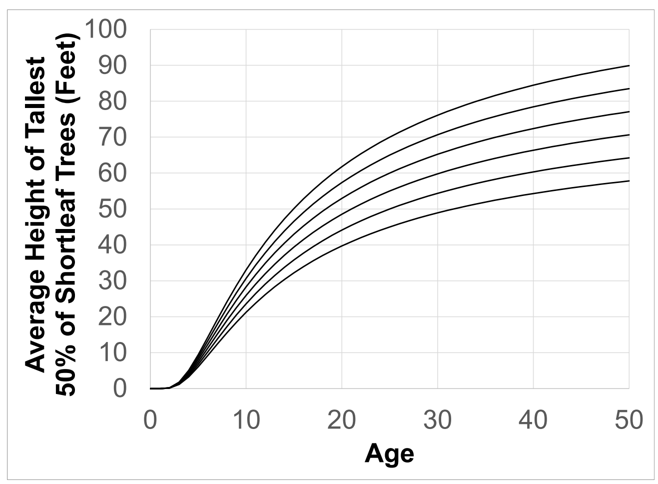


Figure 2—Dominant height across time for site indices of 45, 50, 55, 60, 65, and 70 feet (base age 25).

Mortality equation

Prior to any model fitting, biological considerations were taken into account for the selected model form. Mortality patterns over time of each experimental unit (repeated measurements of a plot) were plotted over age, dominant height (Ht_D), and existing trees per acre to determine if any trends in mortality were visually observable. Some slight relationships were visually observed between mortality rates and stand-level variables; however, the relationships did not appear strong. Hence, model fitting criteria (adjusted R^2) were used to a greater degree to determine the final equation selected. Additionally, the biological appropriateness of parameter estimates was strongly considered. Illogical behavior in the sign or magnitude of parameter estimates could result in the exclusion of a model form. Many equation forms were considered (Zhao and others 2007) and the following equation was selected:

$$N_2 = N_1 e^{-0.06109\,(A_2^{0.546548} - A_1^{0.546548})} \qquad (5)$$

n = 168, adjusted R^2 = 0.9846

where

N_2 = trees per acre at the current time (or time 2)

N_1 = previous trees per acre (or time 1)

A_2 = current age (or age at time 2)

A_1 = previous age (or age at time 1)

Equation (5) assumed the instantaneous rate of mortality changes with age.

All parameter estimates had a significance of $P < 0.0001$. To examine for trends in model residuals, the residuals of equation (5) were plotted over age and previous trees per acre (N_1); residuals were also examined for other acceptable equations. Residual plots appeared similar regardless of the equation tested; however, equation (5) provided a better prediction of those values with large predictive errors.

Future work will concentrate on a more complete analysis of mortality patterns in these stands. Most likely a mortality estimation system that includes two predictive steps will provide more biologically meaningful mortality estimates (Woollons 1998, Zhao and others 2007). In the first step, the probability of any mortality occurring is predicted, and then the second step, if mortality is actually predicted to occur, predicts the reduction in trees per acre (or conversely, the number of trees surviving per acre). This approach is biologically superior because often mortality is sporadic and does not occur in a consistently systematic manner every year over time. Predictive behavior of equation (5) is demonstrated in figure 3.

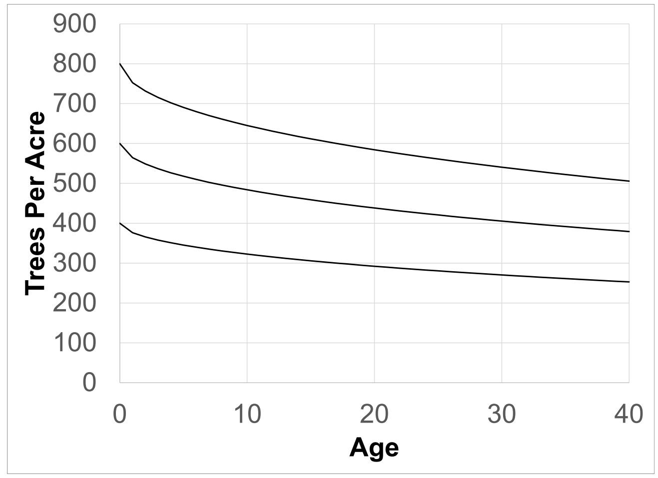


Figure 3—Number of planted shortleaf pine trees surviving per acre for planting densities of 400, 600, and 800 seedlings per acre. Curves of a particular planting density begin at the planting density.

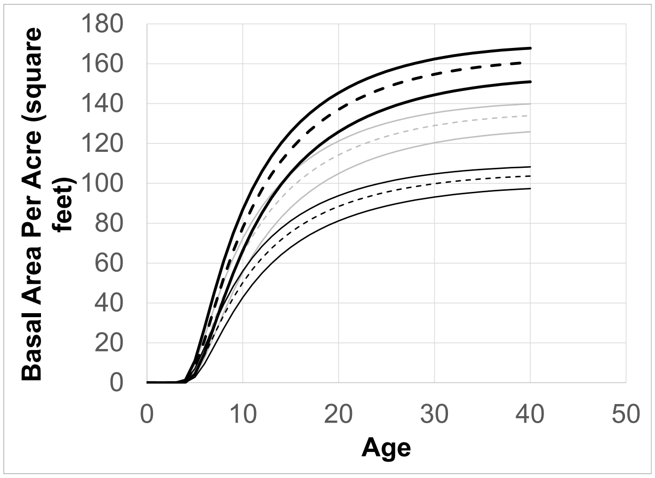
Basal area per acre equation

Several equation forms were initially evaluated and considered to estimate basal area per acre (BAA). An algebraic difference equation, in addition to other variables, includes the lag, or the previous measurement, of basal area to predict the current basal area. Due to reductions in sample size mainly resulting from the lack of diameter measurements at younger ages, and hence basal area lag values (or the basal area per acre at the previous measurement age), no algebraic difference equations were included in the selection of the final equation. Hence, several equations from the literature, not of an algebraic difference form, were tested that included dominant height (a measure of site quality). Ultimately, equation (6) was selected:

$$BAA = e^{1.57094 + 0.63234 \ln N - 5.88784\,(1/A) - 18.6497(1/Ht_D)} \qquad (6)$$

n = 124, adjusted R^2 = 0.8947

All parameter estimates had a $P < 0.0197$. The signs of all parameter estimates seem biologically reasonable. Basal area per acre estimates when using equation (6) are demonstrated in figure 4.



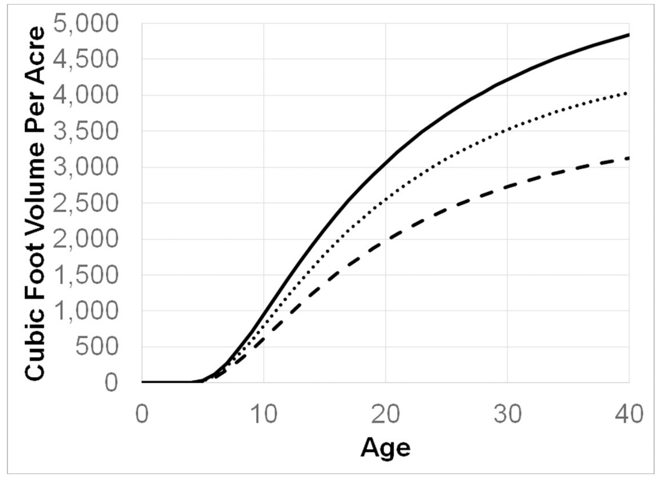


Figure 4—Basal area per acre of planted shortleaf pine for the combination of three site indices and three planting densities per acre. The light black curves = planting density of 400 seedlings per acre; gray curves = planting density of 600 seedlings per acre; and bold black curves = planting density of 800 seedlings per acre. For a particular planting density, the lower solid curve = site index of 45 feet (base age 25); dashed curve = site index of 55 feet; and upper solid curve = site index of 65 feet.

Figure 5—Total cubic foot volume per acre of planted shortleaf pine for a site index 55 feet site (base age 25) and three planting densities of 400 (dashed line), 600 (dotted line), and 800 (solid line) seedlings per acre.

Volume per acre (VAA) equation

Volume per acre can often be expressed as a function of site quality and stocking (Hall and others 2020). Thus, a constant form factor approach can be used, where volume is a function of dominant height, representing site quality, and basal area per acre, representing stocking. Additionally, the use of these two variables can be justified on a functional—even structural—level. This is because volume is a three-dimensional variable, and therefore basal area can be seen as providing two dimensions while dominant height provides the third dimension. Additionally, as expressed in the Schumacher (1939) model, the inverse of age can be included as a regressor. These two model forms were tested. Similar to fitting the basal area per acre equations, due to reductions in sample size mainly resulting from the lack of diameter measurements at younger ages, and hence volume lag values (or the volume per acre at the previous measurement age), no algebraic difference equations were considered for the equation form to be selected. Ultimately, the Schumacher (1939) equation form was selected:

$$VAA = e^{0.606567 + 0.996428 \ln BAA + 0.691797 \ln Ht_D - 3.38497 (1/A)} \quad (7)$$

n = 124, adjusted R^2 = 0.9977

For both equations, parameter estimates had a significance of *P* <0.0014. Equation (7) (Schumacher 1939) was selected because it had a higher fit index, but it also produced better residual error behavior. The maximum residual error was reduced in magnitude, from 967 cubic feet for the equation from Hall and others (2020) to 186 cubic feet for equation (7), and the residuals from equation (7) formed a random pattern unlike those residuals from the equation found in Hall and others (2020) that exhibited a systematic relationship across the domain of the regressors. Cubic foot volume per acre estimates when using equation (7) are demonstrated in figure 5.

Simultaneous estimation of the equation system (Borders 1989) was not conducted. The procedure was not applied because sample sizes would have been reduced for the dominant height and survival equations because d.b.h. was not measured at younger ages, which resulted in smaller sample sizes when fitting basal area and volume per acre equations.

VALIDATION AND DISCUSSION

For all equations, parameter estimates exhibited logical and biologically sound magnitudes and signs.

The model system of equations (4) to (7) were tested using two sources of published data (Williston 1958, 1985). The first validation study

Table 3—Observed values by validation study and age, predicted values for the current system of equations, and predictions from Echinata

Study	Age	Trees per acre			Basal area			Dominant height		
		Obs	Current system Eq. (5)	Echinata	Obs	Current system Eq. (6)	Echinata	Obs	Current system Eq. (4)	Echinata
		~ ~ ~ ~ ~ ~ ~ ~ ~ ~ number	~ ~ ~ ~ ~ ~ ~ ~ ~ ~		~ ~ ~ ~ square feet per acre	~ ~ ~ ~		~ ~ ~ ~ ~ ~ ~ ~ ~ feet	~ ~ ~ ~ ~ ~ ~ ~ ~	
Williston (1958)	13	1,098	1,359	1,432	132	162	134	23	32	30
	23	1,064	1,241	1,153	231	228	198	48[a]	–	–
Williston (1985)	11	692	552	649	99	89	108	32	35	33
	14	692	534	629	133	110	131	38	44	42
	16	685	524	607	147	120	142	44	50	47
	26	499	482	443	162	144	165	67[a]	–	–

Echinata uses the model system presented by Smalley and Bailey (1974).

The Williston (1958) study was planted at 1,742 seedlings per acre; the Williston (1985) study was planted at 692 seedlings per acre.

For equations (5) and (6), estimates used dominant height estimates from equation (4). Similarly, for Echinata, trees per acre and basal area predictions were based on its internal dominant height equation.

Dominant height for the younger ages was predicted separately for the current model system and Echinata using their respective estimated site indices.

Obs = observed values from Williston (1958) or Williston (1985).

[a] Site index was determined separately for the current system (equation 3) and Echinata (Smalley and Bower 1971) by using the dominant height nearest to the base age of 25 years.

– = no data.

(Williston 1958) was from north Mississippi (Benton County) and data are average conditions of 10 plantations at ages 13 and 23 (table 3). Plantations were established at a planting density of 1,742 seedlings per acre. The second validation study was from north central Mississippi in a plantation established at a density of 692 seedlings per acre (Williston 1985). Since equation (7) only predicted total cubic foot volume (all d.b.h., to the tip of a tree), volume predictions could not be validated using these two studies due to differences in the minimum d.b.h. of trees included. Beyond that, both studies reported inside-bark volume to a 3-inch top.

Additionally, the equation system presented here was compared to predictions from an equation system presented by Smalley and Bailey (1974), using the two validation sources mentioned above. The equation system presented by Smalley and Bailey (1974) was incorporated into an Excel (Microsoft, Redmond, WA) spreadsheet to produce a timber decision support system (TDSS) entitled Echinata. Data used in model fitting were from Alabama, Georgia, and Tennessee highlands.

Trees per Acre

It appeared that for higher planting densities (Williston 1958), equation (5) overpredicted survival (table 3). Similarly, so did Echinata. Of 168 observations used in the model fitting of equation (5), only 44 observations were established using a planting density of 1,210 seedlings per acre or greater. A total of 18 observations were established with a planting density of 1,742 seedlings per acre, but all of them were from the Arkansas planting density study. A total of 26 observations had a planting density of 1,210 seedlings per acre, split between the Arkansas and Tennessee studies. Hence, the current model system may not be the most applicable for such high planting densities. For Williston (1985), equation (5) produced better predictions relative to Echinata at age 26, but Echinata produced superior estimates for the younger ages.

Basal Area per Acre

Similar to trees per acre, the current predictive model system overpredicted basal area for higher planting densities at age 13 (Williston 1958). However, Echinata produced a very accurate estimate. At age 23 for Williston (1958), the current model system produced a highly accurate estimate while Echinata underpredicted. Perhaps the current model system predicted accurate carrying capacity values for these high planting densities, just not mortality rates prior to when the carrying capacities are reached.

Based on trees per acre and basal area per acre, the current model system likely overpredicts for younger ages in high planting density plantations. It is difficult to tell if the overpredictions were exclusively due to a greater amount of lower planting density observations (only 18 observations used in model fitting exceeded a planting density of 1,500 seedlings per acre) within the model-fitting dataset or perhaps also due to improved survival rates in the current modeling effort because of better reforestation practices.

For Williston (1985), the current model system and Echinata had essentially the same amount of bias at age 11, but with opposing signs. However, Echinata produced very good estimates at ages 14, 16, and 26. Across all ages, the current model system underpredicted. Across all ages at the Williston (1985) site, at most, the current model system predicted incorrectly by 18.4 percent (age 16). For both sites when using the current model system, for ages in excess of 20 years, errors were 1.3 percent at age 23 (Williston 1958) and 11.1 percent at age 26 (Williston 1985).

Dominant Height

When using equation (4) the estimated site indices (base age 25) were 50 and 66 feet for Williston (1958) and (1985), respectively. Similar estimates were observed when using Echinata and Smalley and Bower (1971) and were 51 feet and 65 feet, respectively. When predicting dominant heights based on the estimated site indices (base age 25) calculated using the oldest ages of 23 (Williston 1958) and 26 (Williston 1985) years, equation (4) and Echinata had very similar predictions. However, Echinata, as a whole, produced slightly better results. The results of most interest, for current plantations, are likely those from Williston (1985). It appears that reasonable predictions will be obtained for dominant height regardless of whether equation (4) or Echinata are used. For Williston (1985), equation (4) had prediction errors of 15.8 percent or less.

One issue with Echinata is that for planting densities less than 500 seedlings per acre, Echinata will often produce illogical estimates of survival. Often, the number of trees per acre surviving at ages 10 and 15 will exceed the number planted. This behavior likely results from the model system by Smalley and Bailey (1974) being fit using only one observation with a planting density less than 500 seedlings per acre and the particular mortality equation form selected. Of 104 plots, 75 of them had planting densities in excess of 1,000 seedlings per acre. Conversely, for the current model dataset, half of the observations have planting densities less than 545 seedlings per acre, and a total of 16 plots have a planting density less than 500 seedlings per acre. For planting densities less than 500 seedlings per acre, predictions of survival are logical when using equation (5).

FUTURE WORK

Future work will need to concentrate on validating using observed plot-level data and not published, plot summarized data. Additionally, validation should be of more recently established plantations. Nonetheless, this brief validation analysis provides some idea of the predictive ability of the current model system [equations (3) to (7)].

This is an initial attempt to summarize shortleaf pine growth and yield literature; identify and consolidate sources of data; gain insight into the biological growth relationships associated with shortleaf pine plantations; and fit, examine, and test various equation functional forms. Future work needs to concentrate on a much more comprehensive dataset. The dataset should be geographically comprehensive in the western half of the Southeastern United States, but also comprehensive in terms of site conditions such as old-field versus cutover, bareroot versus container, genetics of seedlings, planting density, planting configuration, the inclusion of thinning, among many other considerations. It is hoped that this initial work will spark interest in sharing current datasets and establishment of additional studies.

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