

FOREST GROWTH AND YIELD MODELS VIEWED FROM A DIFFERENT PERSPECTIVE

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ABSTRACT. Typically, when different forms of growth and yield models are considered, they are grouped into convenient discrete classes. As a heuristic device, I chose to use a contrasting perspective, that all growth and yield models are diameter distribution models that merely differ in regard to which diameter distribution is employed and how the distribution is projected to future conditions. I describe different diameter distributions, whether they are the classical **continuous** diameter distributions, the implied distributions of whole-stand models, or the discrete diameter distributions of size-class or individual tree models. There are also intermediates between these types of diameter distributions. Aggregation vs. disaggregation describes the alternate poles for how diameter distributions can be projected to future conditions. There are intermediates between these extremes, as well. There are several alternatives that vary from the classical paradigms. One alternative is a continuous analog to stand table projection that employs a “distribution modifying function” to project diameter distributions in time. One discrete variant of stand table projection is termed “**non-naïve**” size class models, as they relax one or more of the assumptions of size class models. An “individual tree kernel” model is one that uses a simple kernel function to distribute probability **around** an individual tree, rather than concentrate the probability at the measured diameter, as in typical individual tree models. Other variants are mentioned. These variants suggest that there is an infinite universe of alternative growth and yield models, and the classic labels for types of growth and yield models do not embrace these alternatives.

1 INTRODUCTION

In textbooks and proceedings, several authors have sought to distinguish different classes of growth and yield models (Avery and Burkhart 1994, Munro 1974, **Vancley** 1994). In such classifications, existing growth and yield models are assigned to convenient, discrete classes. I purposely chose an alternative paradigm—that existing growth and yield models represent individual realizations from an infinite universe of possible growth and yield models. As such, there are infinitely many intermediate models between any “classes” of models.

1.1 Viewing All Forest Growth and Yield Models as Diameter Distribution Models. As a heuristic device, I chose the perspective that all growth and yield models may be viewed as diameter distribution models. This perspective provides a common thread that can be used to link existing growth and yield models. Furthermore, the perspective suggests alternatives that arise logically as intermediates between existing model forms. Although all models may be perceived as diameter distribution models, they may differ in two ways—the diameter distribution that is chosen and the way the diameter distribution is projected. Some distributions are continuous, some are discrete. Some are explicit, some are only implied by the stand-level variables that are predicted. In addition to the choice of a diameter distribution, a modeler must choose how to project the diameter distribution to **future** conditions. Generally, stand-level variables may be projected, and the diameter distribution is disaggregated from the stand, or parts of the stand (individual trees or size classes) can be projected, and the stand level variables obtained by aggregation. An equivalent viewpoint would be that all growth and yield models are population models, and the population is generally described by its size-class distribution. Again, the models would differ according to the specifics of the distribution, and the method to predict the future distribution.

There are also examples of systems that integrate models aggregated at different levels (for example, Daniels and Burkhart 1988, Somers and Nepal 1994, U.S.D.A., Forest Service 1979),

allowing predictions to be made at any of those levels. There are other examples where outputs at a more aggregated level (whole-stand) are used to constrain predictions at a more disaggregated level (Zhang et al, 1997, Matney and Belli 1995).

2 ALTERNATIVES FOR DIAMETER DISTRIBUTIONS

Examples for all of the alternative diameter distributions given below are determined for the same plot. The plot is 0.25 acre, with 54 trees. It is in a naturally-regenerated even-aged loblolly pine (*Pinus taeda* L.) stand that was 74 years old at the time of measurement.

2.1 Whole-stand Models. Although whole-stand models do not have an explicit diameter distribution, they may possess an implied diameter distribution that is identified by the variables that are projected. For example, if a whole stand model consists of a system of equations to predict trees per acre and volume, the implied diameter distribution is a spike of probability at the tree of average volume (figure 1). If the tree of average volume represents the tree of average value very well, then this should be sufficient. This would be most likely when dealing with single-species stands with a simple stand structure (urn-modal, low-variance distribution) where all of the trees produce the same product, without premiums for larger trees.

If “yield” is in terms of basal area rather than volume, the diameter distribution represents a spike of probability at quadratic mean diameter. Given trees per acre, basal area per acre, volume per acre, a local volume equation, and a convenient two-parameter diameter distribution, a diameter distribution can be generated by recovering the parameters using the second moment, and the volume moment (figure 2).

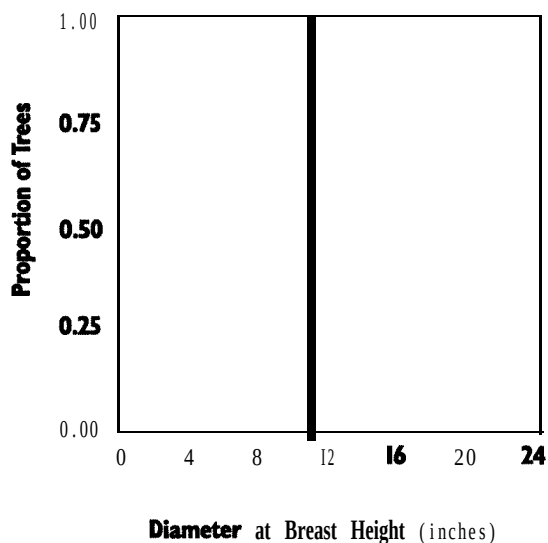


Figure 1. A spike of probability at the diameter of the tree of average volume, the implied diameter distribution for whole-stand models

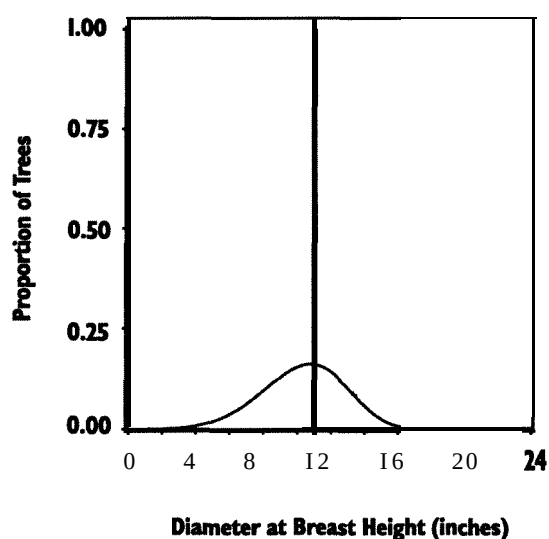


Figure 2. A spike of probability at the diameter of the tree of average volume, and at quadratic mean diameter, and a two-parameter Weibull distribution.

2.2 Classical Continuous Diameter Distribution Models. Classical diameter distribution models estimate parameters of a chosen diameter distribution either directly from a regression function using stand-level variables as predictors, or recover the parameters from the sample moments, order statistics, or percentiles (Bailey and Dell 1973; Hyink and Moser 1983; Burk and Newberry 1984; Zamoch and Dell 1985; Knoebel and Burkhart 1991)(figure 3). The moments, order statistics and percentiles are all merely stand-level variables, but they allow estimation of the parameters without developing regression

equations. Although intricacies of these methods vary, I lump together all classical diameter distribution models for this discussion.

2.3 Discrete Diameter Distributions. Rather than estimating a parametric distribution, the sample diameter distribution can be described by predicting the percentiles (Cao and Burkhart 1984; Borders et al 1987; Borders and Patterson 1990; Droessler and Burk 1994). This can be visualized as a spike of probability at each percentile (figure 4), or alternatively, it can be viewed as a histogram where the area of the different bars is equal, but the height and width of the bars differ.

The classic size-class histogram, or stand table projection paradigm, is depicted in figure 5 (Ek, 1974; Buongiorno and Michie 1980). In this case, two-inch diameter classes are used. Diameter classes could be any width, and need not be of uniform width.

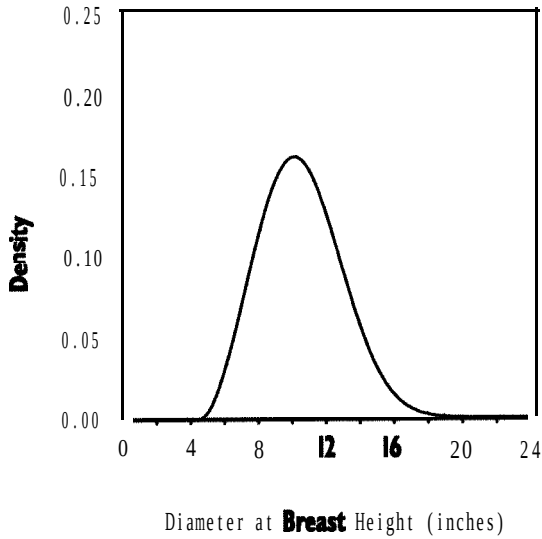


Figure 3. A three-parameter Weibull distribution.

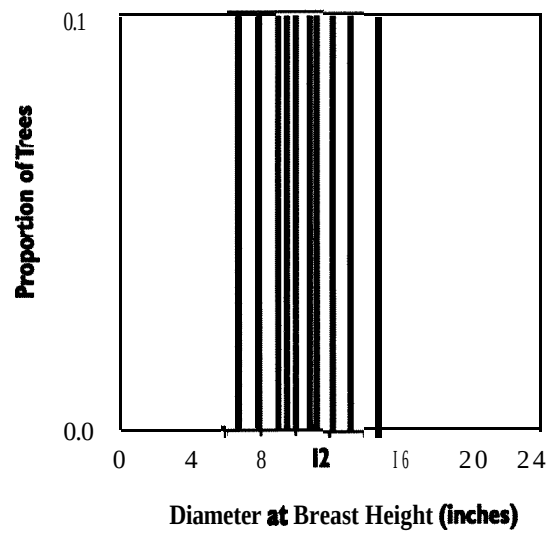


Figure 4. A spike of probability at ten evenly-spaced percentiles.

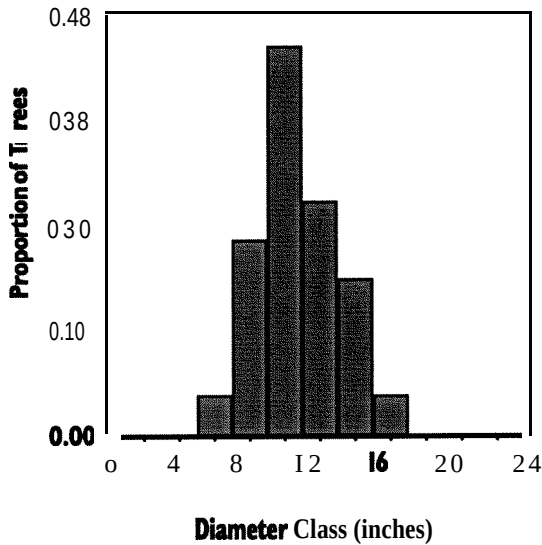


Figure 5. A histogram representing two-inch-wide diameter classes.

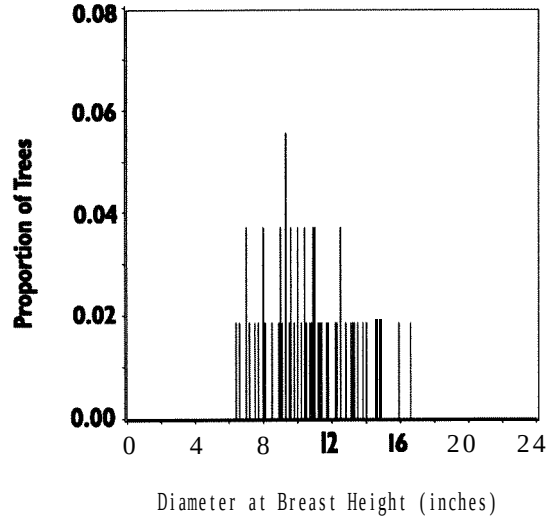


Figure 6. A spike of probability for each individual tree, this is equivalent to a histogram with 0.1 -inch-wide diameter classes.

Individual tree models (Stage 1973; U.S.D.A. 1979; Belcher et al 1983) are also equivalent to histograms (figure 6). In this case, the diameter classes are only 0.1 inches wide, the precision to which trees were measured. Generally, individual tree models include several variables measured on each tree, not merely diameter. This simply implies a multivariate distribution rather than the univariate distribution that is described in figure 6.

2.4 An Individual Tree Model with a Continuous Distribution. Rather than placing all the probability for an individual tree at the measured diameter, the probability could be spread out over a range of diameter using a convenient kernel distribution (figure 7). In figure 7 we show Epanechnikov kernels (Silverman 1986) for 1/6 of the trees for our plot; plotting kernels for all of the trees produces a confusing graph (the Epanechnikov kernel is simply a parabola, zero outside of $\pm 1/2$ the bandwidth, and scaled to integrate to 1). We may desire to spread out the probability so that the resulting stand-level diameter distribution would be smooth. This may be sensible as the inherent population diameter distribution is expected to be smooth, and thus our projected diameter distribution might better reflect the population. This model structure is termed an “individual tree kernel” model.

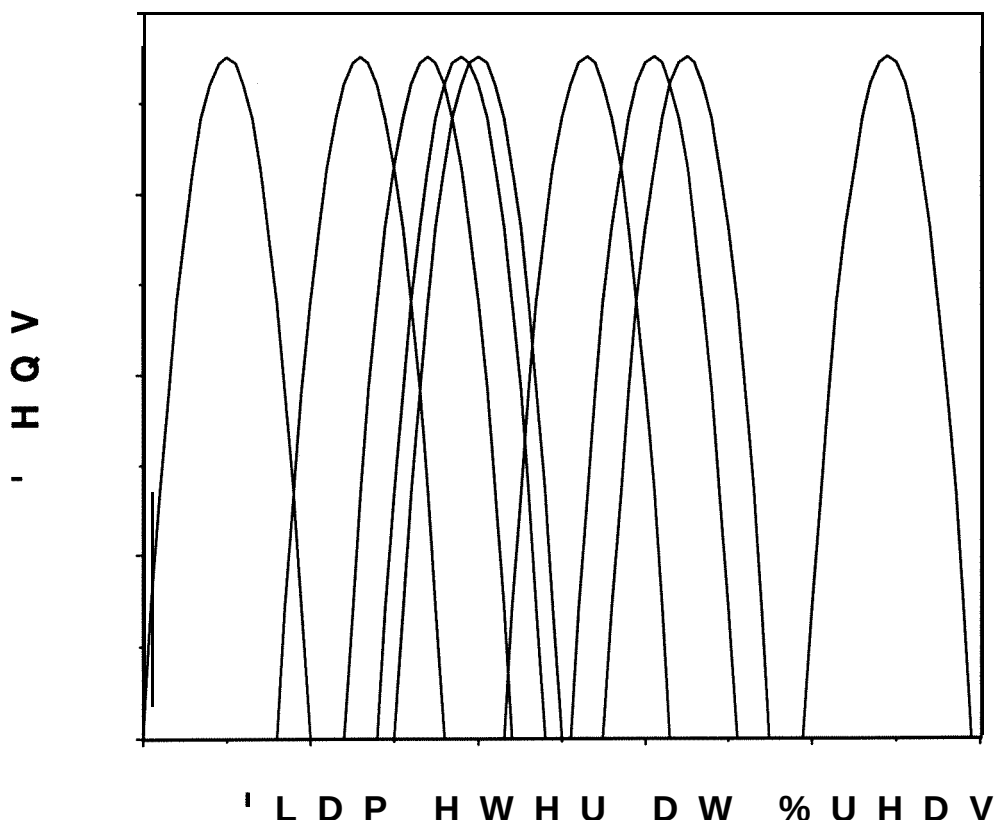


Figure 7. An Epanechnikov kernel for every sixth tree, with a bandwidth of 2 inches.

3 ALTERNATIVES FOR PROJECTING DIAMETER DISTRIBUTIONS

The two main alternatives are: (1) predicting or recovering the diameter distribution given predictions of stand-level variables (which might include percentiles or order statistics), and (2) projecting “part” of the diameter distribution, then aggregating those parts to get the complete diameter distribution. “Part” will typically be individual trees or size classes. Intermediate procedures exist where predictions for the future stand-level variables constrain predictions for the future trees or size classes. These two alternatives, and possibly intermediate procedures, may be employed for any

choice for the diameter distribution. Leary, Holdaway and Hahn (1979) discussed these alternatives for a model that was typically implemented as an individual tree model.

3.1 Predicting or recovering the distribution from whole-stand variables. All of the classical whole-stand and diameter distribution models follow this alternative, regardless of the choice of diameter distribution and method for estimating the parameters. Details of these standard models will not be explored here. The critical issue is that the entire distribution is estimated from whole-stand variables (including moments, percentiles, and order statistics). There are options other than the classical diameter distribution models and two will be mentioned below, a method analogous to a continuous variant of stand table projection, and using stand-level variables to estimate a discrete distribution.

3.2 A Continuous Analog to Stand Table Projection. Bailey (1980) showed there are implied diameter growth functions for some common diameter distributions that will preserve the functional form of the distribution. This is equivalent to transforming the random variable. These functions only hold true if mortality was nonexistent or evenly distributed across diameter, two rare occurrences. Martin et al (1999) explicitly linked a diameter distribution model with the individual tree basal area growth function of Clutter and Allison (1973), allowing projection of a stand table or an initial Weibull distribution. Martin et al (1999) used the mortality relationship of Pienaar and Harrison (1988) to assign mortality to diameter classes. Cao (1997) addressed the survival issue by recovering new Weibull parameters from moments calculated by numerically-integrating over a non-Weibull p.d.f. that resulted from incorporating mortality with the initial Weibull p.d.f. Zhang et al (1993) invoked the terminology of Westoby (1982), by defining a “distribution modifying function” as a function that describes the change of the distribution. This can be mathematically represented as:

$$I [\quad I [P [$$

where $f(x_i)$ represents the p.d.f. of diameter at time i , and $m(x_i)$ represents the modifying function that maps the distribution from time 1 to time 2. The modifying function may include stand-level predictor variables as well as diameter. The modifier function potentially incorporates mortality, and birth (either vegetative or sexual reproduction) (Cochran and Ellner 1992) as well as growth. If the equation is conveniently specified, it represents a continuous analog to the classical stand table projection.

3.3 A Simple Example. A simple graphical example follows, where the distribution modifying function is decomposed into a survival and a growth modifier function. The initial distribution is described with a simple Epanechnikov kernel (Silverman 1986) (figure 8). The survival relationship is described in figure 9. Multiplying the initial distribution by the survival function, then adjusting so that the resulting function is a p.d.f. (i.e. integrates to 1) results in the middle line in figure 10. Note that the smallest diameter has not changed following consideration of mortality. This is due to the survival function being greater than zero throughout the range of diameter, as will be true for typical survival functions. This seems unfortunate, as we would expect minimum diameter to increase as an even-aged stand ages due to growth as well as mortality of the smallest trees. The diameter growth function is represented in figure 11. We show the final diameter distribution as well as the initial and intermediate diameter distribution in figure 10. Probably the easiest way to mentally-picture the effect of the growth function is as a transformation of the x-axis of the c.d.f. (the curve stays the same, the x-axis is merely altered). Note that the growth function did increase the minimum diameter, although the survival function did not.

3.4 The Distribution Modifying Function. In our example, the distribution modifying function is decomposed into separate survival and growth components (birth is ignored), leading to:

$$I [\quad I [V[J [$$

where s and g represent survival and growth modifier functions. There are three options for implementation. It is likely that $f(x_2)$ will be a very complicated function that cannot be analytically integrated. The first option is to endure the potentially awkward function and numerically integrate to obtain yield. This is not such a burden as it might seem; volume is not available as an analytically-

calculated definite integral in the implementation of standard diameter distribution models, although the methodology is known (Strub and Burkhardt 1975). The form of the function would get more awkward **after** each successive growth interval that is projected. The second option would be to devise the functions such that the form of $f(x_2)$ mathematically simplifies to the form of the **original** $f(x_1)$. This seems unlikely except for overly simplistic distributions and growth and survival functions. The **final** option would be to follow Cao's (1997) procedure to **use** $f(x_2)$ to generate moments which would in turn be used to recover parameters in the form of the original $f(x_1)$. Cao (1997) only did this for consideration of survival. It could be done **after** including the effects of both survival and growth. The additional benefit over Cao's (1997) procedure that considers only survival, is that there would be much greater flexibility for the form of the individual tree growth function.

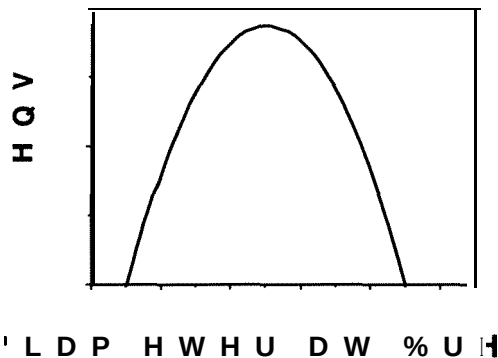


Figure 8. A hypothetical stand diameter distribution using an Epanechnikov kernel for the distribution.

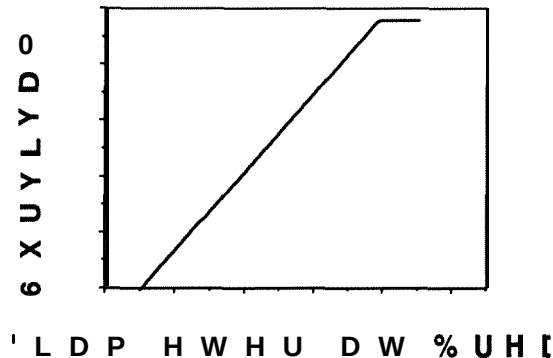


Figure 9. A simple function for individual tree survival, plotted within the range of initial diameters represented in figure 8.

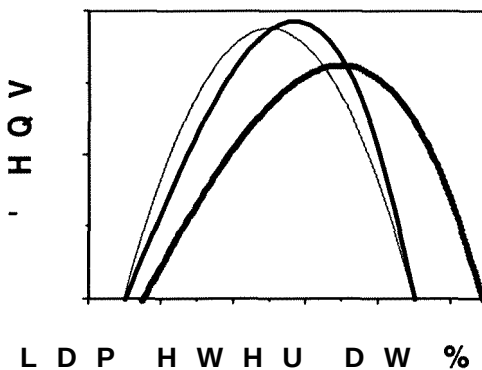


Figure 10. The initial diameter distribution (leftmost, thin line), after modifying for mortality (middle, dark line), and after modifying for mortality and growth (thick grey line).

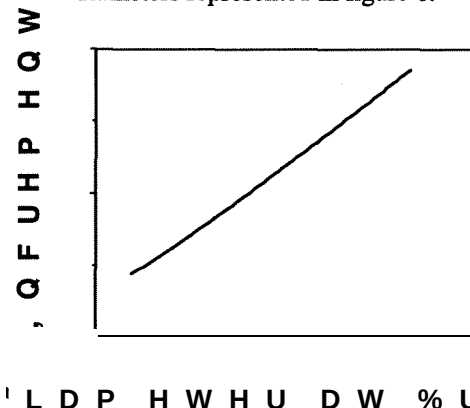


Figure 11. A simple diameter growth function, plotted within the range of the initial diameters represented in figure 8.

3.5 Estimating Discrete Diameter Distributions from Stand-level Variables. Generally, continuous diameter distributions are used when diameter distributions are estimated **from** stand-level variables. However, discrete diameter distributions could also be used. Zhou (1997) [cited in Martin et al (1999)] generated tree lists (i.e. the diameter distribution for individual tree models) from stand-level data. Matney and Belli (1995) and Farrar and Matney (1994) used a Weibull distribution to generate a tree list (i.e. individual tree model, equivalent to each 0.001 quantile), conditioning the tree list to possess the trees per acre, average diameter, and quadratic mean diameter of the stand-level predictions. Stand level data could also be used to determine the proportion of trees in each diameter class. Leduc et al [in press] used an artificial neural network to predict the proportion of trees in each 1-inch-wide diameter

class (figure 12). Percentiles also represent a discrete empirical distribution, and Borders et al (1987) and Borders and Patterson (1990) predicted **them** from quadratic mean diameter and age, two **stand-** level variables. Droessler and Burk (1994) report unsatisfactory results when they attempted to predict change in percentiles over time from stands remeasured with temporary plots. Clutter and Jones (1980) implemented a novel diameter class model where the initial limits (maximum and minimum dbh) of each diameter class could be specified arbitrarily, and the trees/acre, basal area, average dbh, and average height of these diameter classes were projected into the future, and thus the **future** limits of the diameter classes were irrelevant and undefined. Clutter and Jones (1980) method insured that the aggregation of the diameter classes was compatible with whole-stand projections.

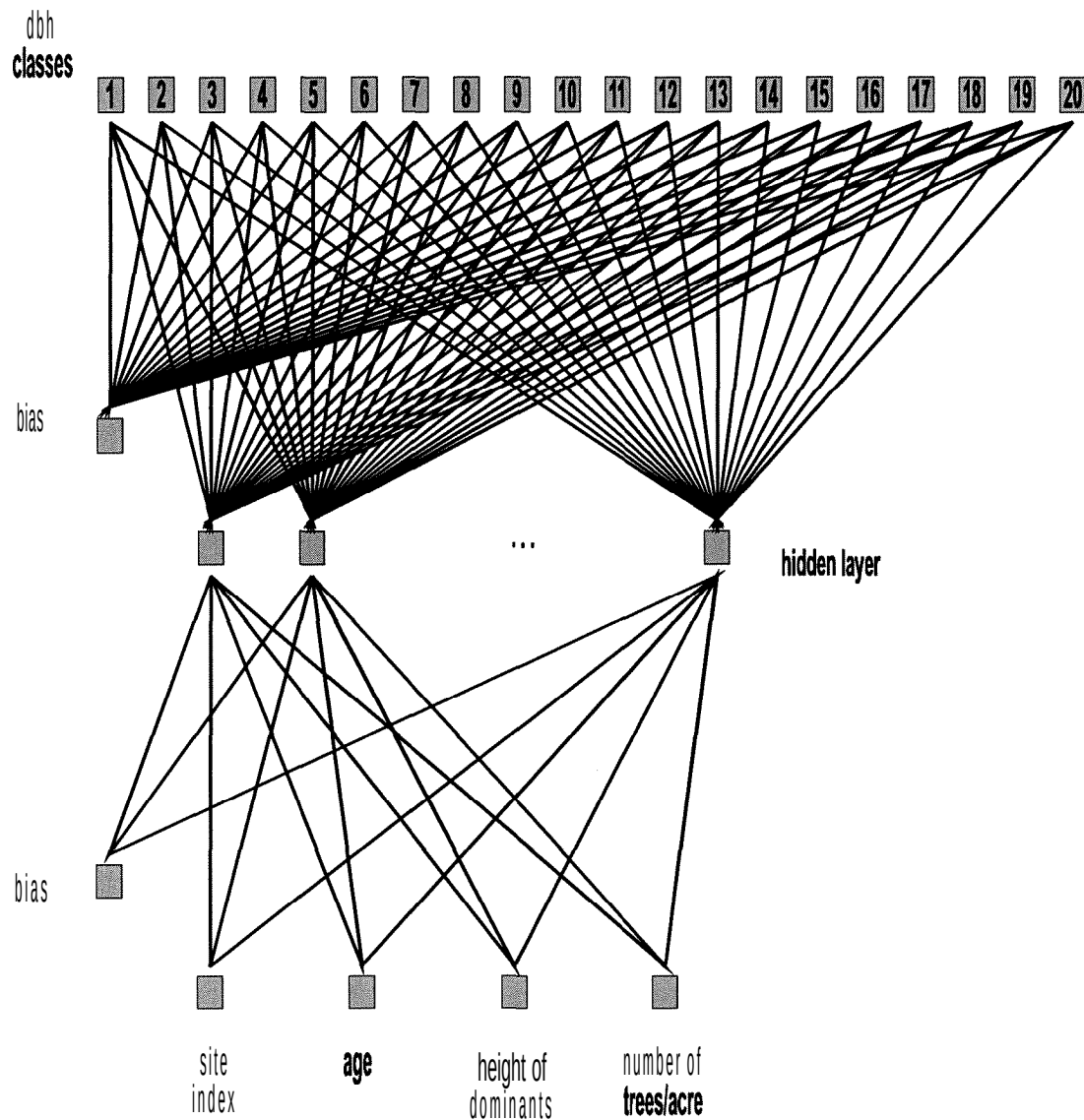


Figure 12. Diagram for the basic structure for a neural network used to predict proportions of trees in one-inch diameter classes. Modified from Leduc et al [in press]

3.6 Projecting “Parts” of the Distribution, then Aggregating . The future diameter distribution for the stand may be assembled following projection of individual trees or classes of trees, whether the classes are defined by size or some other criteria. All of **the** classical individual tree and stand table projection type models operate in this way. The classes may be uniform-width segments of the

diameter distribution, or an algorithm may be used to agglomerate similar trees (Stage et al 1993). Alternatives to the classical methods exist. The two alternatives are to either make the discrete distribution less discrete, or to break-up a continuous distribution.

3.7 Making the Distribution Less Discrete. Both stand table projection models and individual tree models can be altered so that the distributions are not completely discrete. The assumptions of a stand table projection model are that the distribution of trees within the diameter class is uniform, and diameter growth is constant, or related linearly to initial diameter within each diameter class, or alternatively, that the probability of a tree growing into the next diameter class is constant for all trees in the diameter class. These assumptions are not reasonable except where the width of the class approaches zero, at which point the stand table projection model is equivalent to the continuous analog that was mentioned earlier. The inadequacy of these assumptions can be seen in figure 13. A smooth line has been drawn, joining the midpoints of the diameter classes. On the left of the mode, a uniform distribution assumption will underestimate the number of trees that should grow into the next diameter class (although this will depend somewhat on the data used to estimate the equations). It is obvious that there are probably many more trees very close to the next diameter class than there are near the previous diameter class for a diameter class that is to the **left** of the mode. The reverse is true for diameter classes to the right of the mode; the standard assumptions will overestimate the number of trees moving into the next diameter class. This could be particularly troublesome with multiple projection cycles; predictions will contain too many large trees. The magnitude of this problem will depend upon the stand structure of the stand in question as well as the stand structures of the data used to estimate movement probabilities. Haight and Getz (1987) suggested that this outcome of classical stand table projection type models could easily be overcome, but did not attempt to accomplish that task, and did not recognize that it was an inevitable result of the classical methodology. To correct for these problems, either of these assumptions may be relaxed, making projection of trees based upon a more continuous distribution, although the “accounting” of the model may still utilize very discrete classes. Relaxing these assumptions involves estimating continuous diameter growth functions, rather than a constant probability of advancement for each diameter class, or allowing the distribution to be something other than uniform within a diameter class, or relaxing both the diameter growth and uniform probability assumptions. I term these alternatives “**non-naïve**” size class models, as they relax one or more of the **naïve** assumptions of classical stand table projection-type models (“naïve” only suggests simplicity of the assumptions).

Nepal and Somers (1992) relaxed the constant projection probability assumption for stand table projection and used a truncated Weibull distribution to model density within diameter classes. They used an individual-tree diameter growth equation to calculate a tentative **future** stand table, then used the stand-level predictions for basal area and trees per acre to determine survival and adjust growth, conditioning the future stand table to possess the trees per acre and basal area predicted by the stand-level equations. Nepal and Somers (1992) contrasted their method to a method of Pienaar and Harrison (1988). Pienaar and Harrison’s (1988) algorithm was based on projecting relative tree sizes (tree basal area divided by average tree basal area) and was invoked by separable functions to condition for stand-level mortality and growth. Borders and Patterson (1990) compared Pienaar and Harrison’s (1988) method to an alternative derived from Borders et al (1987) method to predict percentiles as functions of quadratic mean diameter and age, and an individual-tree model, and found the individual tree model performed the best for predicting **future** stand tables for their data.

Cao and Baldwin (1999) combined aspects of Nepal and Somers (1992) method with a **least-squares** adjustment to recover the stand-level predictions of trees per acre, average diameter, and basal area per acre. Cao and Baldwin’s (1999) method is further distinguished from Nepal and Somers (1992) method as the former method distributes mortality before the future stand attributes are projected. For their data, Cao and Baldwin’s (1999) fit better than Nepal and Somers (1992).

Tang, et al (1997) developed a method to condition the parameters of individual tree growth and survival functions to recover stand-level average diameter, quadratic mean diameter, and trees per acre. Their model could be implemented as a size-class or individual tree model.

The variants on classical stand table projection have conditioned the projected stand tables to possess the basal area and trees per acre of stand-level prediction equations. This implies that stand structure does not contribute to the prediction of growth and mortality, after correcting for stand-level variables. As this hypothesis cannot be correct, an alternative would be to have stand structure

somehow incorporated into the whole-stand projections. In this way, growth and mortality would be responsive to changes in stand structure.

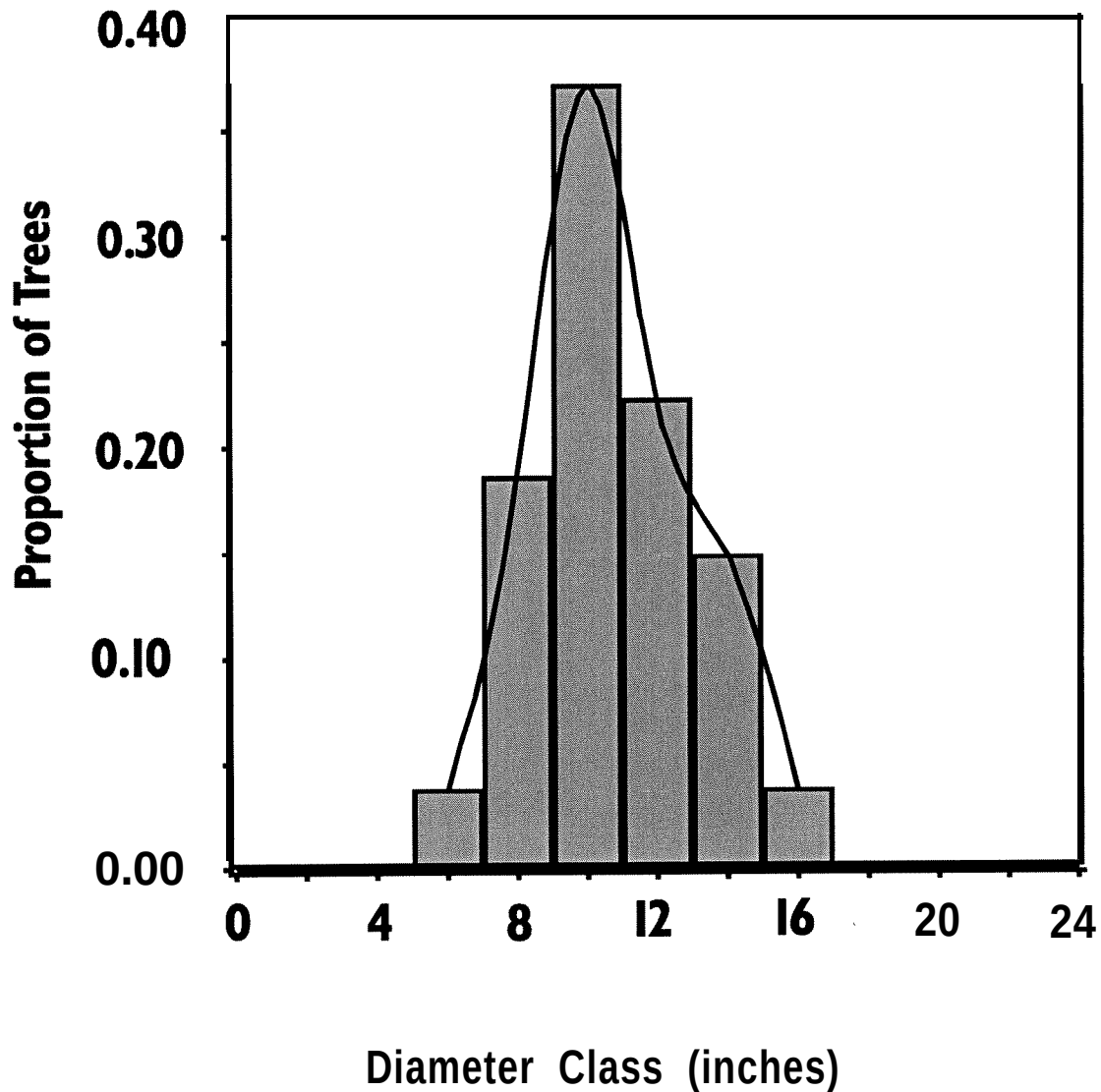


Figure 13. The histogram of figure 5, with the midpoints of the diameter classes joined by a smoothing spline.

It is straightforward to describe a stand table projection algorithm that rejects the uniform distribution hypothesis, but not the constant growth within a diameter class hypothesis.

Let:

P_i = probability of advancing for diameter class i , estimated from many stands.

Implying diameter growth:

$$G_i = P_i w$$

Where w is the width of the class.

If

$f_i(x)$ = probability distribution of trees within class i , in a given **stand**; potentially a straight (but sloping) line for simplicity.

Then the proportion of trees advancing from diameter class i is:

$$3_L I = \int_{8-U_L}^8 I_L I G$$

where U is the upper limit of the diameter class.

Similarly, it is simple to describe a stand table projection algorithm that rejects the constant growth within a diameter class hypothesis, but utilizes a uniform distribution within a diameter class.

Let:

$G(x)$ = some function that describes diameter growth, potentially unique for each diameter class, such that $G(x)$ is nondeclining within a diameter class.

Then the proportion of trees advancing from diameter class i is:

$$3_L = \frac{8 - E}{;}$$

where U is the upper limit of the diameter class, and b is defined as $b=U-G(b)$. Given a convenient formulation for $G(x)$, P_i can be solved analytically. It may make sense to implement a stand table projection-type model even when individual-tree diameter growth functions are available, if, in application, data will arise from tallies by diameter class.

Another example where a discrete model is made less discrete is the individual tree kernel model mentioned previously. Implementation can be performed in at least a couple of ways. Mortality for each kernel can be estimated individually. If mortality is deterministic, mortality will reduce the number of trees per acre represented by each kernel; thus although each kernel initially represented equal number of trees per acre, this would change over time. There could be algorithms specified for joining kernels when one does not represent many trees per acre. The location of the kernel would be projected using a growth **function**. The shape and bandwidth of the kernel could remain constant, or it could change over time. It would be reasonable for the bandwidth to increase over time. Alternatively, the kernels could be accumulated, projection could be made for stand-level variables, then an algorithm could be used to distribute growth and mortality to the kernels at each time-step.

3.8 Making a Continuous Distribution Discrete. Although a continuous distribution may be used to describe a stand, projection may involve projecting pieces of the distribution, then aggregating those pieces into a new continuous distribution. A distribution defined by a **3-parameter** Weibull function has been broken into ten classes of equal cumulative probability in figure 14. Each piece of the distribution could be projected individually. This might be rather straightforward for projecting the stand through one step in time, but it would become problematic if multiple projections are desired. Similar to when distribution modifying functions were considered, moments could be calculated from the resulting distribution, allowing parameters of a new, smooth distribution to be recovered. Thus, the algorithm would be: (1) aggregate to continuous distribution; (2) break the continuous distribution into several parts; (3) estimate the future conditions of these parts; (4) reaggregate into a new continuous distribution by recovering the parameters from the moments.

The concept of projecting parts of the distribution, then reconstituting a smooth distribution might make sense when multi-modal or other complex diameter distributions are considered. The procedure would allow stand structure to be a determinant of growth, and could thus be a better predictor than methods that simply use whole-stand variables to predict growth. It probably only makes sense where the stand structure is more complicated than can be described by a Weibull function.

4 THERE IS A UNIVERSE OF ALTERNATIVE MODELS

It is, perhaps, an **unfortunate** tendency of human minds to segregate and classify rather than seek relationships and **find** common characteristics among different things. Certainly, I classified above, distinguishing discrete from continuous distributions, distinguishing aggregation from disaggregation, and so on. However, the models described above only represent a small sample of the **infinite** universe of all possible growth and yield models. In the discipline of growth and yield modeling there is great diversity among the possible approaches. If we strictly choose to classify and pigeonhole, we may lose

sight of the intermediate models. We may not think of the alternatives. The only purpose of this paper is to provide an alternative viewpoint to the human desire to classify things.

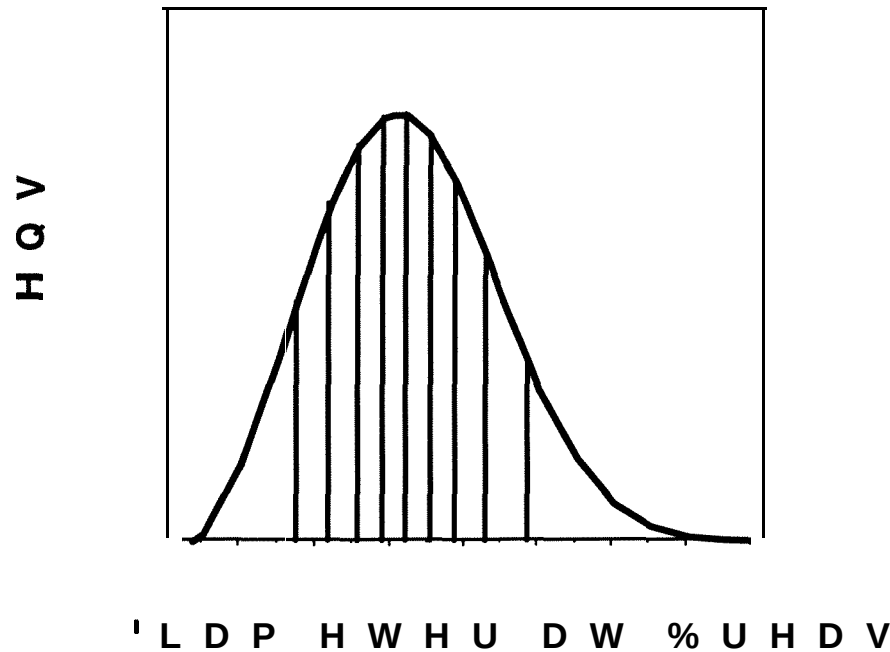


Figure 14. The three-parameter Weibull distribution broken into 10 classes of equal cumulative probability.

The relationships among different model structures are depicted in figure 15. There are processes that link the broadly-stated classes of models. However, there may be **infinite** numbers of intermediate models between those classes. For example, an individual tree model may be equated to a size-class model with a class-width of 0.1 inch. Class width might be incremented gradually, until all trees are in one class, and thus become a whole stand model. Similarly there are a few-to-many intermediate model structures between any two classes depicted in figure 15.

Furthermore, methods to project the model are not detailed in figure 15. Each model, and the multitude of intermediates, could be projected by different methods. For example, an individual tree model could be invoked by standard procedures, or the tree list could be disaggregated **from** future values of stand-level variables. This provides another dimension, **unseen** on the figure. Thus, the alternatives for growth and yield models truly represents a universe of possibilities, with some similar approaches representing “constellations” and warranting a single label such as “whole stand models”, while other individuals have few neighbors, and finally, there is the uncharted blackness awaiting exploration by the intrepid.

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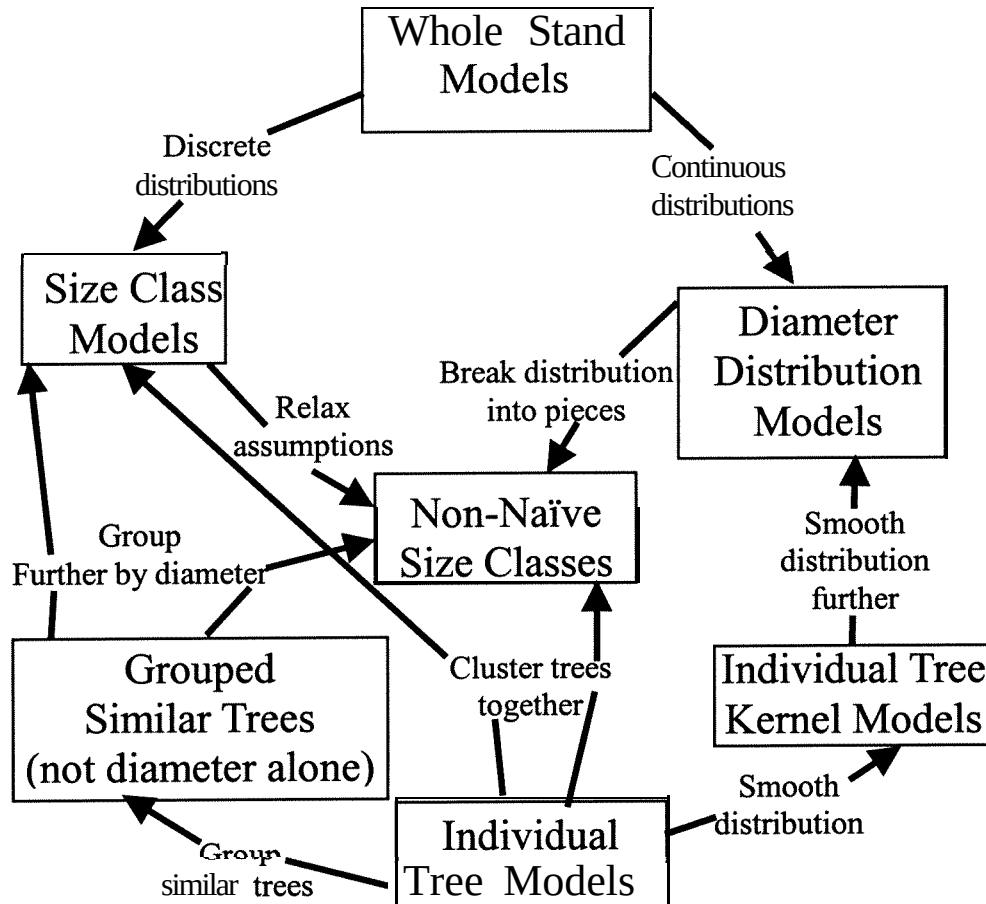


Figure 15. Different structures for forest growth and yield models, and the processes that relate one model form to another.

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