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NONPAR Methodology and Tables

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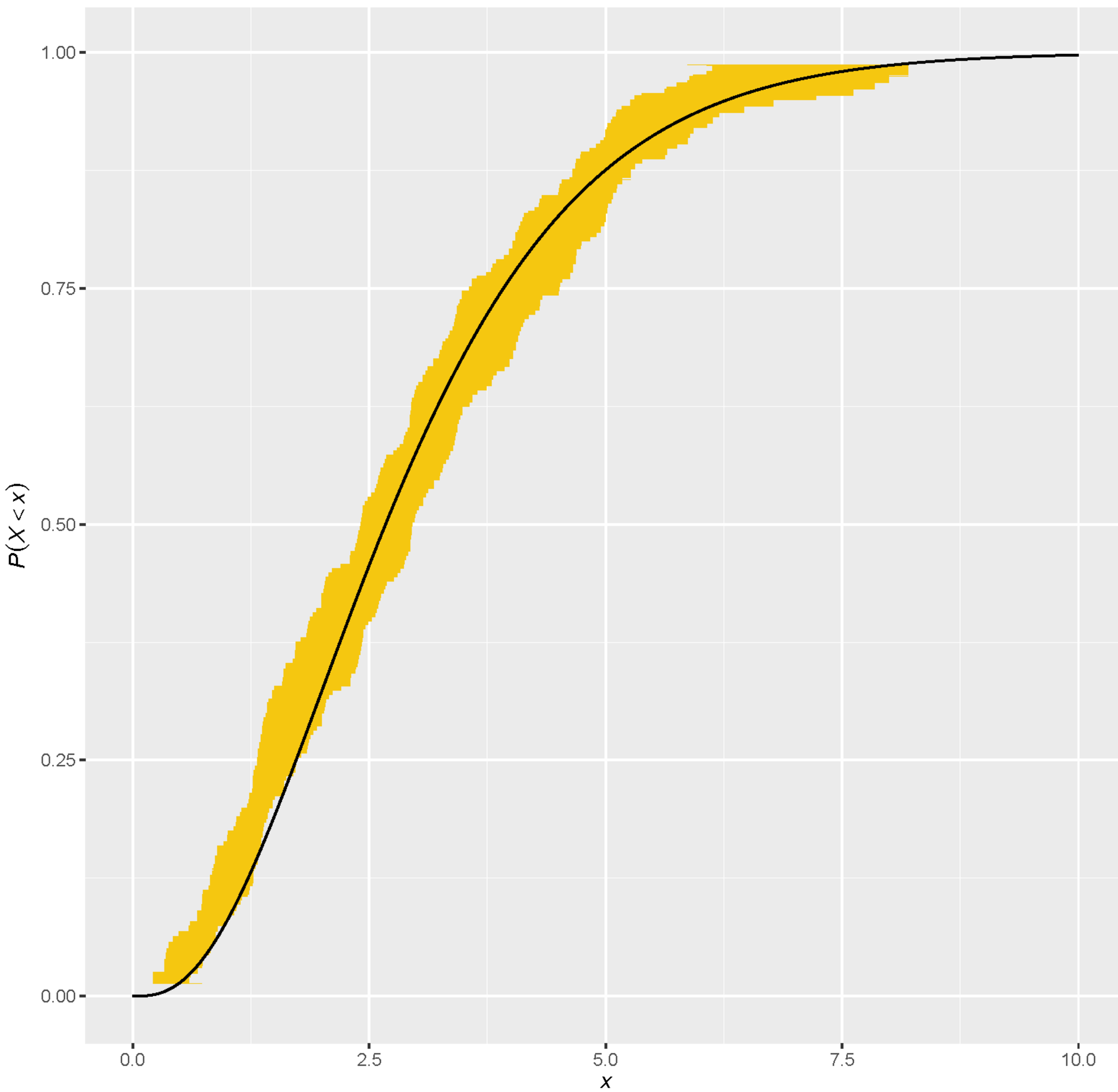

Abstract

The Forest Products Laboratory NONPAR procedure is explained in mathematical detail and demonstrated with examples. The procedure for constructing confidence intervals is updated from a previous version. Tables listing order statistics representing lower tolerance limits and confidence intervals for the 5th percentile are exhibited for sample sizes up to 5,000.

Keywords: Nonparametric, quantile estimation, lower tolerance limit, confidence interval

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Contents

On the cover: The plot is a representation of nonparametric confidence intervals for quantiles. The black curve is the actual (usually unknown) distribution function for the underlying random variable from which a random sample has been collected; in this case it is a Gamma distribution with shape parameter 3 and scale parameter 1. For any given quantile selected from the vertical axis, the horizontal range covered by the yellow–orange shading represents the 95% confidence interval. Granularity and imperfection over this continuum of quantiles is clearly apparent. Represented here is a particular sample of size 225. As the sample size increases, both of these characteristics would reduce; that is, the confidence intervals would become narrower and conform better to the underlying distribution, and the granularity would smooth. Lack of coverage at the tails (top and bottom) is because nonparametric confidence intervals do not exist for sufficiently extreme quantiles; these extremes intensify (get nearer to 0 and 1) as the sample size increases.

NONPAR Methodology and Tables

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1 Introduction

NONPAR (Nonparametric Estimation Program) is a simple procedure that provides nonparametric tolerance limits, confidence intervals, and point estimates for the 5th percentile of a population, based on a sample of observations. Its uses primarily include the determination of design values for dimension lumber properties, namely those based on the 5th percentile of the population. This research note outlines the latest version of the NONPAR procedure and provides tables listing its results pertaining to the 5th percentile. In the previous version of NONPAR, the confidence intervals were determined by way of the commonly used normal approximation method. In this research note, a revised method is introduced for two reasons. First, advances in computing capability have provided the opportunity for more computationally intensive methods. Second, unlike the revised method, the normal approximation method is not purely nonparametric, can give erroneous results when presented with insufficient data, especially for the small sample sizes typically encountered in the context of lumber properties, and does not guarantee coverage probabilities. This revised procedure is constructed in the R programming language, and the publicly available web-based version, found on the FPL statistics web site [URL pending], is rendered using the Shiny package in R. For reference, the original FORTRAN code shall remain indefinitely on the FPL statistics web site with the label "Original NONPAR FORTRAN Code (Obsolete)."

This paper is organized as follows. In Section 2, the methodology for obtaining point estimates, tolerance limits, and confidence intervals for an arbitrary quantile are presented in complete mathematical detail, followed by examples illustrating its implementation specifically for the 5th percentile. Section 3 details how to use tables similar to those found in ASTM standard D2915, table 2 (ASTM International 2022). Also included are extended versions of these tables and similarly constructed tables used to obtain nonparametric confidence intervals using the procedure detailed in Section 2.3. We close with some summarizing remarks in Section 4.

2 Methodology

Consider a random sample, $\boldsymbol{X} = (X_1, X_2, ..., X_m)$, $m \in \mathbb{Z}^+$, obtained from an unknown continuous distribution, $F(x)$. We can restate this as $\boldsymbol{X} = \{X_{(1)}, X_{(2)}, ..., X_{(m)}\}$ such that $X_{(1)} < X_{(2)} < ... < X_{(m)}$. Consequently, the random variable, $X_{(r)}$, is the rth lowest value in $\boldsymbol{X}$ and is called the rth order statistic, for all $r \in \{1, 2, ..., m\}$. Suppose we are interested in the quantile, $x_q = F^{-1}(q)$, $q \in (0, 1)$. This means that $100q\%$ of the population from which $\boldsymbol{X}$ is drawn is less than x_q, and $100(1-q)\%$ of the population is greater than x_q. The value, x_q, is sometimes referred to as the "qth quantile," or "the quantile, q," assuming the context is clear. The most well-known example of a quantile is the median, where $q = 0.5$. (Note that the median should not be confused with the mean. It is defined entirely differently, though there are cases where the mean and the median can be identical, e.g., whenever F has a finite first moment and is symmetric.) The word, "percentile" is just a colloquial metonym for "quantile." Simply put, x_q is the $100q$th percentile, so these terms may (carefully) be used interchangeably.

A well-known result (Thompson 1936, Wilks 1941) provides that, for any $r \in \{1, 2, ..., m\}$,

$$P\left(X_{(r)} < x_q\right) = \sum_{j=r}^{m} \binom{m}{j} q^j (1-q)^{m-j} \tag{2.1}$$

This result can be readily derived from the fact that $F(x_q) = q$, and, consequently, the right-hand side does not depend on F. Hence, it may be used to obtain nonparametric, i.e., distribution-free, inferences about F, particularly about its parameter, x_q. In the following sections, we construct a point estimator and use Eq. (2.1) to obtain a lower tolerance limit (LTL) and a confidence interval (CI) for x_q, based on $\boldsymbol{X}$.

2.1 Point Estimator

A point estimator, $\hat{x}_q$, of x_q is any function of $\boldsymbol{X}$ that will tend to be near x_q with the smallest bias and variance possible. The most well-known nonparametric form of $\hat{x}_q$ is given by

$$\hat{x}_q = (1-\theta)X_{(r)} + \theta X_{(r+1)} \tag{2.2}$$

where $r = r(m, q) = \lfloor (m+1)q \rfloor$, and $\theta = \theta(m, q) = (m+1)q - r(m, q)$. In this context, $\lfloor \cdot \rfloor$ is the floor, or "greatest integer," function. This estimator is essentially a result of linear interpolation between two consecutive order statistics. Although it is not necessarily unbiased, it is a consistent estimator. It is also easy to see that this estimator exists if and only if $1 \leq (m+1)q < m$. Because, in the case that $\hat{x}_q$ does not exist, some nonparametric LTLs and CIs are likely to also not exist (or have little meaning), we assume m and q satisfy this inequality from this point forward.

2.2 Lower Tolerance Limits

For any $p \in (0, 1)$, a $100p\%$ LTL for $100(1-q)\%$ of the population is defined to be any random value, L, that satisfies $P(L < x_q) \geq p$. To use $\boldsymbol{X}$ to find a value for L, we identify the largest value of $r \in \{1, 2, ..., m\}$ for which $P\left(X_{(r)} < x_q\right) \geq p$, according to Eq. (2.1). Then, $L = X_{(r)}$ is a $100p\%$ LTL for $100(1-q)\%$ of the population, based on $\boldsymbol{X}$. It is also stated in this paper as a $100p\%$ LTL for x_q (though, in this context, it is often called a lower confidence bound). It should also be noted that, using this procedure, $X_{(r)}$ can serve as a $100p\%$ LTL for the qth quantile for any $p \leq P\left(X_{(r)} < x_q\right)$, though it is clearly conservative in all cases except equality.

Remark Upper tolerance limits (UTL) are similar to LTLs. In fact, if L satisfies $P(L < x_q) = p$, then L is clearly a $100p\%$ LTL for the qth quantile. In addition, L is a $100(1-p)\%$ UTL for the qth quantile. The latter, however, does not hold if $P(L < x_q) > p$.

2.3 Confidence Intervals

Typically, a CI is constructed by obtaining some point estimate, PE, and "margin for error," ME, based on $\boldsymbol{X}$, and identifying $(\mathrm{PE}-\mathrm{ME}, \mathrm{PE}+\mathrm{ME})$ as the CI, e.g., by assuming PE is normally distributed. In the nonparametric approach, no such method is used. Rather, the very same set of assumptions and equation are used to construct a CI as are for a LTL. To illustrate this, note that applying Eq. (2.1), for any selected $r_1, r_2 \in \{1, 2, ..., m\}$, where $r_1 < r_2$, there are corresponding $p_1, p_2 \in (0,1)$ satisfying

$$P\left(X_{(r_1)} < x_q\right) = p_1 \text{ and } P\left(X_{(r_2)} < x_q\right) = p_2 \tag{2.3}$$

so that $\left(X_{(r_1)}, X_{(r_2)}\right)$ is a nonparametric $100(p_1 - p_2)\%$ CI for x_q, based on $\boldsymbol{X}$. We want a $100p\%$ CI, $p \in (0,1)$. There are potentially many values of r_1 and r_2 satisfying the constraint, $p_1 - p_2 \geq p$, so we apply a reasonable set of restrictions to r_1 and r_2:

1. $r_2 - r_1$ is minimized, and

2. for the minimum value of $r_2 - r_1$, $p_1 - p_2$ is maximized.

Because Eq. (2.1) is based on the binomial distribution, specifically a Binomial(m, q), as long as $P\left(X_{(1)} < x_q\right) - P\left(X_{(m)} < x_q\right) \geq p$, these restrictions will always result in a valid finite-length CI containing $\hat{x}_q$ with (r_1, r_2) satisfying $1 \leq r_1 < r_2 \leq m$.

A simple search algorithm, which exploits the characteristics of the binomial distribution, is applied to find the order statistics for the CI. It proceeds as follows:

1. Compute $p_{\min} = P\left(X_{(1)} < x_q\right)$ and $p_{\max} = P\left(X_{(m)} < x_q\right)$ using Eq. (2.1). If $p_{\min} - p_{\max} < p$, then stop; no nonparametric $100p\%$ CI exists. Otherwise, proceed to Step 2.

2. Set $r_1 = \lfloor (m+1)q \rfloor$ and $r_2 = \lceil (m+1)q \rceil$, where $\lceil \cdot \rceil$ is the ceiling function, and compute $p_1 = P\left(X_{(r_1)} < x_q\right)$ and $p_2 = P\left(X_{(r_2)} < x_q\right)$ using Eq. (2.1).

3. If $p_1 - p_2 \geq p$, then stop; $(X_{(r_1)}, X_{(r_2)})$ is a nonparametric $100p\%$ CI for x_q.

4. If $r_1 > 1$, set $r_1^* = r_1 - 1$ and compute $p_1^* = P\left(X_{(r_1^*)} < x_q\right)$ using Eq. (2.1); otherwise set $r_1^* = r_1$ and $p_1^* = p_1$. If $r_2 < m$, set $r_2^* = r_2 + 1$ and compute $p_2^* = P\left(X_{(r_2^*)} < x_q\right)$ using Eq. (2.1); otherwise set $r_2^* = r_2$ and $p_2^* = p_2$.

5. If $p_1^* - p_2 > p_1 - p_2^*$, then set $r_1 = r_1^*$ and $p_1 = p_1^*$. Otherwise, set $r_2 = r_2^*$ and $p_2 = p_2^*$. Return to Step 3.

This algorithm guarantees the two restrictions above are satisfied.

2.4 Example

Suppose $m = 312$, and $q = 0.05$. Then the order statistic representing the nonparametric 75% LTL for the 5th percentile is the largest value, r, for which $P\left(X_{(r)} < x_{0.05}\right) \geq 0.75$. In this case, since

$$\sum_{j=13}^{312} \binom{312}{j} 0.05^j (0.95)^{312-j} = 0.7858 \tag{2.4}$$

and

$$\sum_{j=14}^{312} \binom{312}{j} 0.05^j (0.95)^{312-j} = 0.6976 \tag{2.5}$$

a 75% LTL for 95% of the population is $X_{(13)}$. For the nonparametric 95% CI for $x_{0.05}$, it can be shown that, according to Eq. (2.1), for $P\left(X_{(r_1)} < x_{0.05} < X_{(r_2)}\right) \geq 0.95$ to hold, $r_2 - r_1$ must be at least 16. We have,

$$P\left(X_{(8)} < x_{0.05} < X_{(24)}\right) = 0.9634 \tag{2.6}$$

which is greater than the same value for any other r_1 and r_2 satisfying $r_2 - r_1 = 16$. So a nonparametric 95% CI for x_q is $(X_{(8)}, X_{(24)})$.

3 LTL and CI Tables for the 5th Percentile

Tables 1 and 2 list the order statistics representing the 75% and 95% LTL for 95% of the population, that is, the 75% and 95% LTLs for $x_{0.05}$, respectively. These two tables are extended versions of those found in ASTM standard D2915, table 2 (ASTM International 2022). Tables 3 and 4 list the order statistics for the 75% and 95% CI for $x_{0.05}$, respectively. All tables are generated by the procedures detailed in Sections 2.2 and 2.3 and cover sample sizes up to at least 5,000. For each of the tables, the sample size column depicts a minimum threshold. For example, suppose m is 417. Then, since $412 \leq m < 433$, the 75% LTL is $X_{(18)}$, and since $417 \leq m < 438$, the 95% CI is $(X_{(13)}, X_{(31)})$. Also note that if $m < 28$, then no nonparametric 75% LTL exists, and if $m < 59$, no nonparametric 95% CI exists.

4 Conclusion

This work provides a detailed explanation of the NONPAR procedure. It is a well-established procedure for obtaining point estimates, tolerance limits, and confidence intervals for a quantile of an unknown distribution based on an observed sample. Although the examples and tables exhibited in this work address specifically the 5th percentile, these procedures are easily applied to any quantile. Refer to FPL's statistics web site for tools related to this procedure.

Table 1: Minimum population sizes (m) for 75% LTL order statistics (r) for $x_{0.05}$.

m	r	m	r	m	r	m	r	m	r	m	r
28	1	921	42	1776	83	2623	124	3465	165	4305	206
53	2	942	43	1797	84	2643	125	3486	166	4325	207
78	3	963	44	1817	85	2664	126	3506	167	4346	208
102	4	984	45	1838	86	2684	127	3527	168	4366	209
125	5	1005	46	1859	87	2705	128	3547	169	4387	210
148	6	1026	47	1880	88	2726	129	3568	170	4407	211
170	7	1047	48	1900	89	2746	130	3588	171	4428	212
193	8	1068	49	1921	90	2767	131	3609	172	4448	213
215	9	1089	50	1942	91	2787	132	3629	173	4469	214
237	10	1110	51	1962	92	2808	133	3650	174	4489	215
259	11	1131	52	1983	93	2828	134	3670	175	4510	216
281	12	1152	53	2004	94	2849	135	3691	176	4530	217
303	13	1173	54	2024	95	2870	136	3711	177	4550	218
325	14	1194	55	2045	96	2890	137	3732	178	4571	219
347	15	1215	56	2066	97	2911	138	3752	179	4591	220
368	16	1235	57	2086	98	2931	139	3773	180	4612	221
390	17	1256	58	2107	99	2952	140	3793	181	4632	222
412	18	1277	59	2128	100	2972	141	3814	182	4653	223
433	19	1298	60	2148	101	2993	142	3834	183	4673	224
455	20	1319	61	2169	102	3013	143	3855	184	4694	225
476	21	1340	62	2190	103	3034	144	3875	185	4714	226
498	22	1361	63	2210	104	3055	145	3896	186	4734	227
519	23	1381	64	2231	105	3075	146	3916	187	4755	228
540	24	1402	65	2252	106	3096	147	3937	188	4775	229
562	25	1423	66	2272	107	3116	148	3957	189	4796	230
583	26	1444	67	2293	108	3137	149	3978	190	4816	231
604	27	1465	68	2314	109	3157	150	3998	191	4837	232
626	28	1485	69	2334	110	3178	151	4018	192	4857	233
647	29	1506	70	2355	111	3198	152	4039	193	4877	234
668	30	1527	71	2375	112	3219	153	4059	194	4898	235
689	31	1548	72	2396	113	3239	154	4080	195	4918	236
710	32	1569	73	2417	114	3260	155	4100	196	4939	237
732	33	1589	74	2437	115	3280	156	4121	197	4959	238
753	34	1610	75	2458	116	3301	157	4141	198	4980	239
774	35	1631	76	2478	117	3322	158	4162	199	5000	240
795	36	1652	77	2499	118	3342	159	4182	200	5020	241
816	37	1672	78	2520	119	3363	160	4203	201	5041	242
837	38	1693	79	2540	120	3383	161	4223	202	5061	243
858	39	1714	80	2561	121	3404	162	4244	203	5082	244
879	40	1735	81	2581	122	3424	163	4264	204	5102	245
900	41	1755	82	2602	123	3445	164	4285	205	5123	246

Table 2: Minimum population sizes (m) for 95% LTL order statistics (r) for $x_{0.05}$.

m	r	m	r	m	r	m	r	m	r	m	r
59	1	1058	42	1963	83	2848	124	3723	165	4591	206
93	2	1081	43	1985	84	2869	125	3744	166	4612	207
124	3	1103	44	2006	85	2891	126	3765	167	4633	208
153	4	1126	45	2028	86	2912	127	3786	168	4654	209
181	5	1148	46	2050	87	2934	128	3808	169	4675	210
208	6	1170	47	2071	88	2955	129	3829	170	4697	211
234	7	1193	48	2093	89	2976	130	3850	171	4718	212
260	8	1215	49	2115	90	2998	131	3871	172	4739	213
286	9	1237	50	2137	91	3019	132	3893	173	4760	214
311	10	1260	51	2158	92	3041	133	3914	174	4781	215
336	11	1282	52	2180	93	3062	134	3935	175	4802	216
361	12	1304	53	2202	94	3083	135	3956	176	4823	217
386	13	1326	54	2223	95	3105	136	3977	177	4844	218
410	14	1348	55	2245	96	3126	137	3999	178	4865	219
434	15	1371	56	2266	97	3147	138	4020	179	4886	220
458	16	1393	57	2288	98	3169	139	4041	180	4907	221
482	17	1415	58	2310	99	3190	140	4062	181	4929	222
506	18	1437	59	2331	100	3211	141	4083	182	4950	223
530	19	1459	60	2353	101	3233	142	4105	183	4971	224
554	20	1481	61	2375	102	3254	143	4126	184	4992	225
577	21	1503	62	2396	103	3276	144	4147	185	5013	226
601	22	1525	63	2418	104	3297	145	4168	186	5034	227
624	23	1547	64	2439	105	3318	146	4189	187	5055	228
647	24	1569	65	2461	106	3340	147	4210	188	5076	229
671	25	1591	66	2482	107	3361	148	4232	189	5097	230
694	26	1613	67	2504	108	3382	149	4253	190	5118	231
717	27	1635	68	2525	109	3403	150	4274	191	5139	232
740	28	1657	69	2547	110	3425	151	4295	192	5160	233
763	29	1679	70	2569	111	3446	152	4316	193	5181	234
786	30	1701	71	2590	112	3467	153	4337	194	5202	235
809	31	1723	72	2612	113	3489	154	4359	195	5223	236
832	32	1745	73	2633	114	3510	155	4380	196	5244	237
855	33	1766	74	2655	115	3531	156	4401	197	5265	238
877	34	1788	75	2676	116	3553	157	4422	198	5287	239
900	35	1810	76	2698	117	3574	158	4443	199	5308	240
923	36	1832	77	2719	118	3595	159	4464	200	5329	241
945	37	1854	78	2741	119	3616	160	4485	201	5350	242
968	38	1876	79	2762	120	3638	161	4507	202	5371	243
991	39	1897	80	2783	121	3659	162	4528	203	5392	244
1013	40	1919	81	2805	122	3680	163	4549	204	5413	245
1036	41	1941	82	2826	123	3701	164	4570	205	5434	246

Table 3: Minimum population sizes (m) for 75% CI order statistics (r_1 & r_2) for $x_{0.05}$.

m	r_1	r_2	m	r_1	r_2	m	r_1	r_2	m	r_1	r_2	m	r_1	r_2
28	1	5	1035	44	61	2096	94	117	3135	143	172	4235	196	229
64	1	6	1055	45	62	2108	94	118	3155	144	173	4255	197	230
74	2	7	1075	46	63	2125	95	119	3175	145	174	4275	198	231
96	3	8	1095	47	64	2145	96	120	3195	146	175	4295	199	232
103	2	8	1115	48	65	2165	97	121	3215	147	176	4316	200	233
104	3	9	1135	49	66	2185	98	122	3235	148	177	4333	200	234
125	4	10	1152	49	67	2205	99	123	3255	149	178	4345	201	235
144	4	11	1165	50	68	2225	100	124	3275	150	179	4365	202	236
155	5	12	1185	51	69	2245	101	125	3295	151	180	4385	203	237
175	6	13	1205	52	70	2265	102	126	3315	152	181	4405	204	238
195	6	14	1225	53	71	2286	103	127	3336	153	182	4425	205	239
205	7	15	1245	54	72	2295	103	128	3348	153	183	4445	206	240
225	8	16	1265	55	73	2315	104	129	3365	154	184	4465	207	241
245	9	17	1286	56	74	2335	105	130	3385	155	185	4485	208	242
258	9	18	1292	55	74	2355	106	131	3405	156	186	4505	209	243
275	10	19	1295	56	75	2375	107	132	3425	157	187	4525	210	244
295	11	20	1315	57	76	2395	108	133	3445	158	188	4545	211	245
315	12	21	1335	58	77	2415	109	134	3465	159	189	4566	212	246
326	12	22	1355	59	78	2435	110	135	3485	160	190	4586	213	247
345	13	23	1375	60	79	2455	111	136	3505	161	191	4600	213	248
365	14	24	1395	61	80	2476	112	137	3525	162	192	4615	214	249
385	15	25	1415	62	81	2489	112	138	3545	163	193	4635	215	250
400	15	26	1435	62	82	2505	113	139	3566	164	194	4655	216	251
415	16	27	1445	63	83	2525	114	140	3582	164	195	4675	217	252
435	17	28	1465	64	84	2545	115	141	3595	165	196	4695	218	253
455	18	29	1485	65	85	2565	116	142	3615	166	197	4715	219	254
476	19	30	1505	66	86	2585	117	143	3635	167	198	4735	220	255
485	19	31	1525	67	87	2605	118	144	3655	168	199	4755	221	256
505	20	32	1545	68	88	2625	119	145	3675	169	200	4775	222	257
525	21	33	1565	69	89	2645	120	146	3695	170	201	4795	223	258
545	22	34	1586	70	90	2666	121	147	3715	171	202	4815	224	259
566	23	35	1595	70	91	2686	122	148	3735	172	203	4836	225	260
577	23	36	1615	71	92	2692	121	148	3755	173	204	4856	226	261
595	24	37	1635	72	93	2695	122	149	3775	174	205	4873	226	262
615	25	38	1655	73	94	2715	123	150	3796	175	206	4885	227	263
635	26	39	1675	74	95	2735	124	151	3816	176	207	4905	228	264
655	27	40	1695	75	96	2755	125	152	3826	176	208	4925	229	265
673	27	41	1715	76	97	2775	126	153	3845	177	209	4945	230	266
685	28	42	1735	77	98	2795	127	154	3865	178	210	4965	231	267
705	29	43	1754	77	99	2815	128	155	3885	179	211	4985	232	268
725	30	44	1765	78	100	2835	129	156	3905	180	212	5005	233	269
745	31	45	1785	79	101	2855	130	157	3925	181	213	5025	234	270
765	32	46	1805	80	102	2876	131	158	3945	182	214	5045	235	271
782	32	47	1825	81	103	2896	132	159	3965	183	215	5065	236	272
795	33	48	1845	82	104	2903	131	159	3985	184	216	5085	237	273
815	34	49	1865	83	105	2905	132	160	4005	185	217	5105	238	274
835	35	50	1885	84	106	2925	133	161	4025	186	218	5126	239	275
855	36	51	1905	85	107	2945	134	162	4046	187	219	5146	240	276
875	37	52	1925	85	108	2965	135	163	4066	188	220	5158	240	277
895	37	53	1935	86	109	2985	136	164	4077	188	221	5175	241	278
905	38	54	1955	87	110	3005	137	165	4095	189	222	5195	242	279
925	39	55	1975	88	111	3025	138	166	4115	190	223	5215	243	280
945	40	56	1995	89	112	3045	139	167	4135	191	224	5235	244	281
965	41	57	2015	90	113	3065	140	168	4155	192	225	5255	245	282
985	42	58	2035	91	114	3085	141	169	4175	193	226	5275	246	283
1005	43	59	2055	92	115	3106	142	170	4195	194	227	5295	247	284
1021	43	60	2075	93	116	3121	142	171	4215	195	228	5315	248	285

Table 4: Minimum population sizes (m) for 95% CI order statistics (r_1 & r_2) for $x_{0.05}$.

m	r_1	r_2	m	r_1	r_2	m	r_1	r_2	m	r_1	r_2	m	r_1	r_2
59	1	10	988	37	64	1987	81	120	3027	128	176	4048	176	231
60	1	9	1003	37	65	2008	82	121	3038	129	177	4068	177	232
93	1	10	1017	38	66	2028	83	122	3058	130	178	4088	178	233
107	2	11	1038	39	67	2048	84	123	3078	131	179	4108	179	234
117	2	12	1058	40	68	2068	85	124	3098	132	180	4128	180	235
138	2	13	1076	40	69	2086	85	125	3118	133	181	4146	180	236
146	3	14	1087	41	70	2097	86	126	3138	134	182	4158	181	237
167	3	15	1107	42	71	2118	87	127	3158	134	183	4178	182	238
176	4	16	1128	43	72	2138	88	128	3168	135	184	4198	183	239
198	4	17	1148	44	73	2158	89	129	3188	136	185	4218	184	240
206	5	18	1158	44	74	2178	90	130	3208	137	186	4238	185	241
228	6	19	1177	45	75	2195	90	131	3228	138	187	4258	186	242
237	6	20	1198	46	76	2207	91	132	3248	139	188	4278	187	243
257	7	21	1218	47	77	2228	92	133	3268	140	189	4297	187	244
273	7	22	1236	47	78	2248	93	134	3288	141	190	4308	188	245
287	8	23	1247	48	79	2268	94	135	3295	140	190	4328	189	246
308	9	24	1268	49	80	2288	95	136	3298	141	191	4348	190	247
312	8	24	1288	50	81	2306	95	137	3318	142	192	4368	191	248
317	9	25	1308	51	82	2317	96	138	3338	143	193	4388	192	249
337	10	26	1322	51	83	2338	97	139	3358	144	194	4408	193	250
354	10	27	1337	52	84	2358	98	140	3378	145	195	4428	194	251
367	11	28	1358	53	85	2378	99	141	3398	146	196	4448	195	252
388	12	29	1378	54	86	2398	100	142	3418	147	197	4457	194	252
401	12	30	1398	55	87	2418	100	143	3431	147	198	4458	195	253
417	13	31	1409	55	88	2427	101	144	3448	148	199	4478	196	254
438	14	32	1427	56	89	2448	102	145	3468	149	200	4498	197	255
450	14	33	1448	57	90	2468	103	146	3488	150	201	4518	198	256
467	15	34	1468	58	91	2488	104	147	3508	151	202	4538	199	257
488	16	35	1488	59	92	2508	105	148	3528	152	203	4558	200	258
500	16	36	1498	59	93	2528	106	149	3548	153	204	4578	201	259
517	17	37	1517	60	94	2539	106	150	3566	153	205	4598	202	260
538	18	38	1538	61	95	2558	107	151	3578	154	206	4613	202	261
553	18	39	1558	62	96	2578	108	152	3598	155	207	4628	203	262
567	19	40	1578	63	97	2598	109	153	3618	156	208	4648	204	263
588	20	41	1589	63	98	2618	110	154	3638	157	209	4668	205	264
606	20	42	1607	64	99	2638	111	155	3658	158	210	4688	206	265
617	21	43	1628	65	100	2655	111	156	3678	159	211	4708	207	266
637	22	44	1648	66	101	2668	112	157	3698	160	212	4728	208	267
658	23	45	1668	67	102	2688	113	158	3710	160	213	4748	209	268
669	23	46	1683	67	103	2708	114	159	3728	161	214	4768	210	269
687	24	47	1697	68	104	2728	115	160	3748	162	215	4774	209	269
708	25	48	1718	69	105	2748	116	161	3768	163	216	4778	210	270
727	25	49	1738	70	106	2768	117	162	3788	164	217	4798	211	271
737	26	50	1758	71	107	2780	117	163	3808	165	218	4818	212	272
757	27	51	1777	71	108	2798	118	164	3828	166	219	4838	213	273
778	28	52	1787	72	109	2818	119	165	3848	167	220	4858	214	274
793	28	53	1808	73	110	2838	120	166	3853	166	220	4878	215	275
807	29	54	1828	74	111	2858	121	167	3858	167	221	4898	216	276
828	30	55	1848	75	112	2878	122	168	3878	168	222	4918	217	277
848	31	56	1868	76	113	2898	123	169	3898	169	223	4935	217	278
861	31	57	1881	76	114	2904	122	169	3918	170	224	4948	218	279
877	32	58	1897	77	115	2907	123	170	3938	171	225	4968	219	280
898	33	59	1918	78	116	2928	124	171	3958	172	226	4988	220	281
918	34	60	1938	79	117	2948	125	172	3978	173	227	5008	221	282
931	34	61	1958	80	118	2968	126	173	3997	173	228	5028	222	283
947	35	62	1978	81	119	2988	127	174	4008	174	229	5048	223	284
968	36	63	1982	80	119	3008	128	175	4028	175	230	5068	224	285

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