Ecological Informatics 90 (2025) 103339

Contents lists available at ScienceDirect

Ecological Informatics

journal homepage: www.elsevier.com/locate/ecolinf

A graph-based optimization framework for firebreak planning in wildfire-prone landscapes

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ARTICLE INFO

Keywords:
Fire propagation tree
Fireplain
Firebreaks
Wildfire spread
Critical node removal
Fire growth model
Mixed integer programming

ABSTRACT

Firebreaks and fuel treatments are critical means for the reduction of wildfire threat and damage to human infrastructure in forest landscapes. However, the uncertain behavior of wildfires makes the planning of firebreaks challenging, especially when available resources are insufficient to treat all locations under wildfire threat. We present a fire propagation graph approach that utilizes directed acyclic graphs to track the possible spread of wildfires from their ignition locations and estimate the impacts of firebreak placement on the possible burn area. The fire propagation graphs depict plausible fire spread within the fire footprints created with a spatial fire growth model. We integrated the fire propagation graph concept into an optimization model that allocates firebreaks in a complex landscape. We compared two firebreak planning strategies. The first strategy reduces the overall connectivity between patches with fuel and minimizes the number of location pairs between which wildfire spread is possible. The second strategy minimizes the possible burn area across the landscape by tracking the impact of firebreaks on the potential fire spread through a large set of fire propagation graphs that depict plausible fire scenarios. We also evaluated the problem that combines both strategies.

We illustrated the approach with the planning of wildfire mitigation measures in the Red Rock-Prairie Creek area of Canada, a complex fire-prone landscape. The firebreak solutions were effectively able to reduce both the potential burn area and the connectivity between locations with fuel. The graph-based depiction of the uncertain wildfire spread helped assess the landscape-level impacts of local firebreak allocation decisions and uncover the tradeoffs between different firebreak planning strategies. The approach could assist wildfire mitigation planning in other regions.

1. Introduction

Wildfires represent the most prevalent type of natural forest disturbance in North America (Dennison et al., 2014; Hanes et al., 2019). The damages from wildfires and associated management costs are expected to increase across the continent, driven by higher frequencies of forest fires due to climate change (Abatzoglou et al., 2021; Delfino et al., 2009; Ellis et al., 2022; Johnston et al., 2012), as evidenced by the record-setting fire season of 2023 (Jain et al., 2024). These circumstances call for the implementation of strategic measures aimed to reduce wildfire threat, including the reduction of fuel amounts and disrupting spatial connectivity of fuels in fire-prone landscapes.

Firebreaks (cf. Ascoli et al., 2018) are common measures to reduce wildfire hazard and prescribe complete or partial removal of vegetation and dead organic fuel, which reduces the ability of fires to spread or decreases their intensity to the level of successful containment (Agee et al., 2000; Ager et al., 2023; Alcasena et al., 2018). An effective system of firebreaks is expected to segment a landscape into compartments where potential damage from wildfires can be localized (Ager et al., 2021; Alcasena et al., 2018; Minas and Hearne, 2016). However, the cost-effective allocation of firebreaks presents a challenging management problem (Collins et al., 2010; Rönnqvist et al., 2015). While firebreak placement decisions are made at the level of individual forest patches, their impact projects to a whole-landscape scale. For example, a well-positioned firebreak could block the spread of a fire through a critical corridor to a large region with forest fuel, thereby preventing

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https://doi.org/10.1016/j.ecoinf.2025.103339
Received 17 April 2025; Received in revised form 17 July 2025; Accepted 19 July 2025
Available online 25 July 2025

significant damage. Thus, the impacts of coordinated firebreak allocation need to be assessed at a whole-landscape scale in terms of the potential reduction of the total burn area or damage. Incorporating this aspect into firebreak planning is difficult because wildfire behavior is highly uncertain in rapidly changing weather conditions and so can be difficult to predict and mitigate effectively (Christiansen, 2015; Jain et al., 2021; Kalies and Kent, 2016; Ott et al., 2023; Syphard et al., 2011).

Firebreak planning as part of a broader wildfire risk mitigation program has been aided by optimization models (León et al., 2019; Rachmawati et al., 2016; Wei, 2012; Wei et al., 2008) that have utilized an array of methods, including mixed-integer programming (Carrasco et al., 2023; González-Olabarria and Pukkala, 2011; Minas et al., 2014, 2015; Nguyen et al., 2024), dynamic and stochastic programming (Kabli et al., 2015; Kim et al., 2009; Konoshima et al., 2008) and game theory (Lagos et al., 2025; Liberatore et al., 2021). Other approaches have included deep reinforcement learning and metaheuristics (Murray et al., 2024; Palacios-Meneses et al., 2023) and multi-objective prioritization (Belavenutti et al., 2023). These optimization models have often relied on the application of simulation models to depict a plausible range of wildfire scenarios (see Chung, 2015; Gillen et al., 2017).

Notable work on optimizing fuel treatments was done by Wei (2012), Wei and Long (2014) and Wei et al. (2008), who applied single-period formulations to a flammable landscape represented by a grid of cells containing fuel. The models used simplified depictions of vegetation composition and made binary decisions on allocating firebreaks in individual cells. A similar landscape representation as a square grid of candidate units (cells) for was implemented in the multi-period model of Minas et al. (2014), which tracked both the status of fuel loads and vegetation regrowth in each cell and calculated the cell's fuel status as dependent on time since the last treatment. The model minimized the number of connected pairs of adjacent cells with fuel. The model of Minas et al. (2015) tracked both fuel treatments and fire suppression decisions by checking whether a forest cell was proximal to the fire management station from where it could receive suppression. Time-dependent fuel accumulation in individual patches was also depicted in the model of Rachmawati et al. (2016), which applied a graph-based approach to divide a landscape into sub-regions based on site adjacency. Rachmawati et al. (2018) proposed a formulation that simultaneously fragmented fuel while maintaining a target level of habitat contiguity defined by the areas with high fuel loads. The model of León et al. (2019) aimed to fragment contiguous areas with fuel while protecting habitat, the quality of which was defined by site age. The model of Carrasco et al. (2023) estimated the impact of both firebreak allocations and wildfire damage on habitat losses in a landscape.

Nguyen et al. (2024) proposed a simulation-optimization approach where a large set of simulated fire footprints was used to allocate firebreaks in a forest landscape setting. For each simulated fire, the feasibility of containment was evaluated by intersecting the fire footprint with the candidate firebreaks and delineating the smallest area that contained the fire's ignition location. The fire's containment was assumed if all firebreak segments around that area were treated at a level that enabled suppression. The objective maximized the total landscape value protected by the firebreak network over a set of simulated fires.

Other approaches have considered timber harvests as fuel treatments. Acuna et al. (2010) and Troncoso et al. (2016) proposed multi-period harvest scheduling models with objectives to reduce landscape flammability and maximize timber revenue. Kabli et al. (2015) formulated a two-stage stochastic mixed integer programming problem where decisions to apply a particular fuel treatment type in each location were made at the first stage, and the second stage estimated the expected losses over a large set of simulated fire scenarios. Similar principles were applied in Bhuiyan et al. (2019), except the objective was to allocate cost recovery among multiple landowners across a flammable landscape.

Liberatore et al. (2021) explored an alternative idea of formulating the fuel treatment problem as a bi-level attacker-defender game that included two players: the land manager (defender) and nature (attacker). First, the land manager allocates the fuel treatments to some patches under a given budget limit. Then, nature burns a subset of patches within the predicted flammability level. Nature's objective is to maximize the burn area and the manager's objective to minimize it. Lagos et al. (2025) extended this formulation to a multi-period case and considered a large set of weather scenarios which defined the damage levels in case of fire.

Fuel treatments have also been conceptualized as minimizing connectivity across a network of forest patches with fuel. The capacity of each patch to support fire spread is defined by whether its fuel load exceeds or falls below a threshold value (Matsypura et al., 2019). The connectivity of the fuel network can be reduced by solving a critical node detection (CND) problem (Veremyev et al., 2014a, 2014b), which identifies those patches (nodes) that maximally degrade connectivity according to some metric of interest. Yemshanov et al. (2021) applied the CND formulation to minimize the number of node pairs with connecting fire spread paths and used the *fireplain* concept (Scott and Thompson, 2015) to delimit, for each ignition node, the number of other nodes in the fuel network with possible fire spread. Later, Yemshanov et al. (2025) conceptualized firebreak allocation as the removal of edges from the network of nodes with fuel. This model allocated linear firebreaks and was easier to solve than the node removal formulation. Note that the firebreak contiguity can be enforced with network flow constraints (see Pais et al., 2021b; Yemshanov et al., 2021). However, these proposed CND models could not track the reduction of the burn area or damage.

In this study, we depict the uncertainty of wildfire ignitions and spread as a large set of directed interconnected graphs that represent the possible propagation of fires across a landscape, and subsequently allow tracking of how a set of firebreaks might impede fire propagation through these graphs. The use of acyclic graphs to characterize possible wildfire spread paths follows the approach of Vilches et al. (2024) and is a departure from the current practice of characterizing fire hazard in landscapes using patch-based metrics such as burn probability or potential spread area from a patch (Pais et al., 2021b; Parisien et al., 2020). Depicting the fire spread as graphs is critical for evaluating how local firebreak decisions translate collectively to a landscape-scale reduction of the burn risk. Placing a firebreak in a location that overlaps the FPT is intended to block the fire propagation flow through that node to other downstream nodes in the FPT (Fig. 1). The uncertain fire behavior in the landscape can be represented by a large set of FPTs. In turn, firebreak placement aims to prune as many FPTs as possible to minimize the total area of the remaining FPTs, which equivalently minimizes the expected burn area in the landscape.

Graph representation is also consistent with a common modelling practice of depicting wildfire behavior as a large set of fire footprints within a given timespan (Carrasco et al., 2023; Nguyen et al., 2024; Pais et al., 2021a, 2021b; Parisien et al., 2020; Riley et al., 2018; Yemshanov et al., 2021). Fire footprints can be created with a fire growth model that factors in influences of terrain, fuel, winds and weather (Pais et al., 2021b; Parisien et al., 2005). For each fire footprint that depicts a possible fire scenario, we compose a fire propagation tree (FPT hereafter): a directed acyclic graph whose arcs define the possible location-to-location fire propagation sequence from the ignition location. The FPTs can be constructed from location-specific data generated by some fire growth models (such as the rates of spread, wind speeds and directions), and some fire growth models can generate FPTs directly (Pais et al., 2021a).

Technically, the FPT concept is compatible with a firebreak planning approach based on the CND problem (Matsypura et al., 2019; Yemshanov et al., 2021), but it applies a different strategy to find an efficient allocation of firebreaks. Their comparison within the same frame of reference could provide practical insights for the planning of wildfire

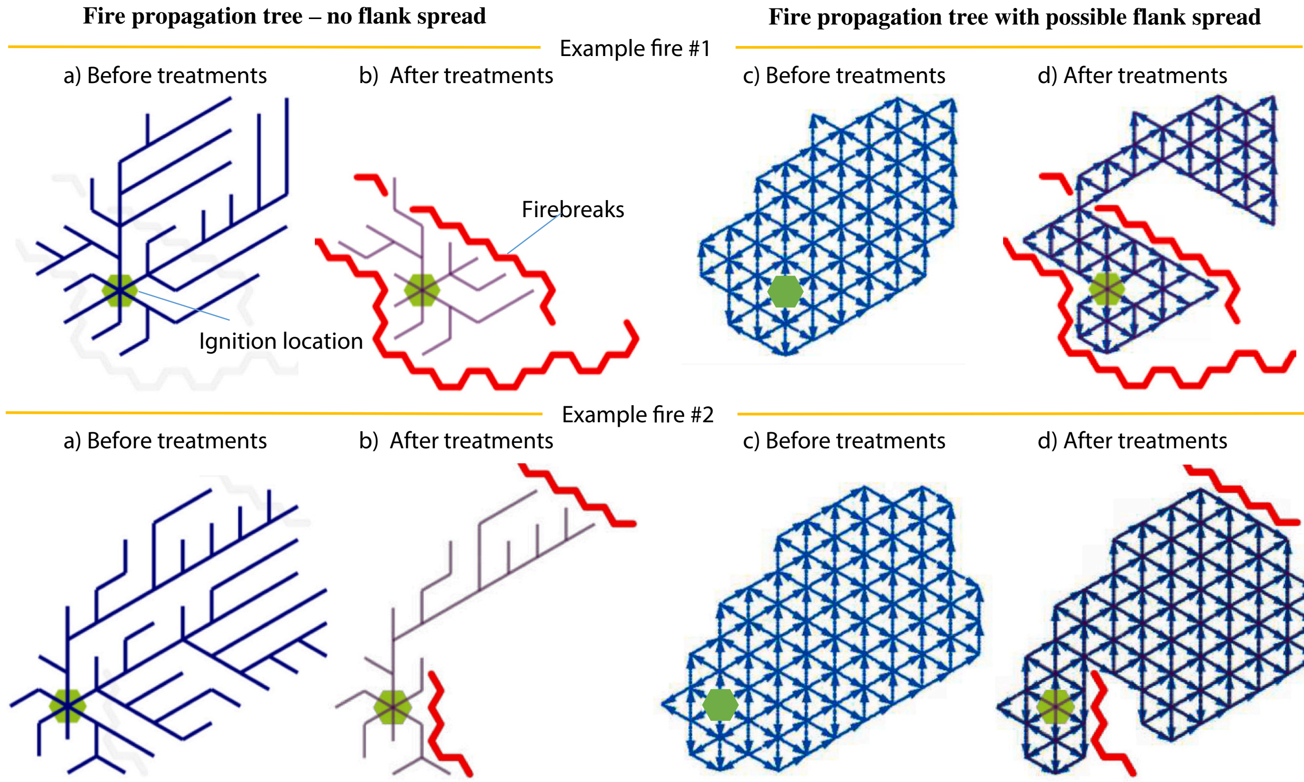

Fig. 1. Examples of fire propagation trees (FPT): a) an acyclic tree-like FPT with the root in the ignition node (green hexagon); b) impact of a firebreak placement on the fire propagation through the FPT; c) an acyclic connected FPT with the option of flank fire spread from multiple upstream locations; d) impact of a firebreak placement on the fire spread through a connected FPT. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

mitigation activities and help understand the key differences between strategies aimed at reducing fuel connectivity and strategies aimed at reducing potential burn damage. In this paper, we compare a firebreak placement problem that conceptualizes the uncertain fire regime in a landscape as a large set of fire propagation trees (FPTs) with a critical node detection (CND) problem, described in our previous work (Yemshanov et al., 2021), that allocates firebreaks to minimize the fuel connectivity in the landscape. We compare the two problems using the same set of fire spread realizations generated with a fire growth model. We compose the FPT graphs to depict the possible propagation of fires in multiple directions via flank spread and spotting in addition to the head fire spread as depicted in Vilches et al. (2024). We further combine the CND and FPT-based concepts into a single optimization model. By integrating the FPT and CND approaches it is possible to develop fire mitigation prescriptions that reduce both the connectivity between patches with fuel and the overall burn risk. This will help develop more robust mitigation plans in regions that may experience large catastrophic fires with significant burn footprints. To reduce the numeric complexity of the FPT-based problem, we propose a problem that aggregates multiple fire propagation trees into fireplain sets (cf. Scott and Thompson, 2015). Since more than one fire may spread from a given ignition location, we aggregate the FPTs which share the same ignition location into fire propagation graphs (FPGs hereafter). The FPGs depict all possible fire spread paths from a given ignition location. We explain the fireplain concept more fully in the Methods section 2 below.

2. Material and methods

2.1. Study area

We applied the fire propagation tree concept to explore fire mitigation strategies in the Red Rock-Prairie Creek region of northwestern Alberta, Canada (Fig. 2), a montane landscape that includes upper foothills and lower subalpine ecoregions of the Rocky Mountains (GoA NRC, 2006; TECO, 2011). The forest composition in the area is dominated by Engelmann spruce (*Picea engelmannii*) and subalpine fir (*Abies lasiocarpa*) at higher elevations, and lodgepole pine (*Pinus contorta*) or mixedwood stands with aspen (*Populus tremuloides*), balsam poplar (*Populus balsamifera*), white spruce (*Picea glauca*) and lodgepole pine at lower elevations. The region is a major timber supply area for local forest industry (TECO, 2011).

Prevailing winds in the area tend to blow from west-southwest to the east (Fig. 3a). In a modelling context, this aspect, in conjunction with an almost continuous distribution of abundant forest fuel, makes the area a suitable testbed for evaluating firebreak allocation strategies (which are expected to align the firebreaks roughly perpendicular to the prevailing winds). The wildfire occurrence risk is characterized as moderate compared to other boreal regions (TECO, 2011). The fire regime is characterized by frequent small to medium-sized fires and infrequent large fires (TECO, 2011), with fire return intervals of 80–300 years in the southern subalpine zone and 45–476 years in lower-elevation areas (Andison, 2000; TECO, 2011). In the lower-elevation areas in the northeastern part of the region, the peak of human-caused ignitions occurs in May. The peak season of lightning-caused ignitions is between May and August (Tymstra et al., 2005). The area includes significant amounts of overmature conifer forest, which is susceptible to fires and will require landscape-level mitigation efforts to protect from future wildfires. The Government of Alberta has initiated planning efforts to reduce wildfire risk in the region (GoA, 2024, 2025).

2.2. Creating the wildfire spread scenarios with the spatial fire growth model

The planning of fire mitigation measures requires a general depiction

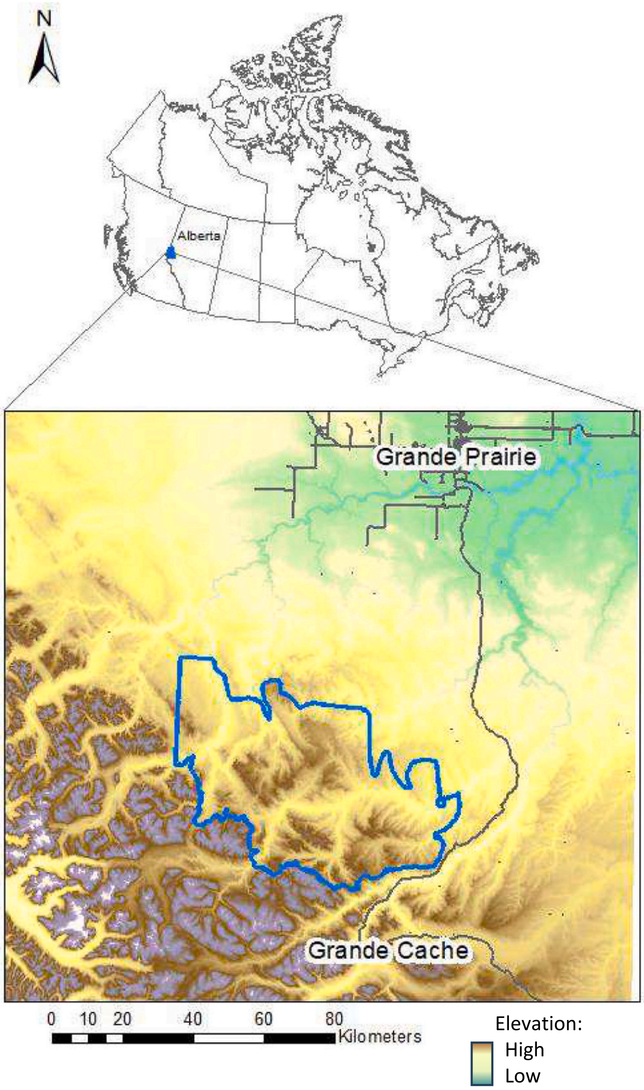

Fig. 2. Study area in the Red Rock-Prairie Creek region, Alberta, Canada. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

of the wildfire regime in the landscape of interest. A common way to describe the fire regime in a forest landscape is through simulation of ignitions and fire spread with a spatial fire growth model (Finney et al., 2011; Pais et al., 2021a; Parisien et al., 2020). Fire growth models use various mathematical algorithms to predict fire progression in space and time across a landscape (Cardil et al., 2021; Sullivan, 2009) and generate a large set of scenarios, each characterized with ignition location(s) and fire footprint(s). Simulated fire footprints have proven useful in optimization modelling of firebreaks and related fuel treatments (Pais et al., 2021b; Wei et al., 2008; Yemshanov et al., 2021, 2023). Fire growth models have been developed for many fire-prone regions, such as the Burn-P3 (Parisien et al., 2005) and Cell2Fire models (Pais et al., 2021a) in Canada and the FSIM model in the USA (Finney et al., 2007, 2011).

We applied the Burn-P3 model to generate plausible fire footprints in the study area. The Burn-P3 model uses the Prometheus algorithm (Tymstra et al., 2010) to simulate fire growth. For each replication that depicts a fire season, Burn-P3 generates ignition locations and footprints of individual fires (see the model details in Appendix A). Technical details about the Burn-P3 model are not especially germane to this study, since our goal was an adequate depiction of fire spread outcomes in the landscape; other fire growth models may provide similarly suitable inputs for the firebreak placement model. We simulated 60,000 independent realizations of a burn year, which we used to build the set of fire propagation trees. This sample size was sufficient to stabilize the fire size distribution and the expected burn area.

2.3. Finding the firebreak configurations with a graph optimization approach

2.3.1. Conceptualizing wildfire spread in a landscape network setting

We depicted a forest landscape as a network G composed of N patches i, $i \in N$, (nodes hereafter) containing combustible fuel. The landscape is divided into hexagonal nodes where each node i is connected to six adjacent nodes j by a pair of directed edge arcs ij and ji, (ij, $ji \in E$) (directed edges hereafter). A fire can propagate through directed edges ij from a node containing the fire's ignition location towards the nodes located at the fire's perimeter (Fig. 4a,c). For this landscape and others like it, the firebreaks must be sufficiently wide to enable effective suppression of fires under firebrand spotting conditions. Firebrand spotting is a common phenomenon in Canadian forests where high-intensity fires produce firebrands that can travel and ignite nearby spot fires (Manzello et al., 2020; Thompson et al., 2022). We set the size of the hexagonal nodes i in fuel network G, which defined the potential firebreak width, to 400 ha with a cross-sectional width of 2.15 km. Based on the spotting distance models of Chase (1981) and Albini et al. (2012), this firebreak width is sufficient to contain fires burning with intensities

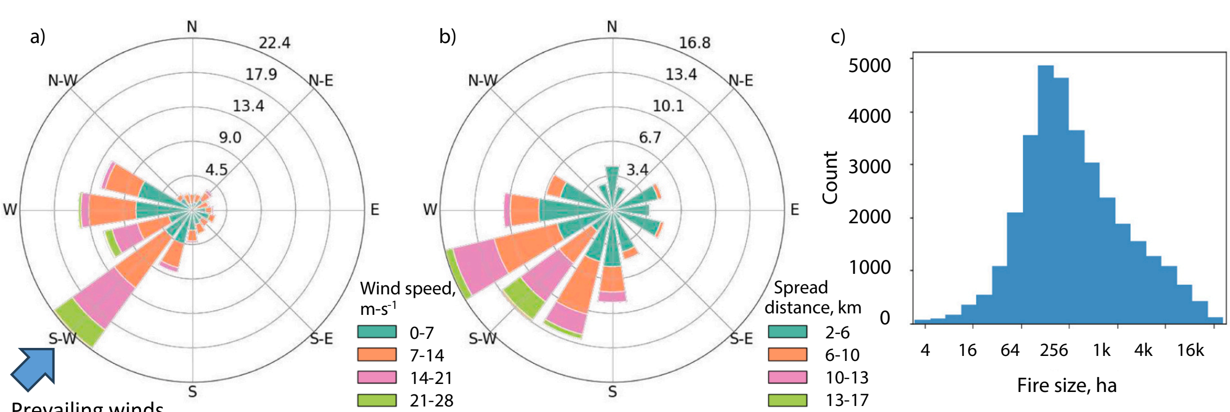

Fig. 3. Wind rose diagrams for the study area: a) prevailing winds; b) fire spread indicators q_{ij} between node pairs i,j based on the fire footprints and ignition locations derived with the fire growth model; c) distribution of the simulated fire sizes in network G. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

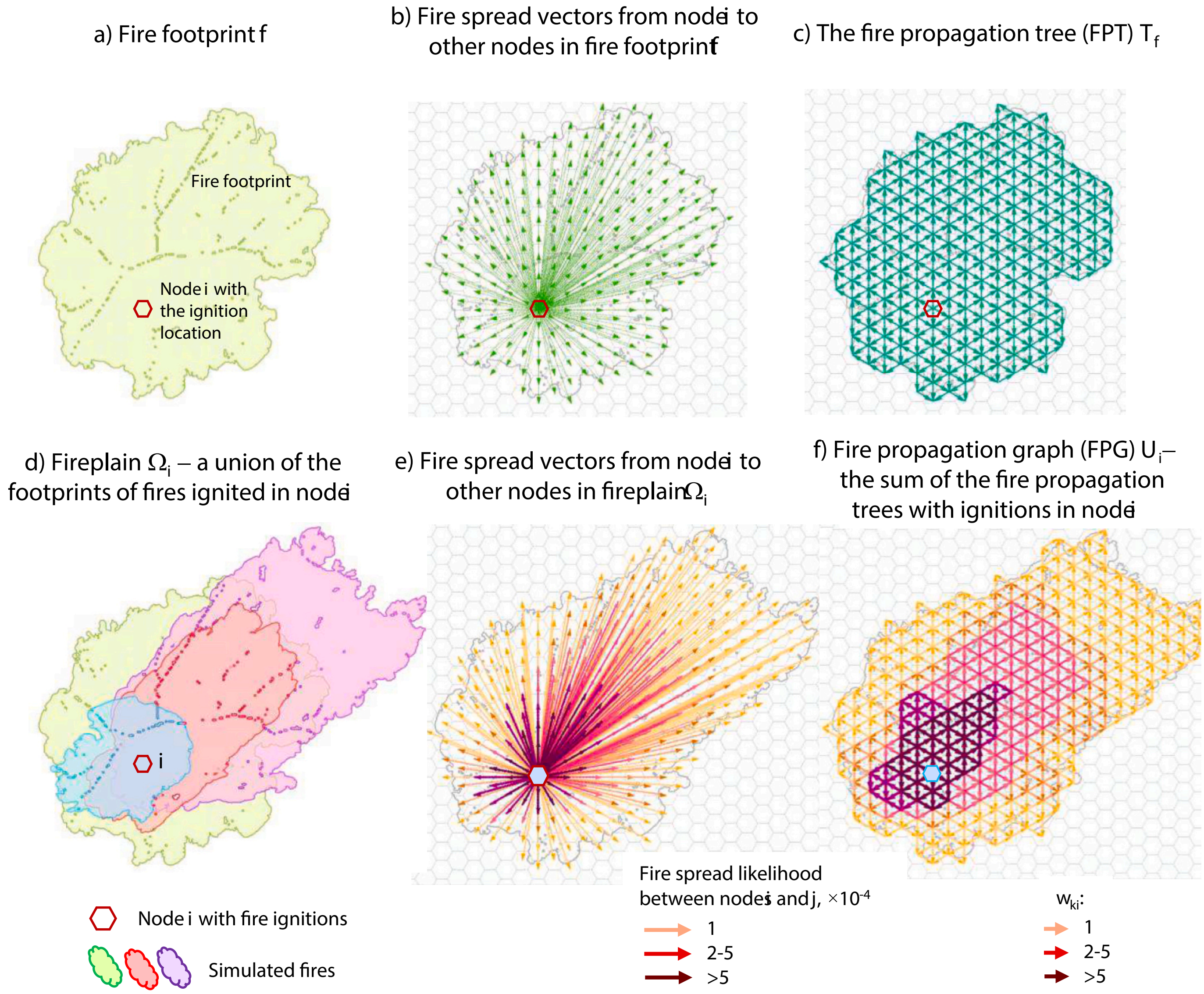

Fig. 4. The fire footprint, fire propagation tree (FPT) and fireplain graph (FPG) concepts: a) a simulated fire footprint (fire scar) f and the ignition node i; b) a landscape depicted as a hexagonal node network G with binary vectors indicating the fire spread from the ignition node to all other nodes within the footprint of fire f; c) an acyclic fire propagation tree (FPT) with a possibility of flank fire spread to a node from all adjacent upstream nodes; d) a fireplain Ω_i around node i – a union of the fire footprints with the ignitions in i; e) fire spread likelihoods between node i and other nodes in fireplain Ω_i around node i (defines the fire spread vectors q_{ij} in the CND problem); f) a fireplain propagation graph (FPG) – the sum of the individual fire propagation trees with ignitions in node i. Edge colors indicate the number of FPTs with ignitions in node i that propagate through an edge ij.

of 55,000 kW/m or less (which constituted 99.9 % of burn locations simulated with the fire growth model in the study area). The mixedwood and shrub vegetation that regenerates naturally after clearcut harvest (i.e., the planned method to create firebreaks in the study area) provides limited fuels for continuous fire spread during the summer fire season (Hély et al., 2001). Fire ignition rates are also greatly reduced in mixedwood forest (Krawchuk et al., 2006), and initial suppression actions in regenerating mixedwood stands for small spot fires – which can be started via firebrands from higher intensity fires in adjacent mature conifer stands – are far more successful than suppression actions in those conifer stands (Tremblay et al., 2018). Furthermore, having sufficient firebreak width is important in remote areas where fire crews' response times may be longer during the peak fire season. This is also consistent with regional management decisions, presented in the draft Upper Smoky Sub-Regional Plan (GoA, 2025), that anticipate the creation of wide strategic fire barriers in the area. This is also consistent with practical considerations, presented in the draft Upper Smoky Sub-Regional Plan (GoA, 2025), that anticipate the creation of wide strategic fire barriers in the area.

We characterized uncertain fire spread in landscape G by a set of F fire footprints f, $f \in F$, (Fig. 4a) created with the fire growth model. We overlaid the simulated fire footprints f on the landscape network of nodes G and, within each fire's footprint, we depicted the fire spread sequence in the form of a fire propagation tree, FPT. For fire f, the FPT depicts the sequence of burned nodes from the node with the ignition location as a directed acyclic graph, T_f, with the root node at the fire's ignition location and the other nodes in T_f located within the fire's footprint, S_f. Directed edges ij connecting adjacent nodes i and j in the FPT, $(i,j) \in S_f$, depict the potential (or realized, based on the fire growth model simulations) fire propagation paths between adjacent nodes within the fire's footprint (Fig. 4c). Conceptually, the FPT graph can be written as a sparce $f \times i \times j$ matrix that lists arcs ij, which are fundamental elements of the FPT T_f for scenario f. Appendix A describes the procedure of composing the fire propagation trees for each fire's footprint.

The multiple FPTs T_f originating from the same ignition node k, $f \in F_k$, can be aggregated to a corresponding fireplain propagation graph (FPG), U_k, where each directed edge ij is assigned the number of individual FPTs that have ignition locations in k and include edge ij (Fig. 4f). Since the FPG can include more than one FPT with the ignition in the same location, we introduced the density parameter w_{ki} that defines the number of FPTs with ignitions in node k, $f \in F_k$, that propagate through node i (Fig. 4f).

2.3.2. Using fire propagation trees for minimizing the potential burn area

Consider a fire f that has burned a portion of landscape G, S_f. Without firebreak placement, all nodes i within the fire's footprint, $i \in S_f$, are predicted to be burned (Fig. 1a). In fire f, the propagation proceeds from the ignition node to other nodes through edges comprising the acyclic graph T_f. Potentially, a set of firebreaks **X**, if allocated to nodes (or edges) within the fire's footprint, could block the spread of the fire to some locations and reduce the total number of burned locations in f from S_f to S'_f (Fig. 1b). We assumed that the firebreaks would be created via clearcut harvesting and involve the complete removal of vegetation to maximize their effectiveness. Since the managed area had full forest canopy dominated by mature forest, we did not track the impacts of initial vegetation conditions on the firebreak creation costs under the assumption that these costs would be similar across the landscape. Each configuration of firebreaks is expected to yield a unique reduction of the burned area of fire f, $S'_f(\mathbf{X})$. Considering that the fire set F includes discrete realizations of the stochastic fire spread process with equal probabilities $1/|F|$, the firebreak placement problem can be formulated as minimizing the total number of remaining burned nodes $S'_f(\mathbf{X})$ across all fires, $|F|$, in landscape G, i.e.:

$$min\sum^{f \in F} S'_f(\boldsymbol{X}) \tag{1}$$

The firebreak problem based on FPTs was proposed in Vilches et al. (2024). A binary decision variable x_i, $i \in N$, defines whether a firebreak is applied to node i and fire spread through i is no longer possible (i.e., $x_i = 0$, and $x_i = 1$ otherwise). The binary decision variable n_{fi}, $i \in S_f$, checks, for each node i in footprint S_f of fire f, whether the fire could spread through the fire propagation tree T_f to i after firebreak placement (i.e., $n_{fi} = 1$, and $n_{fi} = 0$ otherwise). The node with the fire's ignition location serves as a source of fire propagation through edges ij within footprint S_f and is always assigned burned status, $n_{fi} = 1$.

The ability of firebreaks to block fire propagation through the FPT depends on its degree of interconnectedness. For example, the FPTs used in Vilches et al. (2024) only permitted fire propagation to a node from one "upstream" node (Fig. 1a). While this configuration may adequately depict low-intensity fires in some regions, it is not well-suited for the montane forests of North America, where crown fires can simultaneously spread in multiple directions and rapid fluctuations of fire weather may occur within hours (Agee and Skinner, 2005; Thompson et al., 2022). Furthermore, multiple ignitions beyond the main fire front can be created by firebrand spotting (Albini et al., 2012; Manzello et al., 2020; Martin and Hillen, 2016; Page et al., 2019). To depict such circumstances, each node in the FPT, in addition to the directed edges indicating spread to a node from another node at the head of the fire, includes additional edges depicting possible fire spread from other nodes via flank spread and ember spotting (Fig. 1c). The use of interconnected FPTs creates a more conservative firebreak prescription because the firebreaks must block a larger number of possible fire propagation paths to a given node from multiple directions (Fig. 1d). To enable the use of interconnected FPTs, we have modified the firebreak problem of Vilches et al. (2024) (FPT problem 1 hereafter) as follows:

$$min\sum^{f \in F}\sum^{i \in S_f} n_{fi} \tag{2}$$

subject to:

$$\sum^{i \in N}(1 - x_i)c_i \le B \tag{3}$$

$$n_{fi} = 1 \forall i \in S_f f \in F, i \in \phi_f \tag{4}$$

$$\theta_{fj} n_{fj} \ge \sum^{i \in S_f | ij \in T_f}(n_{fi} + x_i - 1) \forall j \in S_f f \in F \tag{5}$$

$$\sum^{i \ne j, ij \in E_i}(1 - x_j) \ge 1 - x_i \forall i \in N \tag{6}$$

Table 1 lists all symbolic notations. Objective (2) minimizes the total number of nodes i in the burned portions of the footprints S_f (nodes with burned status hereafter) in a set of $|F|$ fire scenarios. When unit cost of establishing a firebreak, c_i, is set to one, the budget constraint (3) defines the maximum number of allocated nodes with firebreaks. However, when the establishment cost c_i is allowed to vary, the B value indicates the firebreak establishment budget level. The c_i parameter, while not relevant in our case, makes the formulation usable in regions with significant variation in firebreak establishment costs (e.g., due to differing initial vegetation conditions). For each fire f, constraint (4) assigns burned status to the node with the fire's ignition location (i.e., $n_{ij} = 1 | i \in \phi_f$). For each fire f, set ϕ_f includes the ignition node i only. For each fire f, constraint (4) is applied only to ignition node i and initiates the propagation flow from the ignition node through edges ij in the propagation tree T_f.

Constraint (5) defines the burned status of node j in fire propagation tree T_f after firebreak placement. Fire f can spread to node j from connected upstream nodes i through edges ij if one or more upstream nodes i are burned and do not have allocated firebreaks. The propagation of fire f to node j can only be stopped if all upstream nodes i in fire propagation tree T_f that receive fire from other upstream nodes have firebreaks. If no firebreaks are installed, constraints (4) and (5) ensure that all nodes i in fire footprint S_f are labeled as burned ($n_{fi} = 1$). For each node j in propagation tree T_f, the parameter θ_{fj} defines the number of directed arcs ij connecting the adjacent upstream nodes i to node j, $(ij) \in T_f$. Compared to the model of Vilches et al. (2024), constraint (5) allows the use of FPTs where the fire can propagate to a node from multiple upstream nodes (Figs. 1c,4c). Constraint (6) prevents the allocation of firebreaks to single isolated nodes and ensures that a treated node must be adjacent to at least one other treated node. Subset E_i defines edge arcs ij connecting node i and adjacent nodes j which can receive firebreaks.

Sometimes it is optimal to treat the ignition sources of the largest fires to prevent their possible occurrence. However, this strategy is sensitive to errors in the depiction of the extremely large fires. Since the fire footprints are generated with the stochastic fire growth model, the most extreme scenarios with the largest footprints typically show the greatest variability and can only be considered as approximations of possible extreme events (and would require a significantly larger number of model replications to adequately represent their spatial distribution). By preventing the creation of isolated single-element firebreaks, constraint (6) helps reduce overreliance on the configurations of the most extreme fires and promotes the creation of linear firebreak segments.

FPT problem 1 allocates firebreaks via removal of nodes from the fuel network G. Thus, the firebreaks are assumed to cover the entire node area, which imposes a practical limit on the maximum node size. An alternative formulation conceptualizes firebreaks as the removal of edges connecting adjacent nodes in the fuel network. In this case, a firebreak can be seen as analogous to a linear segment at the border between two adjacent nodes whose width is sufficient to reduce the potential fire intensity to be successfully contained (Agee et al., 2000; Oliveira et al., 2016) (Fig. 5a,b). The firebreak disrupts the edge connecting the two adjacent nodes, thereby preventing fire spread along that edge. Note that it may require only a fraction of the entire node area to accomplish this. Formally, the area around the removed edge can be delineated by a rhombus with its longest diagonal equivalent to the edge length and shortest diagonal equal to the length of the common boundary between the adjacent nodes (Fig. 5b). Under this alternative formulation (FPT problem 2 hereafter), the binary decision variable y_{ij}

Table 1
Summary of the model variables and parameters. (Last column indicates the models in Table 2 where the parameter/variable is used).

Symbol	Parameter / variable name	Description	Problems in which it is used (see Table 2)
Sets:			
N	Nodes (forest patches) i,j,k in landscape network G – potential fuel treatment locations	$i,j,k \in N$	1–8
E	Edges (arcs) connecting adjacent nodes i,j in landscape network G	$E \subset N \times N$	1–8
F	Set of simulated fires / fire propagation trees	$f \in F$	1,2,7,8
F_k	Fires / fire propagation trees with their ignitions in node k (members of fireplain Ω_k and fireplain propagation graph U_k)	$F_k \in F$	3,4
S_f	Nodes – members of the footprint of fire f	$S_f \in N$	1,2,7,8
T_f	Directed edges ij – members of the fire propagation tree depicting the spread of fire f	$T_f \subset N \times N$	1,2,7,8
ϕ_f	Node i with the ignition point of fire f (a singleton set)	$\phi_f \in S_f$	1,2,7,8
$N_G(i)$	Connected neighborhood that includes node i	$N_G(i)\in N$	5–8
Ω_k	Nodes j – members of a fireplain set around node k and potential spread destinations of fires ignited in node k	$\Omega_i \in N$	3–8
U_k	Directed edges ij – members of the fireplain propagation graph for root node k	$U_k \subset N \times N$	3,4
Decision variables:			
x_i	Node deletion binary variable ($x_i =$ 0 if node is removed and $x_i = 1$ otherwise)	$x_i \in\{0,1\}$	1,3,5,7
y_{ij}	Edge deletion binary variable ($y_{ij} =$ 0 if edge ij is removed and $y_{ij} = 1$ otherwise)	$y_{ij} \in\{0,1\}$	2,4,6,8
u_{ij}	Binary variable defining that nodes i and j are *not* removed and there is a path connecting i and j	$u_{ij} \in\{0,1\}$	5–8
n_{fi}	Burned / unburned status of node i in fire f	$n_{fi} \in\{0,1\}$	1–4,7,8
v_{fkj}	Product of n_{fi} and u_{ij}	$v_{fkj} \in\{0,1\}$	7,8
m_{ki}	Burned / unburned status of node i in fireplain Ω_k around node k	$m_{ki} \in\{0,1\}$	3,4
Parameters			
θ_{fj}	Number of directed edges ij connecting the upstream nodes i to node j in propagation tree T_f	$\theta_{fj} \in [0;5]$	1,2
δ_{kj}	Number of upstream nodes adjacent to node j in fireplain graph U_k	$\delta_{kj} \in [0;5]$	3,4
q_{ij}	Fire spread binary indicator between nodes i and j	$q_{ij} \in \{0,1\}$	5–8
c_i	Unit cost to establish a firebreak segment in node i	$c_i = 1$	1,3,5,7
C_{ij},	Unit cost to establish a firebreak segment across edge ij	$C_{ij} = 1$	2,4,6,8
B	Node (or edge) removal budget limit	$B > 0$	1–8
M	Large positive (Big-M) value		5–8
w_{ki}	Number of FPTs with the ignitions in node k which include node i in fireplain graph U_k	$w_{ki} \le \|F\|$	3,4

defines whether directed edge ij connecting adjacent nodes i and j, $ij\in E$, is *not deleted* from network G ($y_{ij} = 1$ and $y_{ij} = 0$ otherwise). Subsequently, FPT problem 2 can be formulated as follows:

minimize objective (2),

subject to constraint (4) and:

$$\sum_{\substack{i,j \in E\\ i<j}} \left(1-y_{ij}\right)C_{ij} \le B \tag{7}$$

$$y_{ij} = y_{ji}\forall(i,j) \in E, i \ne j \tag{8}$$

$$\theta_{fj}n_{fj} \ge \sum^{i\in S_f|ij\in T_f}\left(n_{fi}+y_{ij}-1\right)\forall j \in S_f, f \in F \tag{9}$$

$$\sum_{\substack{i,k \in E\\ k\ne j}}(1-y_{ik}) + \sum_{\substack{j,l \in E\\ l\ne i}}\left(1-y_{jl}\right) \ge 1-y_{ij}\forall(i,j) \in E, i \ne j \tag{10}$$

Eq. (7) is the budget constraint. When the unit cost of establishing a firebreak, C_{ij}, is set to one, constraint (7) defines the maximum number of edges B that can be removed. However, when establishment cost C_{ij} is allowed to vary, the B value defines the maximum firebreak establishment budget level. As with the c_i parameter in Eq. (3), the C_{ij} parameter in Eq. (7), while not relevant in our case, makes the formulation usable in regions with significant variation in firebreak establishment costs.

Constraint (8) implies that the removal of edges between nodes i and j occurs in both directions when a firebreak is installed between i and j. Constraint (9) is analogous to constraint (5) in FPT problem 1 and ensures that fire f cannot spread to node j through fire propagation tree T_f if all directed edges ij connecting adjacent upstream nodes i with burned status (i.e., $n_{fi} = 1$) to node j have firebreaks installed (so $y_{ij} = 0$). The function of constraint (10) is analogous to that of constraint (6) and prevents the removal of isolated single edges. It requires that removal of an edge ij is accompanied by removal of at least one more edge between node i (or node j) and another node adjacent to i or j. Subscript k denotes the other nodes adjacent to node i, $k \ne j$, and subscript l denotes the other nodes adjacent to node j, $l \ne i$.

In FPT problems 1 and 2, the fire set F needs to be very large to adequately represent the possible configurations of large fires, which, for large landscapes, might render the problem intractable. One way to reduce the problem size is to aggregate all FPTs with the same ignition node to fireplain sets. For each node k, fireplain Ω_k defines the subset of other nodes j, $k,j\in N$, to which fires may potentially spread from k (Scott and Thompson, 2015) (Fig. 4d,e). In our FPT context, fireplain set Ω_k is a union of the footprints of all fires ignited in node k, F_k. The members of fireplain set Ω_k are nodes. Note that the fire set F_k translates to both fireplain Ω_k (i.e., a set of nodes) and its corresponding fireplain propagation graph U_k. As shown in Fig. 4e,f, the union of fire footprints portrayed in Fig. 4d can be represented in both ways. We reformulated the FPT problem 1 to the fireplain-based problem (FPG problem 3 hereafter) as follows:

$$min\sum^{k\in N}\sum^{i\in\Omega_k}(m_{ki}w_{ki}) \tag{11}$$

subject to:

constraints (3,6)

$$m_{ki} = 1\forall i \in \Omega_k, k \in \psi_k \tag{12}$$

$$\delta_{kj}m_{kj} \ge \sum^{i\in\Omega_k,ij\in U_k}(m_{ki}+x_i-1)\forall j \in \Omega_k, k \in N \tag{13}$$

Objective (11) minimizes the total number of nodes i with the burned status within the $|N|$ footprints of fireplains Ω_k, $k\in N$, in landscape G. The binary decision variable m_{ki}, $i\in\Omega_k$, checks, for each node i in fireplain Ω_k whether a fire could spread through the fireplain propagation graph (FPG) U_k to i after firebreak placement (i.e., $m_{ki} = 1$, and $m_{ki} =$ 0 otherwise). For each fireplain Ω_k, which defines the footprint of the corresponding fire propagation graph U_k, constraint (12) assigns burned status to the node containing the fireplain's ignition location; this node is also included in subset ψ_k. Constraint (13) calculates the burned status of node j in FPG U_k after firebreak placement. For node j, integer

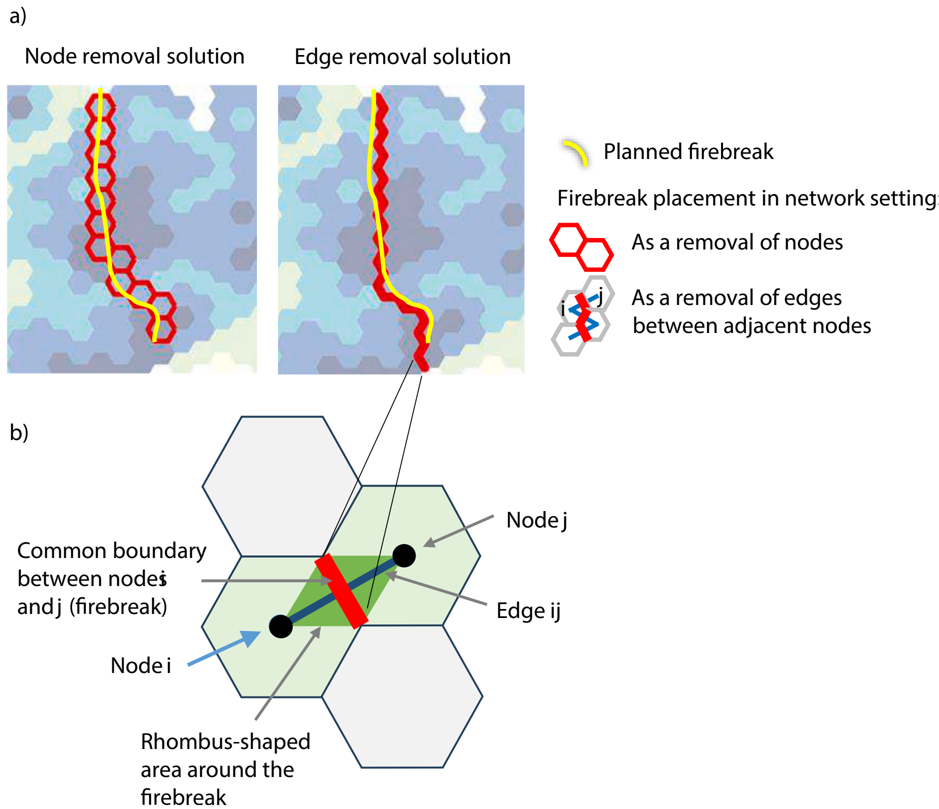

Fig. 5. Firebreak depiction in a network setting: a) the equivalent removal of nodes vs. edges to allocate a linear firebreak; b) an allocation of a linear firebreak by removal of an edge between adjacent nodes i and j.

parameter δ_{kj} defines the number of adjacent upstream nodes in FPG U_k from which a fire could spread to node j. According to constraint (13), node j retains unburned status if none of the adjacent upstream nodes i in U_k are burned or firebreaks are allocated to all burned nodes. The aggregation of individual FPTs with ignitions in node k to a fireplain graph U_k necessitated an additional parameter w_{ki} in objective (11) to track, for node i in fireplain U_k, the number of aggregated FPTs with ignitions in k that included i.

When individual FPTs are aggregated into FPG U_k, it is no longer possible to track the burned status of nodes in the individual FPTs. Since the fireplain is a union of the FPTs ignited in the same location, a node in an FPG may have more connected upstream neighbors than the individual FPTs. Consequently, an FPG may include more fire propagation paths than the equivalent set of FPTs. However, in the case of interconnected acyclic FPTs, where each node is connected to all adjacent upstream nodes (Fig. 4c), differences between the connectivity of individual FPTs and the connectivity of an aggregated FPG are expected to be minor (Fig. 4f). The main benefit of FPT aggregation to fireplain sets is the problem size reduction from an order of $|F*N|$ to $|N*N|$ elements. This reduction can be significant given that a typical fire realization set F in a large landscape may include hundreds of thousands of fires, so $|F|>>|N|$.

A similar approach can be used to reformulate the FPT-based edge removal problem 2 to a fireplain-based FPG problem 4 that allocates firebreaks by removal of edges between adjacent nodes, i.e.:

minimize objective (11).

subject to constraints (7,8,10,12) and.

$$\delta_{kj} m_{kj} \geq \sum\nolimits^{i \in \Omega_k, ij \in U_k} \left(m_{ki} + y_{ij} - 1 \right) \forall j \in \Omega_k, k \in N \tag{14}$$

Constraint (14) is analogous to constraint (13) but uses the directed edge removal variable y_{ij}.

2.3.3. *Applying the critical node detection concept to minimize the fire spread connectivity*

We compared the FPT-based approach with the model from our previous work (Yemshanov et al., 2021) that follows a common fire mitigation strategy of minimizing the connectivity between patches with fuel through interdiction of key fire spread paths (Finney, 2004; Finney et al., 2007; León et al., 2019). That model solved a critical node detection (CND) problem by finding the key nodes in a fuel network whose removal maximally degraded its connectivity according to a chosen metric. However, the CND problem does not track the burn area nor the particular fire spread path(s) because it does not use FPTs. Instead, it checks whether two nodes remain in the same connected component of the fuel network, which allows for possible fire spread between them. For each node pair i,j, the CND problem requires the likelihood (or a binary indicator) that a fire could spread from i to j given that node j is located in the fireplain set Ω_i around i. This information can be extracted from the same fire growth model output that we used to generate the set of FPTs. For each node i, the fireplain sets of nodes j, Ω_i, are used to derive binary indicators q_{ij}, showing whether a fire can spread from i to j (Fig. 4e). Nodes i and j are considered connected if $q_{ij} = 1$ and both nodes are in the same connected portion of the fuel network and j is a part of fireplain Ω_i. We used the Post-BP library (Liu et al., 2024a) to generate the fireplain sets and calculate the fire spread indicators q_{ij} from the simulated fire footprints S_f.

For each pair of nodes i and j in landscape G, we defined a binary variable, u_{ij}, $i,j \in N$, that indicates that nodes i and j are *not removed* from network G ($u_{ij} = 1$) if firebreaks are placed in neither i nor j and there is a fire spread path connecting i and j, such that, for any pair of adjacent nodes k and l on that path, $x_k = x_l = 1$ (and $u_{ij} = 0$ otherwise). Our CND problem 5 minimizes the number of node pairs with possible fire spread path connections (i.e., $u_{ij} = 1$) across network G, i.e.:

$$min \sum\nolimits^{i \in N} \sum\nolimits_{j \neq i}^{j \in \Omega_i} u_{ij} q_{ij} \tag{15}$$

subject to:
constraints (3,6),

$$u_{ij} \geq x_i + x_j - 1 \forall (i,j) \in E, i \neq j, j \in \Omega_i \tag{16}$$

$$u_{ij} \geq \frac{1}{M} \sum_{k \neq j}^{k \in N_G(i), k \in \Omega_i, j \in \Omega_k} u_{kj} - (1 - x_i) \forall i \in N, j \in \Omega_i, i \neq j, (i,j) \notin E \tag{17}$$

Objective (15) minimizes the number of node pairs i,j in fuel network G between which fire spread is possible after firebreak placement. Constraint (16) ensures, for an adjacent connected edge between nodes i and j, that $u_{ij} = 1$ if neither node is allocated a firebreak. Constraint (17) ensures that nodes i and j are connected if there is another non-deleted node $k, k \neq j$, in the connected neighborhood of node i, $N_G(i)$, as well as nodes with connections to i, such that k and j are connected. Parameter M denotes a large positive value. Fires may spread between nodes i and j in opposite directions with different likelihoods, so node j can be a part of the fireplain around node i, but i may not be a part of the fireplain around j.

We also evaluated a CND problem 6 that allocated firebreaks by removal of edges between adjacent nodes as follows:
minimize objective (15),
subject to:
constraints (7,8,10) and

$$u_{ij} \geq y_{ik} + u_{kj} - 1 \forall i \in N, k \in N_G(i), j \in \Omega_i, j \in \Omega_k, i \neq j \tag{18}$$

$$u_{ij} \geq y_{ij} \forall (i,j) \in E, i \neq j \tag{19}$$

Constraint (18) is analogous to constraint (17) and ensures that nodes i and j are connected (i.e., $u_{ij} = 1$) if a non-deleted edge kj exists in the connected neighborhood of i, $k \in N_G(i)$, $k \neq j$, so that k and j are connected. Constraint (19) ensures that adjacent nodes i and j are connected (i.e., $u_{ij} = 1$) if there is a non-deleted edge between i and j, so $y_{ij} = 1$.

2.3.4. Combining the FPT and CND problems

The FPT and CND problems follow distinct goals of minimizing the total burn area versus minimizing fire spread connectivity. Ideally, firebreak allocation should achieve both objectives. We combined the CND and FPT objectives as follows. The objective function estimates the total number of origin-destination node pairs k,j between which fire spread is possible, times the expected burn status of the destination nodes j, i.e.:

$$min \sum^{k \in N} \sum_{j \neq i}^{j \in \Omega_k} \left[u_{kj} q_{kj} \frac{1}{|F|} \sum^{f \in F_k} n_{fj} \right] \tag{20}$$

For each node pair k,j, the binary variable u_{kj}, which indicates the presence-absence of a fire spread path from k to j, is weighted by the expected burned status of the destination node j over $|F|$ fire scenarios, $\frac{1}{|F|}\sum^{f \in F_k} n_{fj}$. This term is computed for node j by tracking fire spread in a set of fire propagation trees F_k with their ignitions in node k after firebreak placement. Note that node j can be a member of multiple fireplains Ω_k with origins in different nodes k. When estimating the burned status of node j for node pair k,j, Eq. (20) only sums up the burned status variables n_{fj} for node j from FPTs with ignitions that originated in node k.

We introduced an auxiliary binary variable v_{fkj} to linearize the product of variables u_{kj} and n_{fj} and formulated the combined FPT-CND problem 7 as follows:

$$min \sum^{k \in N} \sum_{j \neq i}^{j \in \Omega_k} \sum^{f \in F_k} v_{fkj} q_{kj} \tag{21}$$

subject to:
constraints(3–6,16,17) and

$$v_{fkj} \leq n_{fi} \forall k \in N, j \in \Omega_k, k \neq j, f \in F_k \tag{22}$$

$$v_{fkj} \leq u_{kj} \forall k \in N, j \in \Omega_k, k \neq j, f \in F_k \tag{23}$$

$$v_{fkj} \geq u_{kj} + n_{fj} - 1 \forall k \in N, j \in \Omega_k, k \neq j, f \in F_k \tag{24}$$

The binary variable v_{fkj} in objective (21) is the product of decision variables u_{kj} and n_{fj} and was linearized via constraints (22–24).

We formulated the combined FPT-CND problem 8 that allocates firebreaks via removal of edges in an analogous way as minimizing objective (21), subject to constraints (4,7-10,18,19,22–24).

2.4. Scenario assumptions

The placement of firebreaks in montane forests is considered more effective on slopes facing the prevailing winds (Butler et al., 2007). We designated nodes in node removal problem 1,3,5 and 7 formulations (or the areas around edges between two nodes in edge removal problem 2,4,6 and 8 formulations) (Fig. 3b) that had a proportion of south-, west- and southwest-facing slopes facing prevailing winds in the area equal to 20 % or higher as potential candidate locations for firebreaks (Figs.6c,7). We compared eight model formulations (Table 2) using the same sets of FPTs and fireplains (see a description of the model data frames in Appendix A). To compare the node and edge removal solutions, we assumed that removal of a node would be equivalent to installation of a linear firebreak segment extending across the hexagon's width, which means that the segment must be at least two edges in length. We found optimal solutions for a range of firebreak placement budgets between 25 and 100 nodes in the node removal problems 1,3,5 and 7 and, equivalently, between 50 and 200 edges in problems 2, 4, 6 and 8. To compare the CND and FPT-based solutions, we calculated both CND and FPT objective values for all problem solutions. Appendix A provides the model solution details.

3. Results

3.1. Mapping the firebreak solutions

Landscape network G in the study area included 834 nodes and 4788 edges (Fig. 6a,b). Wildfire behavior was characterized by 27,944 indicators q_{ij} for node pairs i,j (Fig. 6d). The burn likelihoods were higher at lower elevations in the northern and eastern portions of the landscape, while the high-elevation southwestern regions had lower burn risk (Fig. 6e). An almost uniform distribution of fireplain sizes Ω_i (Fig. 6f) indicated that large fires could occur in nearly any part of the landscape, however 95 % of fires were expected to spread over distances of 12.9 km or less and extents below 16,000 ha (Fig. 3c).

Respectively, the maps in Figs.7 and 8 and Fig. B.1 Appendix B show the firebreak solutions for the budget levels of 25, 50 and 75 nodes in problems 1,3,5 and 7 (and equivalently, 50, 100 and 150 removed edges in problems 2,4,6 and 8). The prevailing winds from the west-southwest impacted the pattern of simulated fires and caused a layered configuration of firebreaks, which tended to be oriented perpendicular to the prevailing wind directions.

3.2. The FPT problem solutions

Typically, the FPT problems 1 and 2 solutions, which minimized the total burn area in a set of FPTs, allocated the firebreaks around hotspots with the highest burn risk (Fig. 7 callout II). No firebreaks were allocated in the higher-elevation southwestern portions of the landscape with lower burn risk. The FPT solutions allocated firebreaks closer to the areas with the highest burn likelihoods and were most effective in reducing the expected burn area (Table 3). The linear firebreak sequences in edge removal FPT problem 2 solutions were usually not as long as the sequences in edge removal CND problem 6 solutions and

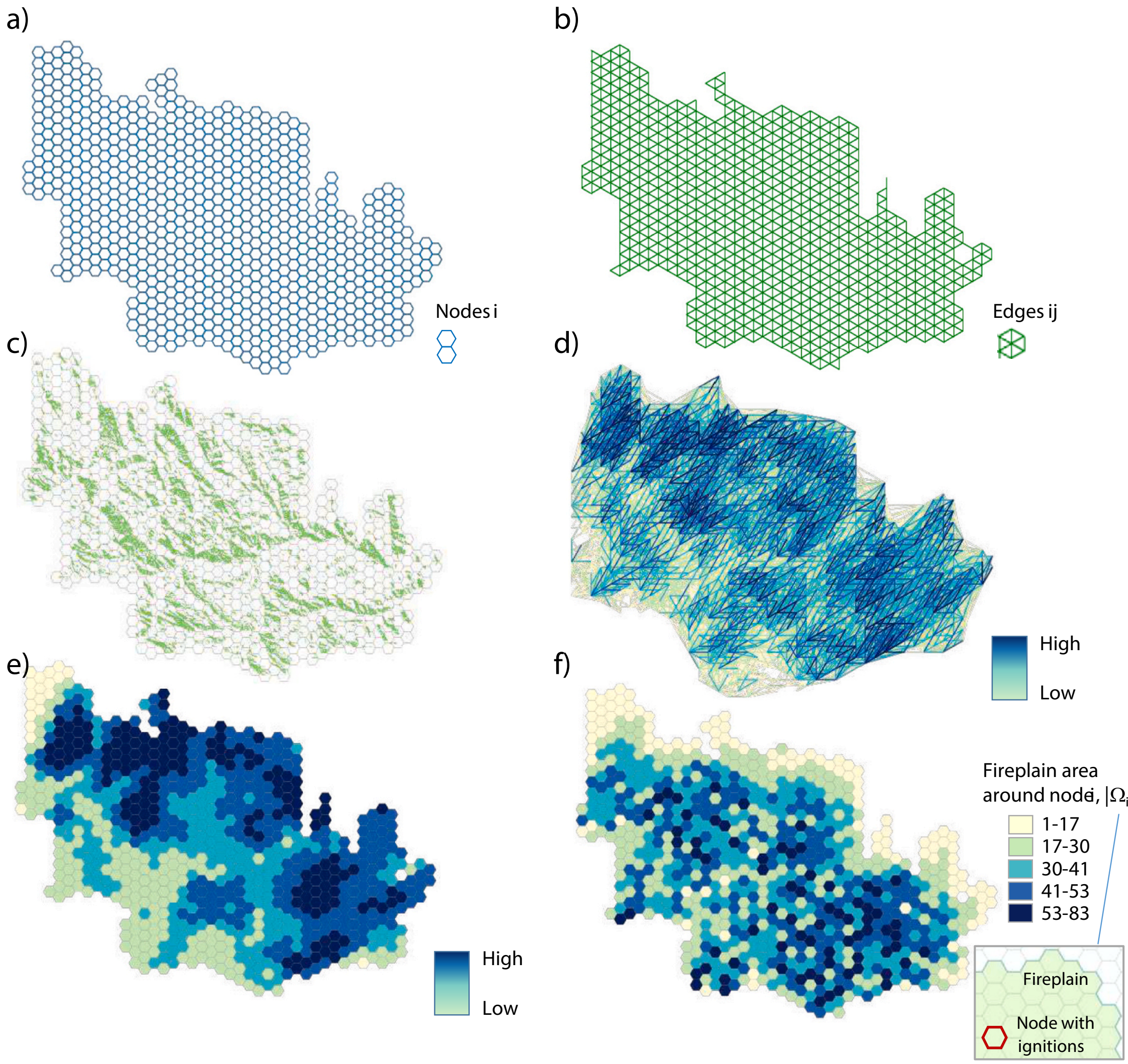

Fig. 6. Landscape network G: a) nodes i; b) edges ij; c) eligible locations for firebreak placement: west-, south- and southwest-facing slopes facing the prevailing winds; d) fire spread indicators q_{ij} between node pairs i,j. Darker colors indicate higher fire spread likelihoods between node pairs. The spread indicators between adjacent nodes are not shown; e) the expected number of fire propagation trees overlapping a node i (equivalent to the burn likelihood in node i); f) fireplain sizes around nodes i, $|\Omega_i|$.

included sporadic gaps (Figs. 7,8 callout III; Table 3).

Aggregation of the FPT-based problems 1 and 2 to the FPG-based problems 3 and 4 yielded generally comparable firebreak configurations and did not change the locations of broad fire risk zones compared to the FPT-based solutions. At small budget levels, the FPG-based problem 4 solutions allocated longer firebreak segments than the FPT-based problem 2 solutions (Table 3) but differences between the FPT- and FPG-based solutions were inconsequential, especially between the node removal solutions (Figs. 7,8). Overall, aggregation of FPTs to a smaller set of FPGs could be a reasonable workaround to reduce the FPT problem size. As noted earlier, the impact of this aggregation on firebreak placement solutions depends on the connectivity of the FPTs.

3.3. The CND problem solutions

Almost all removed nodes and edges in the CND problems 5 and 6 solutions were allocated in contiguous linear sequences (Figs. 7,8). However, allocation was constrained by the pattern of locations where firebreak placement was deemed feasible (Fig. 6c). Notably, the map of feasible locations for edge removals had a more nuanced pattern than the map of feasible locations for node removals (Fig. 7). This is because of the difference between the area in the vicinity of an edge and the area of an entire node. The rhombus-shaped suitable area around an edge ij (Fig. 5b) was three times smaller than the node area and so the corresponding map of suitable edges had three times more eligible locations for firebreak placement than the map of suitable nodes (Fig. 7).

Compared to the FPT problem solutions, the CND model prioritized placing firebreaks to maximally degrade the connectivity between locations with fuel, resulting in a more even spacing of firebreaks across the landscape (Figs. 7,8). Since large fires could occur almost everywhere across the region (Fig. 6f), the CND solutions allocated firebreaks across the entire landscape, including the southwestern portion with lower burn risk (Fig. 7 callout I). At a high budget level, the CND problem solutions tended to divide the landscape into partial compartments (Fig. B.1 Appendix B). When the budget was insufficient to create compartments, the CND solutions allocated shorter linear sequences to interdict the most threatening corridors for fire spread. This strategy also helped effectively reduce average fireplain size (Table 3).

3.4. The combined CND-FPT problem solutions

The firebreak patterns in the combined CND-FPT problems 7 and 8

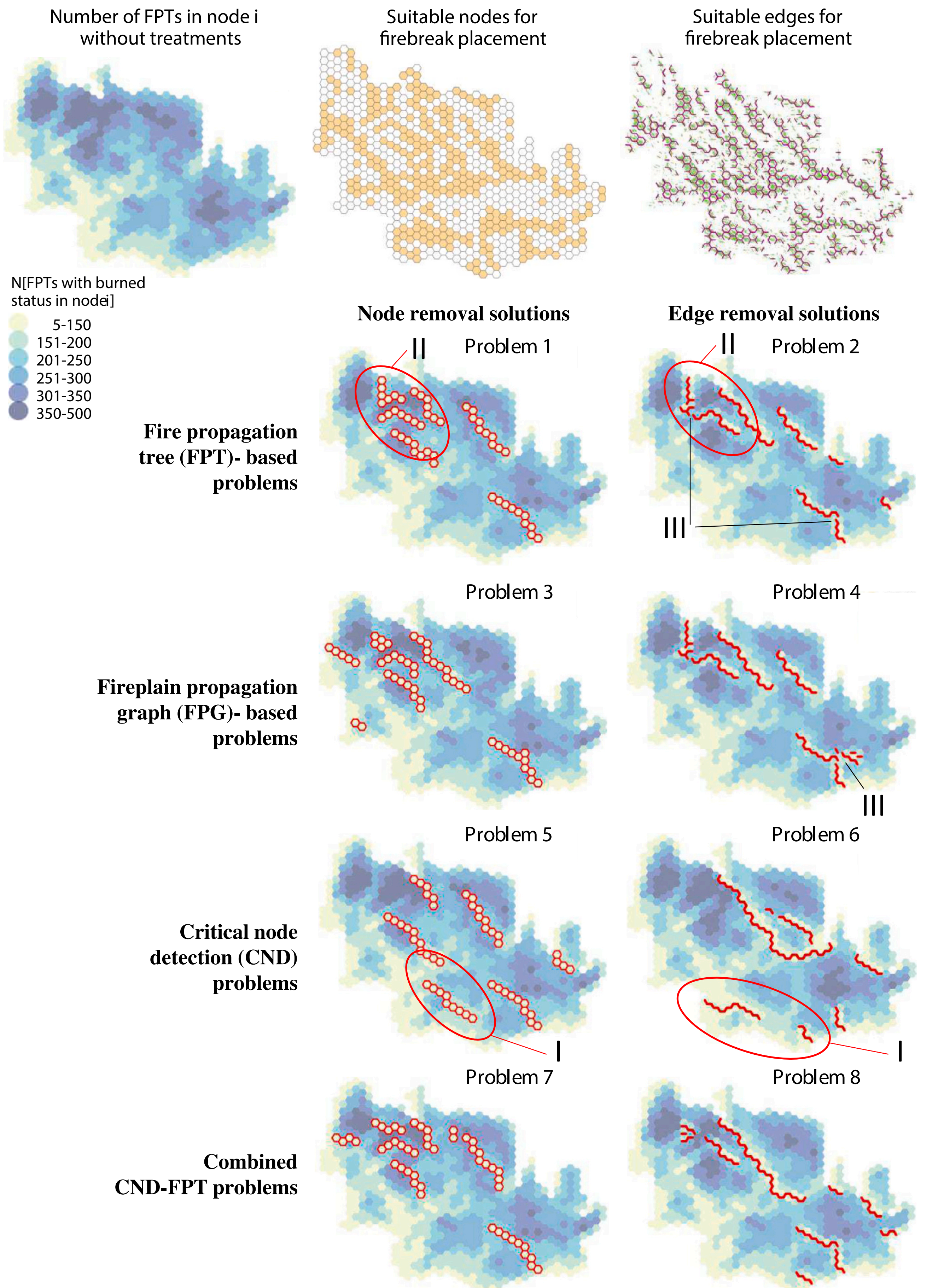

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Fig. 7. Optimal node and edge removal solutions with treatment budgets of 50 removed nodes (or equivalently, 100 removed edges). The maps in the top row show the eligible locations for firebreak placement in the node removal and edge removal problems. The background maps in the firebreak allocation solutions show the expected number of nodes with burned status after the treatments. Callout I shows the firebreak segments located in areas with low burn risk in the southwestern portion of the landscape. Callout II shows the firebreak segments clustered near the hotspot with the highest burn likelihoods. Callout III shows the small gaps between the firebreak segments in FPT-based edge removal solutions.

Table 2
Model scenarios.

Problem type	Concept	Objective	Firebreak allocation	
			Via node removal	Via edge removal
Tracking the burned portions of a set *F* of **fire propagation trees**	FPT	Minimize the expected burn area in a set *F* of fire propagation trees in landscape network *G*	**Problem 1** (Eqs. 2–6)	**Problem 2** (Eqs. 2,4,7–10)
Tracking the burned portions of a set *N* of **fireplain propagation graphs**	FPG	Minimize the expected burn area in a set *N* of fireplain propagation graphs in landscape *G*	**Problem 3** (Eqs. 3,6, 11–13)	**Problem 4** (Eqs. 7,8, 10–12,14)
Critical node detection	CND	Minimize the total number of node pairs *ij* with possible fire spread in network *G*	**Problem 5** (Eqs. 3,6, 15–17)	**Problem 6** (Eqs. 7,8,10, 15,18.19)
Combined problem: critical node detection with tracking of the burned FPT portions	CND-FPT	Minimize the total number of node pairs *ij* with possible fire spread times the expected burn status of the destination nodes *j*	**Problem 7** (Eqs. 3–6,16, 17,21–24)	**Problem 8** (Eqs. 4,7–10, 18,19,21–24)

solutions resembled those in the FPT problems 1 and 2 solutions (Figs. 7,8). Since the combined model weighted the presence of a fire spread path between two nodes by the destination node's expected burn status, it prioritized the interdiction of fire spread paths in areas with the highest burn likelihood (which overlapped the largest number of the FPTs). The firebreaks in the CND-FPT problem 8 solutions resembled the FPT problem 2 solutions, which included small gaps and sporadic clustering (Fig. 7 callout III). The distribution of the firebreak lengths in the combined problem solutions was closer to the aggregated FPG solutions than to the FPT or CND solutions (Table 3).

The trade-off between the CND- and FPT-based strategies was relatively small (Figs. 9,10) and the CND and FPT objective values were close for the same budget level (Table 3). This is because minimizing the number of node pairs with possible fire spread paths also reduces the spread of large fires and so the burned portions of the FPTs. Since the study area was dominated by continuous forest cover, the hotspots with high burn risk always had connecting fuel corridors. The interdiction of these corridors helped reduce the fire threat to the hotspots with high burn risk.

Notably, the reduction of fire spread risk did not taper off as the budget level increased (Fig. 9). Because of the area's almost continuous forest cover, adding more firebreaks resulted in a gradual compartmentalization of the forest landscape. The shape of the cost-effectiveness curve in Fig. 9, which could either follow a negative exponential trend that conforms to the law of diminishing returns or a linear trend that indicates independence of treatment efficacy from the budget level, could help assess whether mitigation solutions can be expected to remain cost-efficient as the budget increases.

The firebreak configurations in the FPT solutions were more sensitive to the locations of sites with high burn risk. However, the FPT problem did not select the nodes with the highest burn likelihoods per se. Instead, the firebreaks were placed at some distance from these potential hotspots, aiming to protect them from fires that may be spread by prevailing winds (Figs. 7,8). All problem formulations were able to allocate contiguous firebreaks, however, for the same budget, the CND and FPT-CND model solutions allocated slightly longer segments on average (Table 3).

3.5. Node removal vs. edge removal solutions

The node and edge removal solutions exhibited slight differences in the fine-scale firebreak configurations. Removal of a node in problems 1,3,5 and 7 prevented a fire from spreading through that node or any of the edges connecting the node to adjacent nodes. By comparison, the removal of individual edges in problems 2,4,6 and 8 allowed for more selective placement of firebreaks, aiming only to remove edges along the most important fire spread paths. Because the area around an edge that is considered suitable for firebreak placement is three times smaller than the area of a node that is eligible for removal in problems 1,3,5 and 7 – and also may be oriented in different directions (Fig. 7) – the distribution of firebreaks in problems 2,4,6 and 8 more closely followed the geographical pattern of suitable candidate sites, with their slopes facing the prevailing winds, than the equivalent distribution of nodes (Figs. 7,8).

4. Discussion

Firebreak planning is commonly undertaken at the scale of individual sites, but the consequences of these local decisions translate to impacts at a whole-landscape scale. Our representation of fire spread via FPTs provides a synoptic view of the combined potential impacts of local fire mitigation actions across a target landscape. Because each simulated FPT depicts a discrete realization of the uncertain fire regime within a burn year, tracing the impacts of firebreak allocation on a large set of FPTs enables estimation of the reduction in the expected burn area across the landscape. Furthermore, our fireplain graph concept captures a wide range of fire spread outcomes at landscape scale and is well-suited for supporting the practical planning of fuel treatments and firebreaks in landscapes with complex fire behavior, such as Potential Wildfire Operational Delineations (PODs) in the USA (Buettner et al., 2023). The benefits of the firebreaks that we modelled in our study (i.e., firebreaks created through large-scale tree removal) are well-supported by implementation projects in montane and boreal forests of North America (Plana et al., 2005; Butler et al., 2013; Beverly et al., 2020; Macauley et al., 2022; Low et al., 2023; Liu et al., 2024b).

The approach that combines critical node detection and expected burn area reduction advanced our previous work for optimizing firebreak allocation in the following ways. The FPT formulation addresses one of the key limitations of the CND problem: its inability to track the degree to which the allocated firebreaks reduce the potential burn area and the corresponding damage. The CND only tracks whether two nodes are in the same connected portion of the network (Veremyev et al., 2014a) and does not enumerate the actual connecting paths between the nodes. By comparison, the FPT problem tracks the fire propagation through each FPT, which allows estimation of the potential spread area after firebreak allocation. In this context, applications of the FPT-based fire spread measures may be more useful for managing the landscape to

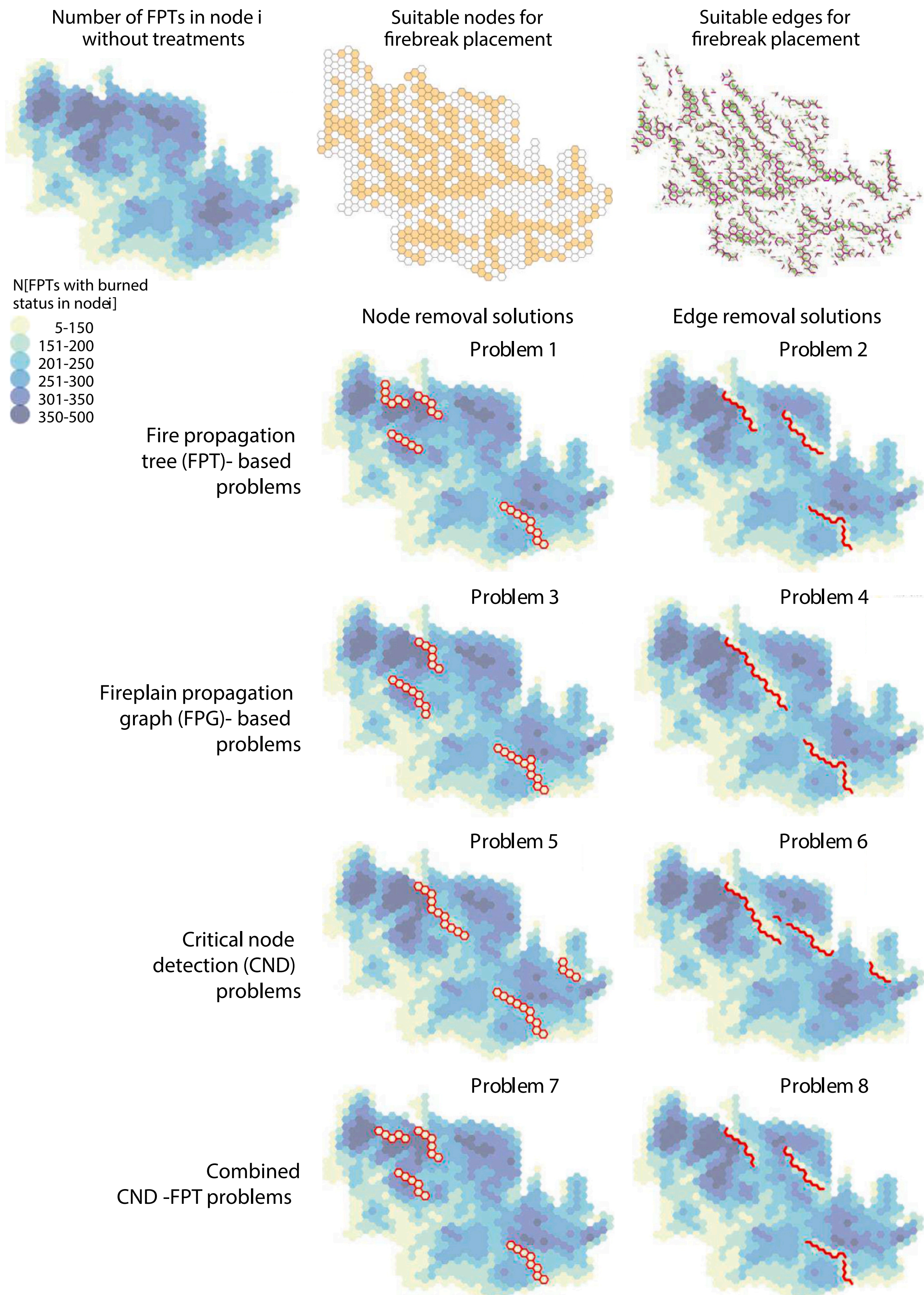

Fig. 8. Optimal node and edge removal solutions with treatment budgets of 25 removed nodes (or equivalently, 50 removed edges). The maps in the top row show the eligible locations for firebreak placement in the node removal and edge removal problems. The background maps in the firebreak allocation solutions show the expected number of nodes with burned status after the treatments.

Table 3
Firebreak allocation summaries in problems 1–8 optimal solutions.

Treatment budget, *B*, equivalent edges/nodes	Node or edge removal solution	Problem type (see Table 2)	Number of node pairs with fire the spread paths (CND problem objective) vs. the no-treatment scenario[a]	Expected burn area (FPT problem objective) vs. the no-treatment scenario[a]	Mean fireplain size, nodes	Budget portion spent on firebreak segments			
						≤3 nodes/ ≤ 6 edges	4–6 nodes/ 7–12 edges	6–12 nodes/ 12–24 edges	>12 nodes > 24 arcs
25 nodes /		FPT problem 1	93.2 %	88.4 %	31	–	64 %	36 %	–
	Node	FPG problem 3	93.5 %	86.6 %	31	–	24 %	76 %	–
	removal	CND problem 5	93.7 %	85.6 %	30	–	16 %	84 %	–
		CND-FPT problem 7	93.3 %	87.7 %	31	–	32 %	68 %	–
50 edges		FPT problem 2	93.9 %	93.7 %	32	–	36 %	64 %	–
	Edge	FPG problem 4	94.0 %	92.9 %	32	–	8 %	30 %	62 %
	removal	CND problem 6	94.9 %	90.6 %	31	4 %	16 %	80 %	–
		CND-FPT problem 8	94.1 %	93.3 %	32	–	38 %	62 %	–
50 nodes /		FPT problem 1	88.0 %	76.8 %	28			100 %	
	Node	FPG problem 3	89.0 %	77.9 %	29	4 %	20 %	76 %	
	removal	CND problem 5	89.0 %	73.4 %	27		20 %	80 %	
		CND-FPT problem 7	88.2 %	77.4 %	28	4 %	42 %	54 %	
100 nodes		FPT problem 2	89.3 %	90.8 %	30	12 %	15 %	48 %	25 %
	Edge	FPG problem 4	89.4 %	91.2 %	31	13 %	15 %	47 %	25 %
	removal	CND problem 6	90.9 %	84.3 %	29	8 %	26 %	17 %	49 %
		CND-FPT problem 8	89.7 %	88.7 %	30	22 %	20 %		58 %
75 nodes /		FPT problem 1	83.3 %	68.5 %	26	4 %	12 %	61.3 %	22.7 %
	Node	FPG problem 3	84.4 %	67.4 %	26	2.7 %	14.7 %	57.3 %	25.3 %
	removal	CND problem 5	84.8 %	63.6 %	24		12 %	38.7 %	49.3 %
		CND-FPT problem 7	84.3 %	68.5 %	26	16 %	28 %	56 %	
150 nodes		FPT problem 2	85.0 %	86.9 %	30	12.7 %	28.0 %	33.3 %	26 %
	Edge	FPG problem 4	85.2 %	86.9 %	30	7.3 %	29.3 %	38 %	25.3 %
	removal	CND problem 6	87.2 %	79.2 %	27	1.3 %	18 %	45.3 %	35.3 %
		CND-FPT problem 8	85.4 %	83.4 %	29	8 %	27.3 %	30 %	34.7 %

[a] Lower values indicate better efficacy of treatments.

minimize potential burn damage than to minimize fuel connectivity.

The FPT problem requires generating a large set of fire propagation trees, which may render the problem intractable for large landscapes if large fires are common in the area. The aggregation of individual FPTs into FPG graphs helps reduce the problem size, making it more tractable, but may create more conservative firebreak solutions because it may increase the number of connections between adjacent upstream-downstream nodes. Since we used maximally connected FPTs with fire propagation to a node from all adjacent upstream nodes, the impact of this aggregation was relatively minor. Alternatively, the use of tree-like FPTs, such as presented in Vilches et al. (2024), would produce different solutions in fireplain-based and FPT-based problems. This is because FPT aggregation would create an FPG with a higher degree of connectedness than the connectedness of individual FPTs and so would create a more conservative firebreak allocation to protect against fires spreading to a location from multiple directions. Potentially, the over-conservatism from fireplain aggregation could be partially mitigated by introducing a parameter that tracks, for each edge in an FPG, the number of times the edge was included in the fire footprints comprising that FPG. This parameter could be used analogously to the node-based parameter w_{ki} in the objective function (11).

The FPT and CND models have different data requirements to characterize the fire behavior in the landscape. The FPT-based model requires a set of fire propagation trees, whereas the CND problem only needs the estimates of fire spread indicators between all pairs of locations. These indicators can be calculated from the ignition points and the fire footprints (Yemshanov et al., 2021), which are standard outputs of many fire growth models (Finney et al., 2011; Pais et al., 2021a; Parisien et al., 2005). Therefore, the CND may be easier to apply in knowledge-poor situations where the data for specifying the possible fire propagation paths within the fires' footprints are limited.

Our models explored the capacity of the FPT-based approach to

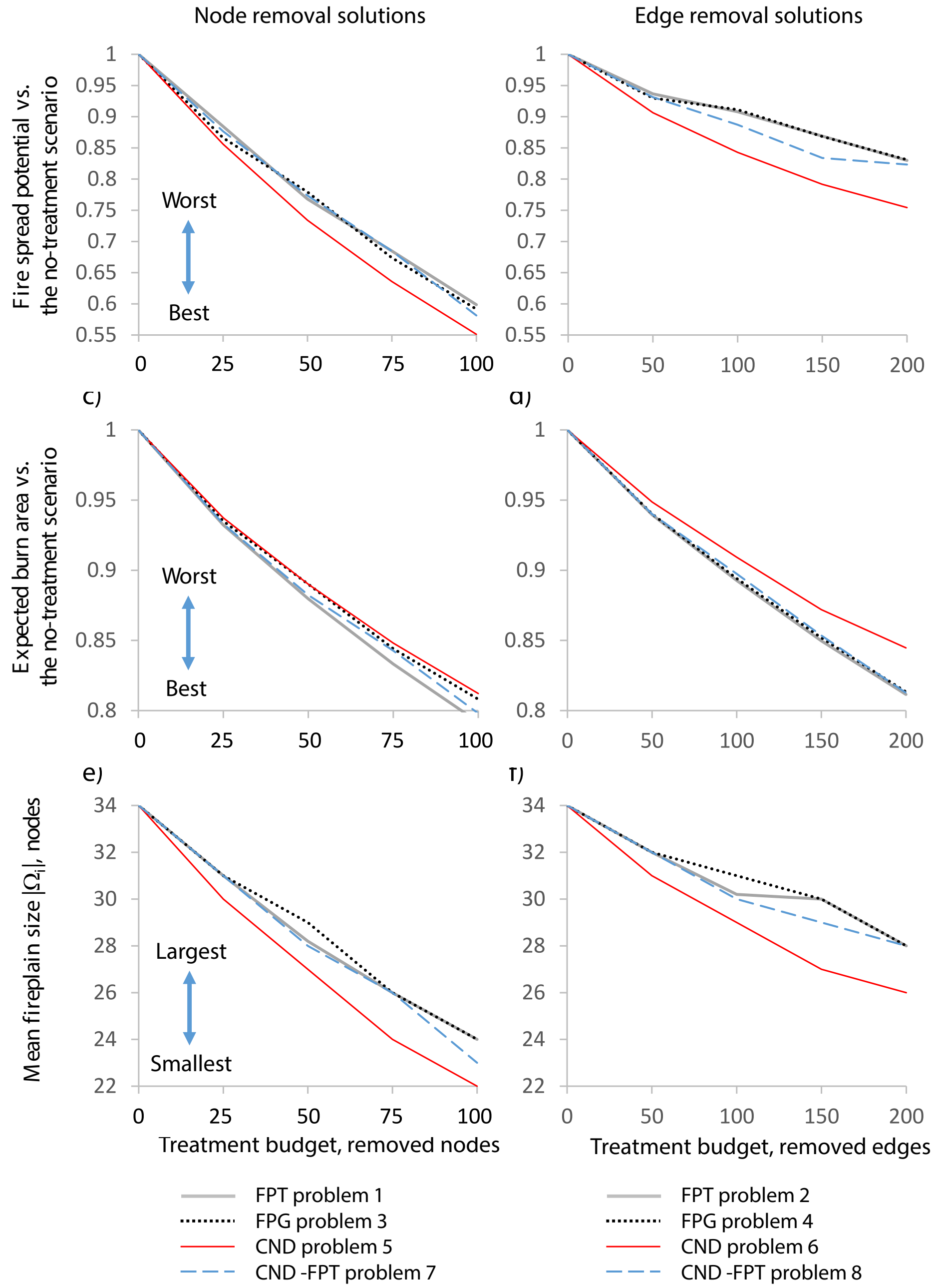

Fig. 9. Objective value vs. the treatment budget. X-axis denotes the treatment budget level in the equivalent number of treated nodes or edges: a, b) fire spread potential (the CND problem objective) vs. the no-treatment scenario; c, d) expected burn area (the FPT-based problem objective) vs. the no-treatment scenario. Lower values on the Y-axis indicate better outcomes; e, f) mean fireplain size (characterizes the potential worst-case fire spread area around each node).

create linear firebreaks on its own and did not enforce the creation of fixed-length contiguous segments. Among the formulations tested, the CND problem produced the most contiguous firebreaks. The FPT-based solutions tended to allocate shorter firebreaks which could get sporadically clustered. This behavior is expected because the FPT-based problem does not track fuel connectivity by checking the presence of fire spread paths between pairs of locations.

The FPT-based approach required the construction of fire propagation trees for all fire footprints simulated with the fire growth model. While some models, such as the Cell2Fire model (Pais et al., 2021a, 2021b), can generate the FPTs on their own, other fire growth models do not offer this capability. In this situation, the FPTs can be reconstructed from other available outputs, such as the location-specific wind directions and speeds and rates of fire spread (if such data can be generated by the fire growth model).

In our case, we assumed a worst-case local fire propagation scenario where a fire could spread to a forest patch from all adjacent upstream locations via spotting, which is common in Canadian forests (Page et al., 2019; Thompson et al., 2022). Firebrand spotting is a highly uncertain process driven by rapidly changing weather and fuel conditions. We assumed that firebreaks must be effective under these circumstances (i.e., when fires may arrive to a node from multiple sources) and used fully interconnected FPTs to create more contiguous firebreak sequences. However, this may create overly conservative firebreak configurations in regions with low wind volatility and no spotting. In these situations, more sophisticated algorithms may be necessary to build FPTs that track individual edges indicating potential fire spread paths. This may require generating a larger set of FPTs because, for each simulated fire footprint, one would need to depict the possible multitude of local fire spread paths.

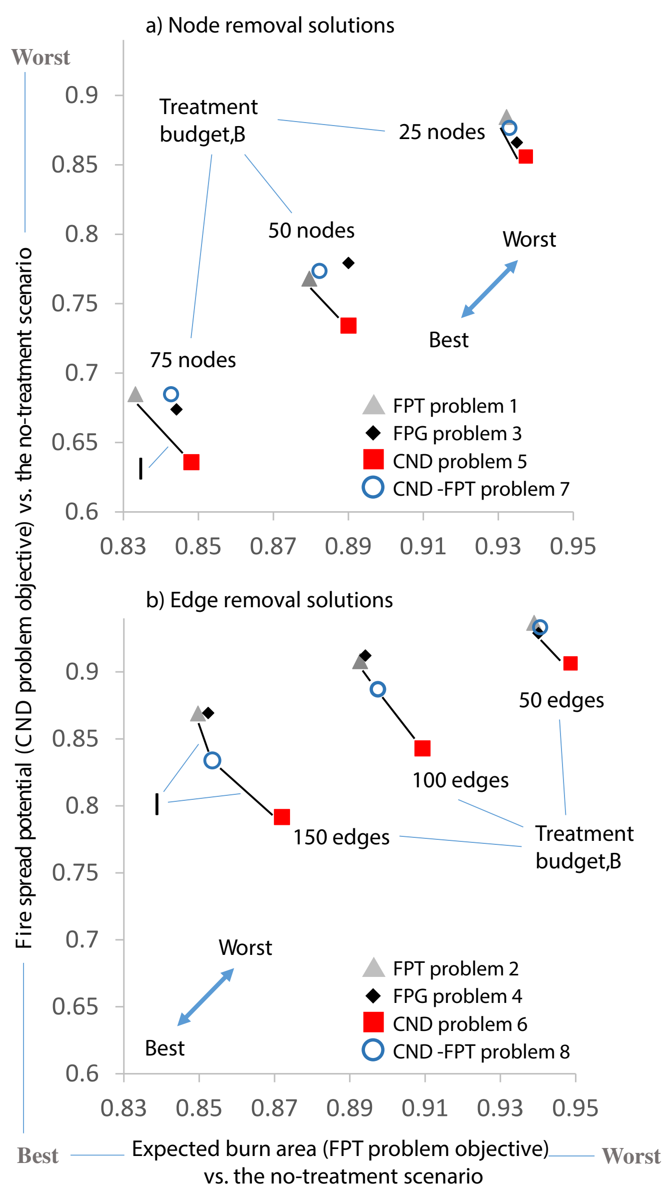

Fig. 10. The trade-off between reduction of the fire spread potential (the CND problem objective) and decrease of the expected burn area (the FTP problem objective). The point clusters show the solutions at three treatment budget levels of 25 nodes (50 edges), 50 nodes (100 edges) and 75 nodes (150 edges). Callout I shows the trade-off frontiers between the CND problem solutions that minimize the fire spread potential and the FPT-based solutions that minimize the expected burn area. Lower values on the X- and Y-axes indicate better performance.

The prevailing wind pattern in our study area allowed us to use directed FPTs, which helped reduce the problem size. In regions with omnidirectional wind patterns, the FPTs could include the backward fire spread vectors (i.e., in all directions) within a fire footprint. This is likely to change the optimal solutions by concentrating the firebreaks around hotspots with high burn risk, aiming to protect these hotspots from fires that may arrive from multiple directions.

4.1. Methodological aspects

Our models applied binary firebreak allocation decisions without specifying the treatment level and delineated the treatment feasibility for the candidate sites as a binary mask. Potentially, the FPT-based formulation could be modified to enable the selection of the treatment type or the firebreak width. For each location in the FPT, one would need to provide the relevant estimates of the fire intensity level and the corresponding assumptions about the level of treatments (or firebreak width) that would be required to contain the fire at that intensity level, a relationship that has been documented in the literature (Wilson, 1988) but contains factors not captured here given its partial dependence on suppression resources, such as aircraft and ground crews (Gannon et al., 2023). Given a standardized relationship between firebreak width and success rate, the FPT problem then could be modified to choose from the alternative treatment types and track the proportion of fire propagation trees with the contained fires. This formulation could also help track possible firebreak failures and undertake robustness tests for the spatial resolution of fuel network *G*. For each FPT, the firebreak failure conditions could be estimated via comparison of the distribution of burn intensities in the chosen firebreak location against the intensity limit that a given firebreak type is able to contain.

In our model, the firebreak width was defined by the selected node size, which we determined to be sufficiently large to stop most predicted fires in the area under spotting conditions. A coarse node size could be a limiting factor in landscapes dominated by small fires and with an absence of firebrand spotting. Furthermore, there is always a chance that even wide firebreaks may fail to stop the spread of high-intensity fires under extreme weather conditions. The issue of firebreak failure can be addressed by reformulating the firebreak problem as the selection of a portfolio of risky investment assets, where each asset depicts a candidate firebreak location and is characterized with a likelihood of failure. Notably, our scenario-based formulation of the FPT problem enables us to explore the risky portfolio concept through minimization of the expected worst fire outcomes that arise from the largest, most intense fires (e.g., depicted by the largest fire propagation graphs).

The approach can also assist with integrating wildfire risk mitigation strategies into regional land management (cf. Hirsch et al., 2001; Wollstein et al., 2022), particularly in regions with developed human infrastructure. However, the full implementation of integrated management in large regions or units, such as Potential Operational Delineations in the USA (Buettner et al., 2023; Thompson et al., 2002), may require extending the model to account for multiple land uses and economic actors. Nevertheless, our approach, thanks to its capacity to track both fuel connectivity and the potential burn area, can support the development of such integrated fire mitigation projects if required.

The approach can be further adapted to plan fire mitigation activities in areas of wildland-urban interface (WUI). One critical aspect in WUI applications is the selection of an appropriate node resolution to depict high heterogeneity of fuel in urban and suburban areas. In fact, WUI applications may also require adjusting the fire growth model to capture the spread of fires in locations with large amounts of non-forest fuels and human structures. The model can also integrate other landscape management constraints, such as the need to protect wildlife corridors or stratify fire mitigation activities by land tenure. Maintaining wildlife corridors would require the addition of habitat connectivity constraints, such as the network flow constraints used in St John et al. (2018) and Yemshanov et al. (2023). This would enable estimation of the trade-off between the reduction of wildfire risk and the loss of critical habitat.

4.2. Concluding comments

To our knowledge, this is the first study to consider the application of both the fire propagation graph and critical node detection concepts in a firebreak optimization context, a feature that has allowed us to tease apart the goals of reducing the fuel connectivity and the landscape-level burn risk. The proposed formulations provide the implementation ideas for developing firebreak solutions in forest landscapes under an uncertain fire regime. Our case study revealed a minor trade-off between the solutions minimizing fuel connectivity versus those minimizing the burn area, which is a result of the homogeneous distribution of forest fuel across the study area. We also explored an option to reduce the problem complexity by aggregating multiple fire propagation trees into fireplain graphs. However, we caution that this reduction may produce overly conservative firebreak solutions in regions dominated by low-severity fires with no volatile wind patterns or spotting. Our approach provides a workable solution for the landscape-level planning of mitigation measures in response to increasing wildfire frequencies, especially given the perspectives of future climate change and human developments in North American forests.

CRediT authorship contribution statement

Denys Yemshanov: Conceptualization, Formal analysis, Funding acquisition, Methodology, Writing – original draft, Writing – review & editing. **Ning Liu:** Data curation, Formal analysis, Methodology, Visualization. **Eric W. Neilson:** Conceptualization, Data curation, Formal analysis, Funding acquisition, Methodology, Writing – review & editing. **Daniel Thompson:** Formal analysis, Methodology, Writing – review & editing. **Frank H. Koch:** Conceptualization, Formal analysis, Methodology, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

Funding for this work was provided by Natural Resource Canada's Wildfire Risk Management Program (DY, NL, DT, EN). Additional support was provided by the US Department of Agriculture Forest Service (FHK). The authors express gratitude and special thanks to Wendy Crosina and Lyle Dechief from Weyerhaeuser Inc. for their assistance with the geospatial data characterizing the study area.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.ecoinf.2025.103339.

Data availability

The code to generate the fire propagation trees and other relevant model data is available on GitHub from https://github.com/nliu-cfs/FPT_toolkit.

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