

Risk Assessment: a Forest Fire Example

David R. Brillinger, Haiganoush K. Preisler and John W. Benoit

Abstract

The concern of this paper is obtaining baseline values for the number of forest fires as a function of time and location and other explanatories. A model is developed and applied to a large data set from Federal lands in the state of Oregon. To proceed the data are grouped into small spatial-temporal cells (voxels). Fires are rare so there are many of these voxels with no fires. In fact there are so many such cells that in the analyses presented a sample is taken to make the work manageable. The paper sets down a likelihood for the sampled data and fits a generalized additive model involving location, elevation and day of the year as explanatories.

Keywords: Forest fires; generalized additive model; Oregon; risk analysis; sampled data; wildfires

1 Introduction

Forest fires represent a problem of considerable societal importance. We mention the following report that appeared in the San Francisco Chronicle of 7/16/2002,

... Nearly two weeks ago, the Forest Service used up the entire \$321 million budgeted for firefighting in 2002. It is expected to spend another \$645 million by the end of the year. ...Wildfires have already burned more than 3.3 million acres this year, more than twice the yearly average over the last decade.

The concern here is the development of a risk model for use in estimating the probability of a forest fire taking place at a particular location and time as a function of those and other explanatory variables. The work is implemented for the case of a fine grid of cells and an accompanying large data set. An analysis is carried out for a region surrounding the state of Oregon, henceforth referred to as Oregon, and employing: location, elevation and day of year as explanatories. The elements of the approach are:

1. a spatial-temporal point process and associated covariates,
2. likelihood-based inferential methods developed for such processes,
3. approximation of the point process by a 0-1 valued process on a lattice,
4. a sampling of the 0; *i.e.* no-fire cells,
5. generalized additive model technology.

Oregon was chosen for this pilot study because of its approximate rectangular geographic shape and its high rate of fires. The work presented here has as its goals model development and the estimation of baseline values for future work. The model contains but a few explanatory variables. Employing a sample of the no-fire cases rather than all the no-fire cases available is an unusual aspect of the approach presented.

The sections of the paper are: Introduction, Risk assessment, Previous statistical work, Model and analysis development, The data, Results and Discussion.

Risk assessment is familiar to Terry Speed. His papers include: reviews of the procedures that have been employed in the nuclear industry [24], [27], an analysis concerning a ship following a specified sea route [26], and an analysis of risk to levees from adverse weather conditions [25]. The first mentioned contains the following wonderful exchange with the Director of Technology, U.S. National Transportation Safety Board,

Dear Professor Speed,

In response to your aerogramme of April 5, 1977, the Chairman's statement concerning the chances of two jumbo jets colliding (6 million to one) has no statistical validity nor was it intended to be a rigorous or precise probability statement. The statement was made to emphasize the intuitive feeling that such an occurrence has a very remote but not impossible chance of happening.

Thank you for your interest in this regard.

Sincerely yours ...

From these papers and personal observation it is clear that Terry knows a lot about taking risks. May he continue to have fun doing so for many years.

2 Risk assessment

Probabilistic risk assessment can be defined as the process of estimating, for some class, the probabilities of hazardous events taking place within a specified time period and in a specified context. Such an assessment often proceeds by reducing a particular complex system to its simpler components. This is followed by the fitting and validation of stochastic models associated with the components. Typically large doses of substantive subject matter are required in such modeling and data analysis projects.

This paper is concerned with the case of forest fires. Wildfires are a natural disturbance in virtually all the world's ecosystems and the annual losses are staggering, see the *Chronicle* report above. It seems clear that fire occurrence depends on local conditions such as: location, elevation, wind velocity, precipitation, temperature, air humidity, topography, litter type, level of suppression amongst other explanatories. In our work fire ignition will be viewed as a random phenomenon. A pertinent conceptual model is: when the temperature (or some related latent variable) at a given location exceeds a threshold, depending on the local conditions, a fire breaks out. The latent

variable will depend on explanatory variables such as those listed above. In the work below the logit transform will be employed. It corresponds to a latent variable with a logistic distribution.

3 Previous statistical work

Often a Poisson model has been employed for the number of fires, see Dayananda [3], Poulin-Costello [17], Mandallaz and Ye [8, 9]. In other cases it is a logistic: Chou *et al.* [2], Poulin-Costello [17], Martell *et al.* [11]. In another approach McKenzie *et al.* [12] use multiple regression and regression trees. Markov chain models are employed in Martell [10]. Peng and Schoenberg's works, [22], [16], relate wildfire incidence to temperature, precipitation, fuel moisture and fire history for Los Angeles County. These researchers find that time elapsed since a location has burned previously appears important. Roads *et al.* [20] use a regression model for fire occurrence with 6 fire danger indices by fuel type as explanatory variables. There is also spatial autocorrelation. [18] provide a review.

4 Model and analysis development

4.1 The spatial-temporal conditional intensity function

To begin suppose that the space-time domain is broken up into voxels $(x, x + dx] \times (y, y + dy] \times (t, t + dt]$. Consider the spatial-temporal point process, N , with conditional intensity function assumed to exist and defined by

$$\lambda(x, y, t) = \text{Prob}\{dN(x, y, t) = 1 | H_t\} / dx dy dt$$

where $dN(x, y, t) = N(dx, dy, dt)$ counts the number of fires in the voxel where H_t is the history of the process N up to and including time t . Supposing that λ contains a parameter θ the log-likelihood function will be written

$$L(\theta) = \int_0^T \int_x \int_y \log[\lambda(x, y, t | \theta)] dN(x, y, t) - \int_0^T \int_x \int_y \lambda(x, y, t | \theta) dx dy dt \quad (1)$$

see Fishman and Snyder [5]. Explanatory variables may appear in λ but are presently suppressed in the notation.

Asymptotics of maximum likelihood estimates based on point process likelihoods have been developed in Ogata [15], Sagalovsky [21], Rathbun [19], Schoenberg [23]. The last two papers focus on the spatial-temporal case.

Consider next practical approaches to using the log-likelihood (1) in practice.

4.1.1 Approach 1

The second term of (1) is the awkward one in our case because it covers such a large area in space-time. It will be approximated. One way to do this is by sampling points from an independent of N spatial Poisson process, $M(dx, dy, dt)$, of rate π on the space $\mathcal{R} \times [0, T]$. Then the log-likelihood (1) may be approximated by

$$\int_0^T \int_x \int_y \log[\lambda(x, y, t | \theta)] dN(x, y, t) - \int_0^T \int_x \int_y \lambda(x, y, t | \theta) dM(x, y, t) / \pi. \quad (2)$$

Averaging over M , but holding the process N fixed, the expected value of (2) is (1). The expected number of points in the approximating sum of the second term is π times the volume of $\mathcal{R} \times [0, T]$.

4.1.2 Approach 2

A model that is often simpler to deal with follows. It is an approximation to Approach 1. Replace the spatial-temporal point process, $N(dx, dy, dt)$, by a 0-1 valued process $N_{x,y,t}$ on a lattice with $N_{x,y,t} = 1$ if there is a fire in the corresponding voxel and by 0 otherwise for (x, y) in $\mathcal{R}$; $t = 0, \dots, T-1$. (This idea was used to advantage in [1].) Suppose then that

$$\text{Prob}\{N_{x,y,t} = 1 | H_{t-1}\} = \lambda_{x,y,t}$$

with H_t the history up to and including t . A Bernoulli approximation to the log likelihood (1) is now

$$\sum_{x,y,t} N_{x,y,t} \log(\lambda_{x,y,t}) + \sum_{x,y,t} (1 - N_{x,y,t}) \log(1 - \lambda_{x,y,t}) \quad (3)$$

In the present case there are many voxels for which $N_{x,y,t}$ is 0 and so the second sum in (3) contains many terms. To deal with this, randomly select 0-voxels with probability π and include them alone in the analysis. (In what follows there will actually be two-stage sampling with $\pi = \pi_1 \pi_2$. This is to possibly obtain more efficient estimates. The estimation procedure then involves two types of uncertainty; one from the fire process itself and a second from the sampling of data points. The latter component will decrease with increasing π .)

To simplify the notation for the moment, index the voxels by k rather than x, y, t . Let S denote the collection of the voxels that had a fire and the sample of those that did not. In what follows it will be assumed that the N_k are independent given the explanatories. This condition will be relaxed in future work. Using the identity

$$\text{Prob}\{A|B\} = \text{Prob}\{B|A\} \text{Prob}\{A\} / \text{Prob}\{B\},$$

one has the conditional probability

$$\text{Prob}\{N_k = 1 | H_{t-1}, k \text{ in } S\} = \gamma_k = \lambda_k / (\lambda_k + (1 - \lambda_k)\pi)$$

with the complimentary probability for the event $N_k = 0$. Now by elementary algebra

$$\text{logit } \gamma_k = \text{logit } \lambda_k + \log 1/\pi.$$

Using the indicated assumption of independence the log-likelihood based on the N_k for k in S is

$$\sum_{k \text{ in } S} [N_k \log(\gamma_k) + (1 - N_k) \log(1 - \gamma_k)], \quad (4)$$

i.e. a Bernoulli likelihood. Appropriate uncertainty measures are often available for estimates obtained by maximizing (4), see the Appendix.

In the analyses $\eta = \text{logit } \lambda$ will be a "linear predictor" based on explanatories. To obtain estimates one can use a generalized linear model program, such as *glm()* of Splus, with an offset of $\log(1/\pi)$ to carry out an analysis. In fact the generalized additive model program *gam()*, [6], will be employed below.

Above an assumption of independence was made. For the present this will be made reasonable by the inclusion of explanatories. For example, location is meant to handle the similarity of nearby values.

4.1.3 Approach 3

Another method begins, again, by approximating the log-likelihood (1) by one based on 0-1 variates. Further suppose once again the 0-voxels have been sampled. Let $\{\delta_{x,y,t}\}$ denote independent Bernoulli variates corresponding to the sampling with parameter π . They are to be independent of the fire process, N . Consider the approximate log-likelihood

$$\begin{aligned} & \sum_{x,y,t} [N_{x,y,t} \log(\lambda_{x,y,t}) + (1 - N_{x,y,t}) \log(1 - \lambda_{x,y,t}) \delta_{x,y,t} / \pi] \\ & = \sum_{k \text{ in } S} w_k [N_k \log \lambda_k + (1 - N_k) \log(1 - \lambda_k)]. \end{aligned} \quad (5)$$

The w_k are weights that equal 1 at locations with fires and equal $1/\pi$ at the sampled locations with no fire. If $\pi = 1$ this reduces to the log-likelihood (3).

4.2 Assessing fit

Suppose a particular link function has been employed, e.g. the logit, and one wishes to assess it. The fitted linear predictor values may be assigned to the cells of a histogram. For each cell some of the corresponding N 's will be 0 and some will be 1. The "number of 1's" divided by "the number of 1's" plus "the number of 0's" weighted up by $1/\pi$ provides a nonparametric estimate of the function $\lambda(\eta) = \text{Prob}\{N = 1 | \eta = 1\}$ with η representing the linear predictor of the generalized additive employed model. This estimate may be compared to $\lambda(\eta | \hat{\theta})$ where $\hat{\theta}$ is the maximum likelihood estimate. An example is provided below, see Figure 6.

4.3 Predictions

Quantities of substantial interest to the Forest Service managers, for planning purposes, include probabilities such as

$$\lambda_{x,y,t+1} = \text{Prob}\{N_{x,y,t+1} = 1|H_t\}.$$

The fitted linear predictor and its statistics may be used to compute pertinent estimates. Provided some explanatories in the model are leading variables, and they can be reasonably forecast, one would have useful predictions.

The work presented here has in mind obtaining baseline values using models involving location, elevation and day of year only. The next stage of research will study improvements obtained when leading variables, *e.g.* based on meteorology, are included.

5 The data

In this pilot study fire occurrence data from Federal lands in the state of Oregon were used. The data consisted of locations and dates of every fire greater than 0.1 acre that occurred between April 26, 1989 and December 31, 1996. (There were 15,786 such fires.) The date refers to the day the fire started. The source of the fire location data was the USDA Forest Service, National Fire Occurrence Data Base [28]. Figure 1 provides an elevation map of Oregon with federal fire locations for 1990. The dark winding horizontal line at the top of the figure is the Columbia River. It provides the border of Oregon with Washington. Figure 2 shows the so-called "Federal Mask", *i.e.* the Federal lands in Oregon. Their area accounts for approximately 56% of the state.

Figure 3 is a time series plot of the square roots of fortnightly counts of fires over the time period of the study. A clear annual effect corresponding to the fire season may be seen. There is no serious suggestion of a trend.

Data on response variables and explanatories were selected as follows. The fires were those inside Federal lands, delineated by Figure 2. To get the quantities for beginning data analyses a two-stage sampling procedure was employed for the voxels without a fire because of the data management problem. In the first stage of the sampling a collection of days was selected, with each day having probability $\pi_1 = 0.1$ of being picked. In the second stage a proportion $\pi_2 = 0.0012$ of cells inside the Federal mask was picked. This resulted in a total of 73880 cases of which 15,786 were fire occurrences N_k of (4) equals 1 and 58094 were with N_k equal 0. The overall rate of fires experienced for the period of the study was $15786/(2760*209490) = .0000273$ per km^2 per day. (There were 2760 days and a region of area 209490 km^2 in the study.)

To obtain the estimates, the function *gam()* of Splus, [6], was used to fit a Bernoulli model with an offset of $\log 1/\pi$, a linear predictor

$$\eta_k = \text{logit } \lambda_k = g_1(x_k, y_k) + g_2(d_k) + g_3(e_k) \quad (6a)$$

Elevation map and 1990 Federal fire locations

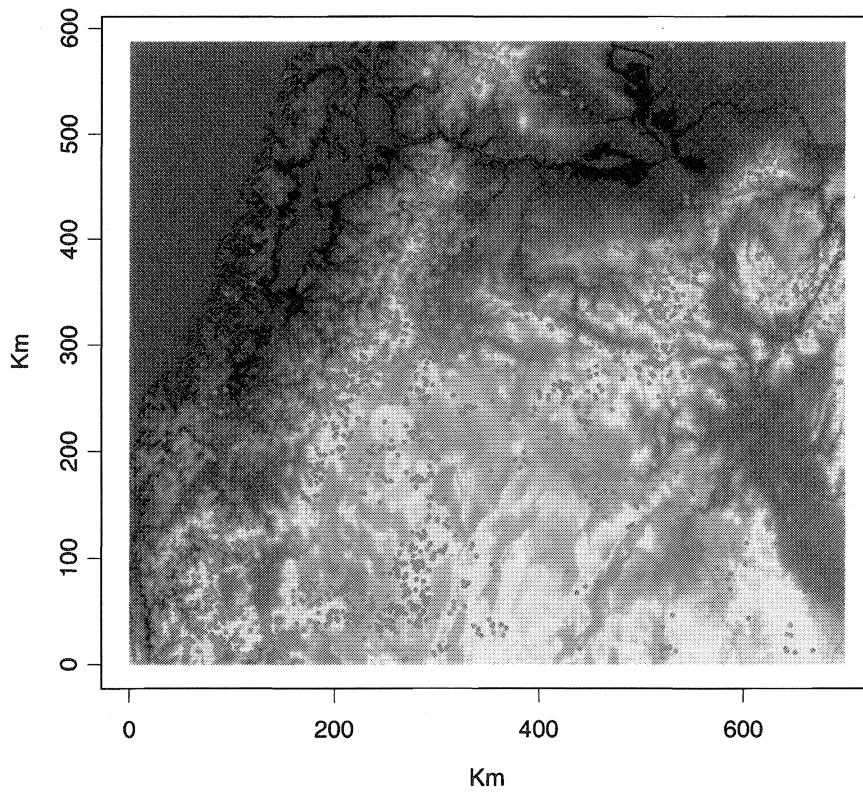


Figure 1: Federal fire locations in 1990. Dark blue corresponds to water and red circles to fire locations.

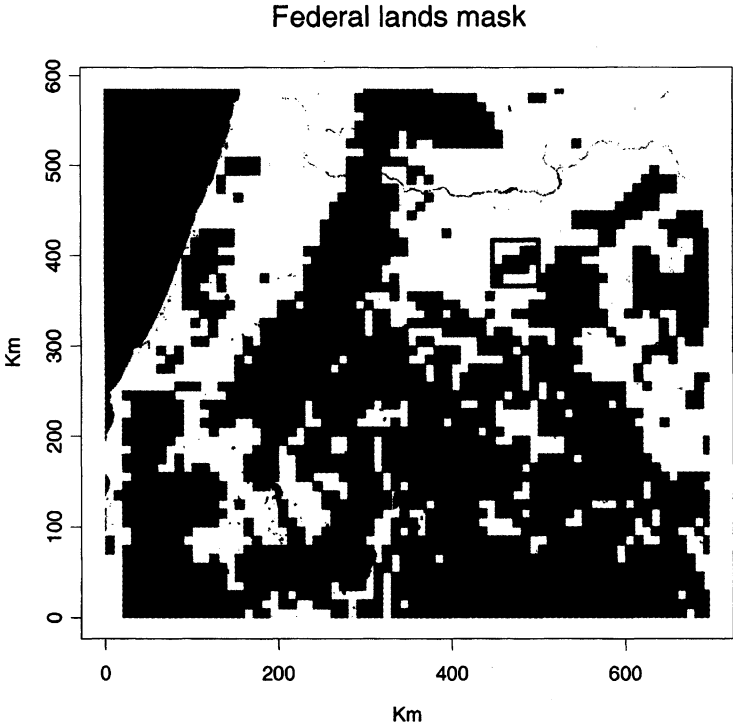


Figure 2: The Federal lands in Oregon. The box refers to the region taken for an example in Section 6. It is referred to as Region B.

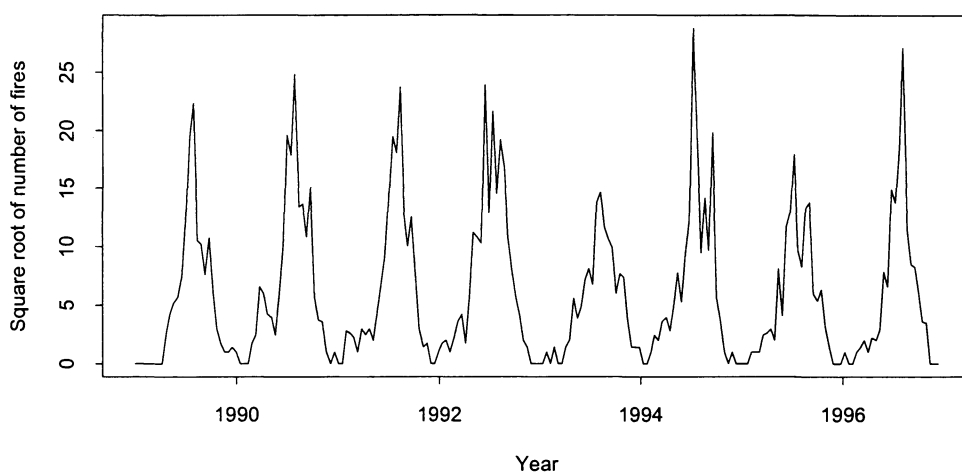


Figure 3: Square root of fortnightly count of Oregon fires in Federal lands 1989-1996.

and a probability mass function of

$$Prob\{N = 1|\eta\} = \exp\{\eta\}/(\pi + (1 - \pi)\exp\{\eta\}). \quad (6b)$$

Here (x_k, y_k) corresponds to the locations (in meters) of coordinates of the k -th response, d_k corresponds to day in year and e_k refers to elevation at the location. The $g(\cdot)$ are (nonparametric) smooth functions to be estimated. The functions g_2 and g_3 were splines with g_2 having period 1 year, [13]. The term g_1 was estimated via the scatterplot smoother lo .

One of the interesting aspects of the work is that the data set is quite large and awkward. Maps for the state of Oregon were extracted from the CD's, converted to ASCII files and then a sample was selected in two stages. Locations of fires were recorded in units of latitude and longitude. These were converted to metric units. Even with a small fraction ($\pi=0.00012$) of the available voxels, each gam run required approximately 10 minutes to run on a 900MHz computer. This meant that it was not easy to do exploratory data analysis.

6 Results

6.1 The fit

Figure 4 provides the estimated spatial effect for model (6a,b). It was obtained by the second approach described above. The results are presented in both perspective and contour form. Some general features apparent in this map are the low intensity level in the south-eastern region of Oregon, contour levels -2 and -3. This region is mostly high desert with few forests. Regions with the historically highest level, contour level 1, of fire appear to be in the Cascade Mountains.

Figure 5 provides plots of the estimated effect of elevation and day of the year. Unsurprisingly, the latter indicates amongst other things more fires in the summer as was evident in Figure 3. The elevation effect increases for those less than 2000m. The approximate 95% bounds are obtained by the jackknife procedure as discussed in the Appendix. Generally the bounds are small as might be expected given the large amount of data involved. The uncertainty bounds for the higher elevations are wide because there are not many high locations in the data set.

Figure 6 implements the method of Section 4.2 for assessing the appropriateness of the model (6a,b). It is a plot of the empirical relative frequencies of fires as points. These are computed after grouping the data into classes based on the linear predictor. Also provided, as a solid line, are the estimated probabilities from model (6a,b) and, as dashed lines, approximate binomial confidence bounds. The fit appears reasonable, but one does notice points above the curve on the right. This departure may disappear when other explanatories are included in the model.

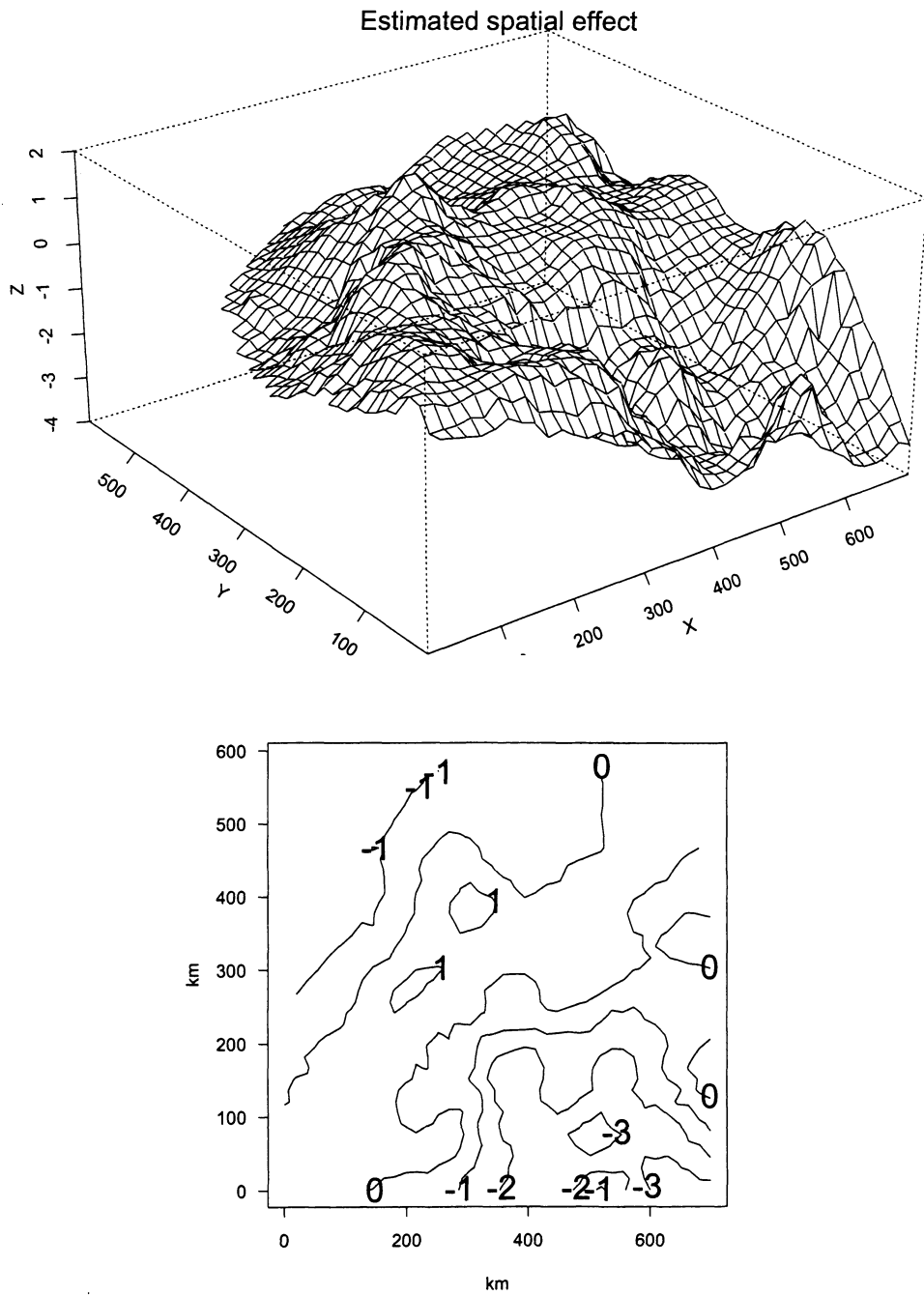


Figure 4: Estimated spatial effect, $\hat{g}_1$, for model (6a,b).

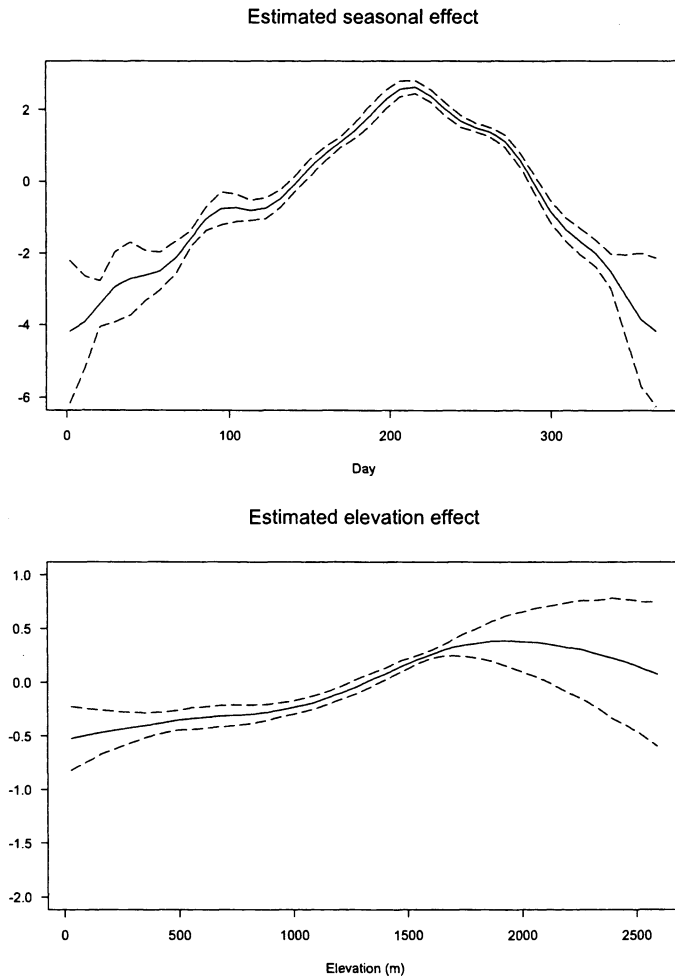


Figure 5: Estimated effects $\hat{g}_2$ of day in year and $\hat{g}_3$ of elevation for the model (6a,b). The dashed lines provide approximate marginal 95% bounds computed by a jackknife procedure.

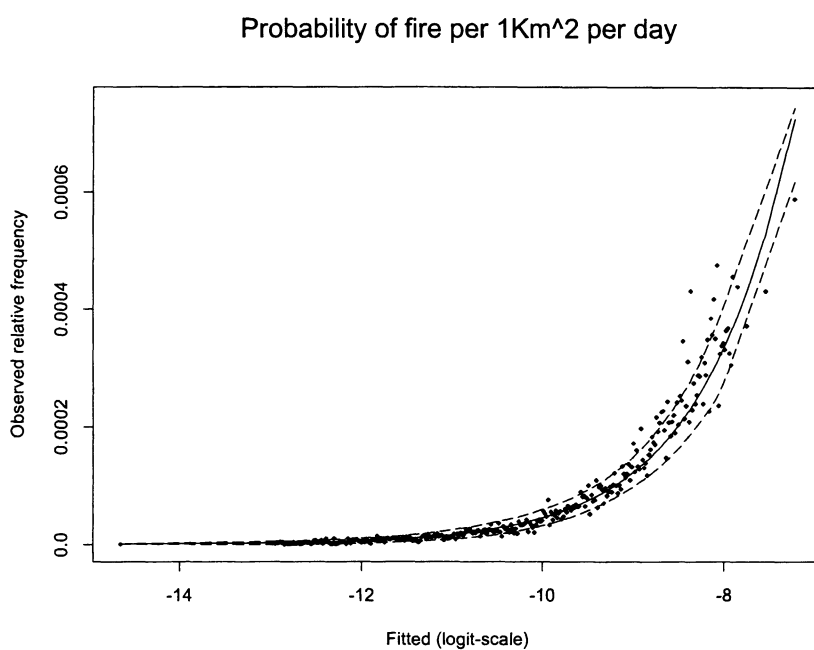


Figure 6: Observed relative frequencies of fire, after grouping the data into classes based on the fitted linear predictor, $\hat{\eta}$. The solid curve is the fitted logistic curve. The dashed lines are smoothed approximate 95% limits obtained via a binomial approximation.

6.2 An example

Forest managers are interested in estimates of the number of fires likely to occur in a given region during a given fire period. Amongst other things these estimates will help them allocate resources. As an example, using the output above, estimated fire probability values were produced for the shaded region within the box of Figure 2. This is the Heppner Ranger District of the Umatilla Forest in Oregon. It will be referred to as Region B. It is taken as representative of the sort of region of interest in wildfire risk estimates for the U.S. generally.

Figure 7 and Table 1 present some of the results. The solid line in Figure 7 gives the estimate of the monthly rates obtained by fitting the model with location, elevation, and day of year as explanatories. The shaded region gives approximate 95% marginal confidence limits for the estimate. These are obtained in the same jackknife computations as produced the errors bounds of Figure 5. In this case the linear predictions are perturbed by $\pm 2SE$ and converted to probabilities by the transform (6b). The resulting fire rates are seen to peak in the summer season with an estimated value of 6.74 fires for the month of August.

The dots in Figure 7 refer to the naive estimate of the rates of fires obtained by dividing the total number of fires for a month by the number of years of observation. The vertical lines give approximate 95% limits employing a Poisson approximation. One notes differences, but it may be remarked that the only physical characteristics of Region B being used in the present estimation are its elevations.

To be specific, managers are interested in things like

$$Prob\{i \text{ or more fires in July}\}, i = 1, 2, \dots$$

Table 1 provides estimates of such together with approximate 95% confidence limits for the estimates. The count is based on the sum of Bernoulli variables. When the probabilities of the Bernoullis differ the distribution is called the Poisson-Binomial, [7]. These authors show that this distribution is well approximated by a Poisson in the case that the largest of the individual pixel probabilities is small. This seems to be the case in the present situation where the risk of fire is low, particularly since the pixels are small. For example the estimated probability of 7 or more fires in July for Region B is .324 with an approximate confidence interval of (.176, .520).

Here the only serious explanatory employed beyond location is elevation. The goal of future work is prediction using leading explanatories. It will be interesting to see the extent to which the confidence interval shrinks as these are brought into the model.

7 Discussion

The model, (6a,b), for the breaking out of wildfires has been set down and fit. The model was motivated by threshold considerations and is of nonparametric character.

Empirical and Fitted Fire Rate for Region B

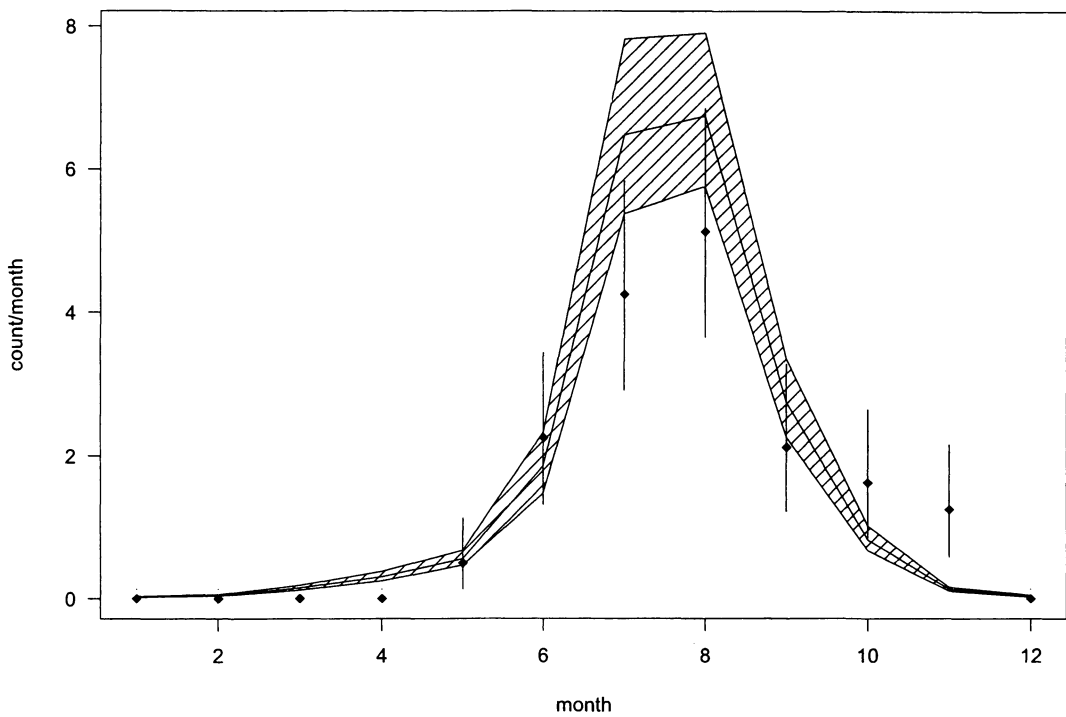


Figure 7: The solid central line gives the fitted rate of fires by month. The shaded region gives ± 2 SE limits. The points are the monthly empirical rates of fires. Vertical lines are ± 2 SE limits for the points.

<i>i</i>	<i>probability</i>	<i>confidence interval</i>
1	.989	(.967,.996)
2	.956	(.895,.983)
3	.887	(.771,.948)
4	.774	(.610,.883)
5	.628	(.441,.784)
6	.470	(.291,.658)
7	.324	(.176,.520)
8	.206	(.097,.386)
9	.121	(.049,.268)
10	.066	(.023,.174)
11	.033	(.010,.106)
12	.016	(.004,.060)
13	.007	(.001,.032)
14	.003	(.001,.016)
15	.003	(.000,.007)
16	.001	(.000,.003)
17	.000	(.000,.001)
18	.000	(.000,.001)
19	.000	(.000,.000)
20	.000	(.000,.000)

Table 1: Estimated probability of i or more fires and approximate 95% confidence limits for the month of July and Region B.

The assumptions include smooth dependence of the risk probability on location, elevation, day of the year. A logistic link function is employed.

There are many sources of variability present in addition to the random nature of wildfires. The simplest is that arising from using a sample of the no-fire voxels. The sampling was two-stage without replacement. Finite population correction factors were ignored in computing uncertainty estimates, but they are negligible. The sampling fraction, π , will be increased in future studies.

The principal source of the variability that shows itself in Figure 7 and Table 1 is possibly that arising from missing explanatories. To the extent possible this will be dealt with in future studies. Additional data sources to be included then are weather data from the Weather Information Management System [14], fire danger indices from NIFMID [14] and more topographic data, *e.g.*, aspect and distance to nearest population center.

This has been a pilot study with the purpose of obtaining baseline values. These will be used as standards for later forecasting models.

One difficulty needs to be mentioned. In using *gam()* it was found that the results of prediction runs depended on which other predictions were carried out at the same time. This may be due to the accumulation of roundoff error. Studies are continuing. The results presented in this paper are for runs of all the predictions being carried out at the same time. The reason for this choice is that the computing time required was less than, say, running one prediction at a time.

Acknowledgements

We thank Bob Burgan, Francis Fujioka, Rich Kimberlin, David Martell and the Referee for their helpful comments. Bob Burgan and Carolyn Chase supplied the data. Trevor Hastie and Phil Spector helped out when trouble arose employing *gam()*. Alan Aager chased down some geography.

D. R. Brillinger, Department of Statistics, University of California, Berkeley,
brill@stat.berkeley.edu

H. K. Preisler, USDA, Forest Service, PO Box 245, Berkeley, CA 94701,
hpreisler@fs.fed.us

J. Benoit, USDA, Forest Service, 4955 Canyon Crest Dr., Riverside, CA 92507,
jbenoit@fs.fed.us

References

- [1] D. R. Brillinger and H. K. Preisler. Two examples of quantal data analysis: a) multivariate point process, b) pure death process in an experimental design. In *Proc. XIII International Biometric Conference, Seattle*, pages 94–113, 1986.
- [2] Y. H. Chou, R. A. Minnich, and R. A. Chase. Mapping probability of fire occurrence in San Jacinto Mountains, California, USA. *Environmental Management*, 17:129–140, 1993.
- [3] P. W. A. Dayananda. Stochastic models for forest fires. *Ecological Modelling*, 3:309–313, 1977.
- [4] B. Efron and R. J. Tibshirani. *An Introduction to the Bootstrap*. Chapman and Hall, New York, 1993.
- [5] P. M. Fishman and D. L. Snyder. The statistical analysis of space-time point processes. *IEEE Transactions on Information Theory*, IT-22:257–274, 1976.
- [6] T. Hastie. Generalized additive models. In J. M. Chambers and T. J. Hastie, editors, *Statistical Models in S*, pages 249–307. Wadsworth, Pacific Grove, 1992.
- [7] J. L. Hodges and L. M. Le Cam. The Poisson approximation to the Poisson Binomial distribution. *Annals of Mathematical Statistics*, 31:737–740, 1960.
- [8] D. Mandallaz and R. Ye. Statistical model for the prediction of forest fires. Report Project Minerve II, ETH Zurich, 1996.
- [9] D. Mandallaz and R. Ye. Prediction of forest fires with Poisson models. *Canadian Journal of Forest Research*, 27:1685–1694, 1997.
- [10] D. L. Martell. A Markov chain model of daily changes in the Canadian forest fire weather index. *International Journal of Wildland Fire*, 9:265–274, 1999.
- [11] D. L. Martell, S. Otukol, and B. J. Stocks. A logistic model for predicting daily people-caused forest fire occurrence in Ontario. *Canadian Journal of Forest Research*, 17:394–401, 1987.
- [12] D. L. McKenzie, D. L. Peterson, E. Alvarado, J. K. Agee, and R. A. Norheim. Spatial models of fire frequency in the Columbia River Basin. <http://silvae.cfr.washington.edu/people/dmck/crb.html>, 1998.
- [13] H. A. Nielsen. Summary: periodic spline basis. *s-news*, June 29, 1999.
- [14] NIFMD. <http://www.fs.fed.us/fire/planning/nist/kcfast.html>.

- [15] Y. Ogata. The asymptotic behaviour of maximum likelihood estimators for stationary point processes. *Annals of the Institute of Statistical Mathematics*, 30:243–261, 1978.
- [16] R. Peng and F. Schoenberg. Estimation of wildfire hazard using spatial-temporal fire history data. Technical report, Statistics Department, UCLA, 2001.
- [17] M. Poulin-Costello. People-caused forest fire prediction using poisson and logistic regression. Master's thesis, Dept. of Math. and Stat., University of Victoria, Victoria, Canada, 1993.
- [18] H.K. Preisler and D. Weise. Forest fire models. In A. H. El-Shaarawi and W. W. Piegorsch, editors, *Encyclopedia of Environmetrics*, pages 808–810. Wiley, Chichester, 2002.
- [19] S. L. Rathbun. Asymptotic properties of the maximum likelihood estimator for spatio-temporal point processes. *Journal of Statistical Planning and Inference*, 51:55–74, 1996.
- [20] J. O. Roads, F. M. Fujioka, and R. E. Burgan. Development of a seasonal fire weather forecast for the contiguous United States. In *Third Symposium of Fire and Forest Meteorology*, pages 99–102. American Meteorological Society, 2000.
- [21] B. Sagalovsky. *Maximum likelihood and related estimation methods in point processes and point process systems*. PhD thesis, University of California, Berkeley, 1983.
- [22] F. Schoenberg, R. Peng, Z. Huang, and P. Rundel. Exploratory analysis of Los Angeles County wildfire data. Technical report, Statistics Dept. UCLA, 2000.
- [23] F. P. Schoenberg. *Assessment of point process models*. PhD thesis, University of California, Berkeley, 1997.
- [24] T. P. Speed. Negligible probabilities and nuclear reactor safety: another misuse of probability. Technical report, Mathematics Dept., University of Western Australia, 1977.
- [25] T. P. Speed. The possibility of future damage to the Leslie Salt Company concentrator and diversion levees caused by adverse weather conditions: an evaluation of probabilities. Technical report, Mathematics Dept., University of Western Australia, 1980.
- [26] T. P. Speed. Tow route risk assessment. Technical report, University of Western Australia, 1980.
- [27] T. P. Speed. Probabilistic risk assessment in the nuclear industry: Wash 1400 and beyond. In L. M. LeCam and R. Olshen, editors, *Neyman-Kiefer Conference*, Pacific Grove, 1983. Wadsworth.

- [28] USDA. National Fire Occurrence Data Base. <http://www.fs.fed.us/fire/fuelman>.

Appendix

The use of the jackknife for the model (6a,b) may justified by assumptions that K is large, that the values

$$(N_k, x_k, y_k, d_k, e_k), \quad k = 1, \dots, K$$

may be treated as an i.i.d. sample from some distribution and that the statistics computed are approximately additive in some i.i.d. variates. The variances estimated are overall as opposed to conditional. One reference to the jackknife is Chapter 11 in Efron and Tibshirani [4] where it is discussed as an approximation to the bootstrap.

In the implementation employed, the standard errors of the estimates of $g_1(x, y)$, $g_2(d)$, and $g_3(e)$ are obtained by splitting the 7 years of data into 7 random segments of 365 days. The values obtained do not differ greatly from those produced by *gam*.