SPLINE CURVE MATCHING WITH SPARSE KNOT SETS

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ABSTRACT

This paper presents a new curve matching method for deformable shapes using two-dimensional splines. In contrast to the residual error criterion [7], which is based on relative locations of corresponding knot points such that is reliable primarily for dense point sets, we use deformation energy of thin-plate-spline mapping between sparse knot points and normalized local curvature information. This method has been tested successfully for the detection and database retrieval of deformable shapes.

1. INTRODUCTION

Curve matching is one of the fundamental tasks in computer vision and has been used to detect and recognize two- or three-dimensional (2D or 3D) objects [22]. Because of its importance, several approaches have been proposed in pursuit of finding an efficient and general curve-matching algorithm [3,10,14,15,16,17,21]. The detection and recognition of non-rigid, deformable curves (or shapes) are particularly well suited to curve-matching methods [5,12,17], and active contour models [18] called "snakes" have been popular for over a decade for modeling and detecting deformable curves in a plane. Meanwhile, curvature scale space representation [24] has been selected for MPEG-7 standard shape descriptor in database retrieval application. Among them, B-spline snakes [19] have good potential to be extended into computer-aided geometric modeling and database query. In contrast to their utility for detection purposes, however, B-spline curve representations have rarely been used for curve recognition mainly because of the non-uniqueness of spline parameters [9].

Addressing this non-uniqueness in their pioneering work, Cohen et al. [7] used B-spline knot-point matching for curve classification and recognition. In order to account for affine invariance, they estimated the underlying affine transformation between two curve representations by utilizing moment invariants, and aligned the curves by undoing the transformation [16]. A similar approach is presented in [15], in which both affine-invariant features and B-spline object curves are used for coarse-to-fine matching. However, both approaches have dealt only with affine transformed objects, still leaving in doubt their capability to deal adequately with deformable objects. Furthermore, these approaches used oversampled spline knot points and/or other features to minimize negative effects due to point mismatches.

Motivated by the need to develop a general (especially B-spline based) curve matching algorithm that works on affine transformed objects, as well as on deformable objects, we present a new approach that incorporates deformable mapping and geometric characteristics (strain energy) of spline curves. Our approach operates with small knot sets, and avoids the need for oversampling. A thin plate spline (TPS) model [4] is used in order to extract the deformable mapping parameters between two sets of corresponding points determined by a match matrix method [10] combined with deterministic annealing. Similar point correspondence problems are addressed in [1,6]. A major difference of our point-correspondence method compared to others is that, when constructing the match matrix, geometric information of curves is incorporated at accompanying points in addition to distances. The result is that good matching results are often obtained even with sparse point sets.

Based on the point correspondences that are found, strain energy is calculated to evaluate dissimilarity for each corresponding pair of curve segments. Strain energy is often used as an internal constraint of snakes [18]. Here, however, a mean strain energy is used, which is normalized according to arc length. Along with the deformation energy that is computed when the point correspondence is considered, the strain energy is incorporated into our cost function for curve matching.

Section 2 overviews spline curve representation and estimation, especially for closed curves, and section 3 addresses point correspondence problem and deformation energy. A new matching cost function is formulated in section 4. Section 5 experiments the method, and section 6 concludes the paper.

2. B-SPLINE CURVE AND FITTING

As a widely used function approximation tool, splines have been extensively used in computer aided geometric design and computer graphics. They have also proven to be useful for curve representation in computer vision and image analysis applications [2,13,15].

Given a set of knots $\{t_0 < t_1 < \cdots < t_g\} \subset [t_0, t_g] \subset R$, an n-degree B-spline function $N_i^{n+1}(t)$ over $[t_i, t_{i+n+1}]$ is defined with the concept of divided difference [9],

$$N_i^{n+1}(t) = \left(t_{i+n+1} - t_i\right) \sum_{j=0}^{n+1} \left(t_{i+j} - t\right)_+^n \Big/ \prod_{\substack{l=0 \\ l \neq j}}^{n+1} \left(t_{i+j} - t_{i+l}\right), \quad (1)$$

where $(x-c)_+^n$ has value $(x-c)^n$ if $x \geq c$, and is otherwise 0. The set of B-splines, denoted $\{N_i^{n+1}(t)\}_{i=0}^{g-n-1}$, have following properties: positive for all t, local support, and partition of the unity property. The B-splines are C^{n-1} continuous at the knots, and any C^{n-1} planar curves $\mathbf{f}(t)$ over $[t_n, t_{g-n}]$ has a unique representation

$$\mathbf{f}(t) = \left[x(t) \;\; y(t)\right] = \sum_{i=0}^{g-n-1} \mathbf{c}_i N_i^{n+1}(t), \qquad t \in [t_n, t_{g-n}], \quad (2)$$

where $\mathbf{c}_i = [c_i^x \;\; c_i^y]$ are points in 2D, called control points.

To describe closed curves, Flickner et al. [13] have constructed a periodic spline basis by extending the knot sequence, $\{\tilde{t}_i\}_{i=-\infty}^{\infty}$ with $\tilde{t}_i = t_{i \bmod g}$, and accordingly the basis functions by

$$\widetilde{N}_i^{n+1}(t) = \sum_{j=-\infty}^{+\infty} N_{i+jg}^{n+1}(t) \quad (3)$$

where $N_{i+jg}^n(t) = N_i^n(t - g(t_g - t_0))$. The basis still satisfies the above properties. A closed spline curve is then represented as

$$\mathbf{f}(t) = \left[x(t) \;\; y(t)\right] = \sum_{i=0}^{g-1} \mathbf{c}_i \widetilde{N}_i^{n+1}(t), \qquad t \in \mathbb{R}. \quad (4)$$

Given a periodic set of knots $\{\tilde{t}_0 < \tilde{t}_1 < \cdots < \tilde{t}_{g-1}\}$, a least-squares spline fit $\hat{\mathbf{f}}$ of N data points can be obtained by finding least-square solution of the coefficient vector $\mathbf{c} = [\mathbf{c}_0^T, \cdots \mathbf{c}_{g-1}^T]^T$ as follows

$$\hat{\mathbf{c}} = \arg\min \left\|\mathbf{v} - \mathbf{E}\mathbf{c}\right\|^2 = \left(\mathbf{E}^T\mathbf{E}\right)^{-1} \mathbf{E}^T \mathbf{v}. \quad (5)$$

where $\mathbf{E}$ is a linearly independent $N \times g$ matrix with whose element is $E_{ij} = \widetilde{N}_j^{n+1}(t_i)$ with $t_i = t_0 + i(t_g - t_0)/N$, i = 0,1, …,N-1. The vector $\mathbf{v}$ is a representation of the data points. Consequently, the fitted spline curve corresponding to the estimated control points is given by

$$\hat{\mathbf{f}} = [\hat{\mathbf{x}} \;\; \hat{\mathbf{y}}] = \mathbf{E}\hat{\mathbf{c}}. \quad (6)$$

Usually, the number of data points is much greater than the number of knots. Here, the parameterization t_i is obtained by the uniform (equidistant) parameterization under the assumption of completely connected boundary data. However, a chord length parameterization method[11] is used in practical situation to cope with the cases that some boundary points are missing, because it preserves the geometry of data points.

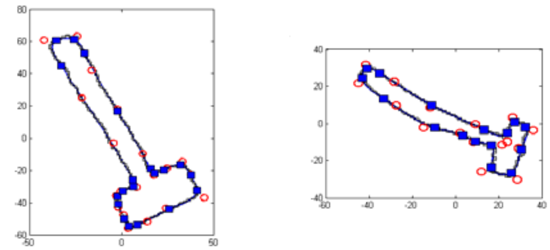


Figure 1. B-spline boundary representations for deformed objects. Squares and circles represent knot and control points, respectively.

Determining the number of knots and their locations, known as the free-knot problem, is a much harder problem to which there currently exists no general optimal solution, though there are several proposed practical techniques. Recent research has addressed this problem from a statistical point of view [8,12]. For the simplicity of the implementation, a practical technique is adopted, by which the number of knots is iteratively increased based on fitting error calculation. Starting with a minimum number of uniformly distributed knots, a new knot is inserted in every iterations until the fitting error tolerance is satisfied. Knot location is determined by selecting a point that causes maximum fitting error. Hausdorff distance between $\mathbf{v}$ and $\hat{\mathbf{f}}$ is used for the error calculation. One constraint used in locating knots is to avoid both multiple or extremely close knots that may disrupt smoothness of the fitted spline curves. Figure 1 shows examples of spline curve fitting. With a few knot points, the splines approximate object boundaries to a given error bound. Only cubic splines (n = 3) is considered throughout the paper.

3. KNOT POINT CORRESPONDENCE

Finding point correspondence is a fundamental task in curve matching and recognition. It is particularly difficult when deformable shapes are involved, and becomes even worse if the point sets are sparse. As a small number of knots leads to reasonable spline approximation of object boundaries, the paper considers a point-correspondence problem for sparse knot points of spline curves.

3.1. Sparse Knots Correspondence

The general point correspondence problem is to find a match matrix M between two point sets, A and B, such that a cost function, consisted of a shape distance and a cost for outliers, is minimized. Outliers are points with no correspondence. Sets A and B may have different numbers of points, i.e., $A = \{A_j\}_{j=1}^p$ and $B = \{B_k\}_{k=1}^r$, and the match matrix $M = \{M_{j,k}\}_{j=1,k=1}^{j=p,k=r}$ is defined as follows

$$M_{j,k} = \begin{cases} 1 & \text{if point } A_j \text{ corresponds to point } B_k \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Typically, the shape distance $d_M(A, B)$ is defined as

$$d_M(A,B) = \frac{1}{|M|}\sum_{j=1}^{p}\sum_{k=1}^{r} M_{jk} d(A_j, B_k),$$

where $|M|$ is the number of 1's in M, and d represents Euclidean distance between two points. The cost for outliers is ignored here if the matched points outnumber a minimum required number of pair.

In spline curve matching, finding a point correspondence between control points or knots is not straightforward, because a B-spline curve is not uniquely described by a single set of control points. That is, with each different choice for the placement of the knot points, a different set of control points can be induced, still describing the same curve. Furthermore, in practical situations a few control points can represent outlines of most smooth objects, resulting in a few knots that worsen the matching problem. Therefore, direct comparison methods between control points or knots are not appropriate for spline curve matching. To overcome this drawback and to apply a direct comparison, a careful rearrangement of knot position has been studied in [16], but still the method requires dense knot distribution.

This paper provides a means that resolves above-mentioned drawbacks with sparsely distributed spline knots. First of all, a correspondence is found between two different sets of knot by means of match matrix technique. Each element of match matrix is assigned to

$$\begin{aligned} m_{jk} &= (1-\eta(T))e_1(d) + \eta(T)e_2(\kappa_j^A - \kappa_k^B) \\ e_1(d) &= \exp(-\alpha d^2(A_j, B_k)) \\ e_2(\kappa_j^A - \kappa_k^B) &= \exp(-\beta(\kappa_j^A - \kappa_k^B)^2) \end{aligned} \quad (8)$$

where a temperature variable T is used for deterministic annealing that is explained in later section. The symbols κ_j^A and κ_k^B represent signed curvature of spline curves $\mathbf{f}_A(t)$ and $\mathbf{f}_B(t)$ at knot location j and k, respectively, and are defined as

$$\kappa = sign(\dot{\mathbf{f}} \times \ddot{\mathbf{f}}) \left\| \dot{\mathbf{f}} \times \ddot{\mathbf{f}} \right\| \big/ \left\| \dot{\mathbf{f}} \right\|^3. \quad (9)$$

The parameter $\eta(T)$ is a function of temperature and monotonically decreases from 0.8 to 0.2 as T does. Parameters α and β should be carefully selected to regularize the scale differences of d and k.

Equation (8) incorporates both Euclidean distance and curvature at knot locations, by which two geometrically different points would not match only because they are spatially close. In this formulation, the correspondence is mostly dependent on the curvature similarity at first, but as the temperature decreases the distance factor becomes more important. Here, by making β a function of Euclidean distance of two points, it is possible to consider the correspondence only for points within a given radius. As a similar method, there is a "softassign" technique [20]. Final match matrix is obtained by binarizing m_{jk}.

3.2. Thin Plate Spline and Deformation Energy

The thin plate spline (TPS) is a commonly used basis function for representing coordinate mappings from a point set $\{\mathbf{v}_i = (x_i, y_i)\}_{i=0}^p$ to its corresponding points $\{\mathbf{v}'_i = (x'_i, y'_i)\}_{i=0}^p$. Here, the locations (x_i, y_i) must be all different and are not collinear. Then, the TPS interpolant $f(x, y)$ minimizes the bending energy

$$I_f = \iint_{\mathbb{R}^2} \left(f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2\right) dxdy \quad (10)$$

and has the form

$$f(x,y) = \mathbf{a}_1 + \mathbf{a}_x x + \mathbf{a}_y y + \sum_{i=1}^{p} \mathbf{w}_i U\left(\left\|(x_i, y_i) - (x,y)\right\|\right) \quad (11)$$

where $U(r) = r^2 \log r$. The TPS parameters $\mathbf{a}_1$, $\mathbf{a}_x$, and $\mathbf{a}_y$ contribute to affine transform of the point set, while $\mathbf{w}_i$'s are responsible for deformation of them. For the computation of the parameters, refer to [4]. Then, the bending energy, sometimes called deformation energy, is proportional to

$$I_f \propto \mathbf{w}^T \mathbf{K} \mathbf{w}, \quad (12)$$

where $K_{ij} = U(\|(x_i, y_i) - (x_j, y_j)\|)$. When noise is present between the corresponding points, the exact interpolation condition is relaxed by introducing a regularization parameter λ, and the minimization problem is given

$$\xi(f) = \sum_{i=1}^{p}\left(\mathbf{v}'_i - f(\mathbf{v}_i)\right)^2 + \lambda I_f. \quad (13)$$

The λ controls the amount of smoothing; the limiting case of λ=0 reduces to exact interpolation. The regularized case can be solved by simply replacing the matrix $\mathbf{K}$ by $\mathbf{K} + \lambda\mathbf{I}$, where $\mathbf{I}$ is the $p\times p$ identity matrix [23]. Since the regularization parameter λ is dependent on object scale, the size of objects is normalized to have maximum length of 1 and set λ to 1.

One drawback of the TPS model is that its solution requires the inversion of a large, dense matrix of size $(p+3)\times(p+3)$, where p is the number of points in the data set. However, our curve matching algorithm performs the correspondence search only for spline knot points, which is a small set compared to the number of data points. This means that the size of the matrix can be much smaller than typical cases.

3.3. Deterministic Annealing Method

TPS has been also used for point correspondence problems of non-rigid objects [1,6]. A popular technique in solving such a non-rigid problem is to use deterministic annealing method. At a given high value of temperature T, the match matrix is extracted by the equation (8). At this stage, the correspondence mostly depends on geometric information of object boundaries. Then, the TPS parameters are estimate based on the match matrix, and knot points are warped by the equation (11). As the temperature reduces, by which the correspondence becomes more relying on physical distance, the above procedure is repeated until M converges.

Figure 2 shows an example for knot correspondence. Object A is the same as in Figure 1a, and object B is an affine transformed version of one in Figure 1b. Affine parameters are estimated by the means presented in [16]. Figure 2c shows the resulting knot correspondence. Outliers are depicted as open circles and squares.

4. CURVE MATCHING

4.1. Cost Function

Match matrix M estimated in the previous section gives us one-to-one correspondence between two sets of knot points of B-spline curves. Let $\mathbf{f}_A(t)$ be a p-knot periodic B-spline representation of curve A with knots $\{t_j\}_{j=1}^{p}$, and let $\mathbf{f}_B(s)$ be a similar representation of a curve B, having r knots $\{s_k\}_{k=1}^{r}$. Sets of knot points for each curve are calculated at each knot location and represented as $\{A_j = (x_j^A, y_j^A)\}_{j=1}^{p}$ and $\{B_k = (x_k^B, y_k^B)\}_{k=1}^{r}$. Then, the match matrix M represents the correspondence of a group of points in $\{A_j\}_{j=1}^{p}$ to a group of points in $\{B_k\}_{k=1}^{r}$, yielding new ordered knot locations: $\{t_i'\}_{i=1}^{p'}$ and $\{s_i'\}_{i=1}^{p'}$, so that intervals $[t_i', t_{i+1}')$ and $[s_i', s_{i+1}')$ represent matching portions in curve $\mathbf{f}_A(t)$ and $\mathbf{f}_B(s)$, respectively.

A matching cost between curve A and B, $\xi(A, B)$, is a sum of total deformation energy, ξ_d, and strain difference, ξ_s, and defined as

$$\begin{aligned} \xi(A,B) &= \xi_d(A,B) + \xi_s(A,B) \\ \xi_d(A,B) &= \sum_l \mathbf{w}_l^T \mathbf{K}_l \mathbf{w}_l \\ \xi_s(A,B) &= \lambda_s \sum_{i=1}^{p'} \left(\mu^A \int_{t_i'}^{t_{i+1}'} \left(\kappa^A(t)\right)^2 dt - \mu^B \int_{s_i'}^{s_{i+1}'} \left(\kappa^B(s)\right)^2 ds \right)^2 \end{aligned} \tag{14}$$

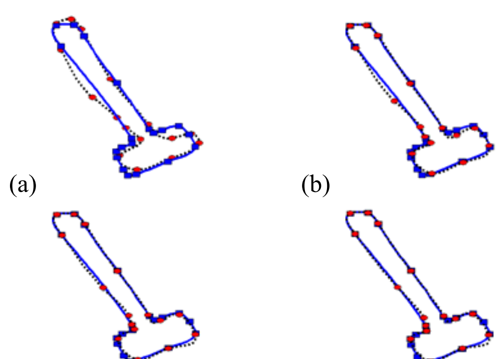


Figure 2. Example of sparse knots correspondence.

where

$$\mu^A = \frac{C_i^A}{t_{i+1}' - t_i'} \qquad \mu^B = \frac{C_i^B}{s_{i+1}' - s_i'}$$

The total deformation energy penalizes high deformation when the curve A deforms onto the curve B. The deformation energy is calculated when searching the point correspondence as described in section 3. If the deterministic annealing repeats l times, the total deformation energy is the sum of each deformation.

The strain difference measures the geometric dissimilarity between two curves by the squared sum of difference of strain energy. For each portion as shown in Figure 3a and b, mean strain energy is used as the dissimilarity measure. Here, $t_{p'+1}' = t_1'$ and $s_{p'+1}' = s_1'$. A special care is required when $t_p \in [t_i', t_{i+1}')$ or $s_p \in [s_i', s_{i+1}')$ for $i = 1, \ldots, p'$. Convexity of curve segments is reflected in the parameters C_i^A and C_i^B, having 1 for convex and -1 for concave. Figure 3c and d illustrate this.

The overall procedure is as follows:

1. Perform contour extraction from an image.
2. Fit periodic B-spline curve to the contour points.
3. Find corresponding knots between the extracted B-spline curve and a stored model.
4. Calculate the matching cost.
5. Repeat steps 3 and 4 for each stored model, and determine a model having the least cost.

4.2. Initial Alignment

The proposed matching algorithm, which requires point correspondences between sparsely distributed spline knot points, relies substantially on initial alignment of two objects to be matched. One means of achieving initial alignment is with an affine parameter estimator that uses moments calculated from spline curves [16]. However, it

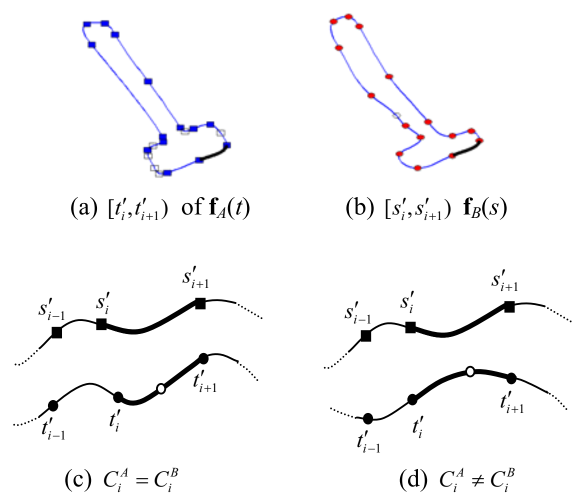


Figure 3. Curve matching.

has been experienced that the estimator is sensitive to noise and often comes up with poor results, especially for heavily transformed objects.

To improve capability of the same estimator, an initial alignment is performed first by the use of multilevel approximation capability of spline curves [15]. The alignment repeats rotation and scaling as the boundary goes from coarse to fine scale by the incremental knot insertion. As an example, the coarsest (dotted) and the finest approximations of a model and an input object are depicted in figure 4a and b, respectively. Eigenanalysis of knot points of two spline curves determines rotation and scaling amount. Figure 4c shows a rotated and scaled version of the input object. At the coarsest level, the alignment would not be accurate enough and need more steps as shown in figure 4d-f.

With the rough alignment, affine parameters are estimated based on the method described in [16]. Figure 4g shows the final alignment. Knot correspondence is also depicted in the figure. Note again that the correspondence does not solely depend on Euclidean distance; instead that curvature information leads to correct point matching.

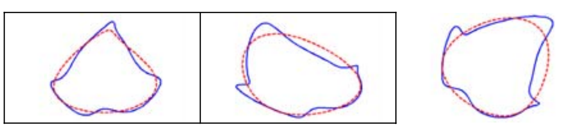
5. EXPERIMENTAL RESULTS

The proposed curve matching method has been tested successfully for model-based shape detection and database retrieval. First, the spline curve matching method proves an excellence on model-based shape detection. For this experiment, an image of blobs with different shapes, shown in Figure 5a, is used. Let the spline curve depicted in Figure 5b be a model for matching. The model is simply a spline approximation of a typical shape of interest, and contains a few knot points. Each blob in the input image is extracted and its boundary is approximated with a spline curve. Figure 5c shows the matching cost for nine blobs numbered in the input image. The values in the table correspond to total deformation energy (ξ_d), a strain energy difference (ξ_s), and matching cost (ξ), respectively. λ_s is set to 100 for all experiments presented here. Blobs numbered from B_1 to B_6 have relatively low matching cost, representing a good match to the model, while the others have high matching costs. Blob B_2 has a relatively high cost compared to other detected ones due to its slightly displaced indentations.

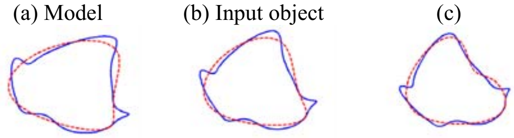
The spline matching method is tested for database retrieval application. A part of MPEG-7 benchmark dataset and silhouettes of object images are collected, and then they are affine transformed or deformed to build a database of 25 categories 380 silhouettes. Boundary curves of each object are approximated offline with B-spline, and the hierarch of knot insertion is stored with each spline curve. For a given query shape, the spline curve matching is performed to find least cost objects from the database. The first column in Figure 5 shows query shapes, and following columns gives silhouettes of four low cost objects and one high cost object. Their matching costs are shown underneath. The experiment shows that matching cost within the same category is mostly less than 10 while matching between different categories produces higher cost.

6. CONCLUSION

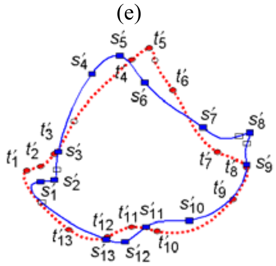
A new curve matching method that uses sparse spline knot points is presented. Corresponding knot points are first detected automatically, and then deformation energy and strain differences of the spline approximations are calculated. Despite sparse distributions of knot points, the experimental results show that the method is promising for model-based shape detection and database retrieval.



(a) Model (b) Input object (c)

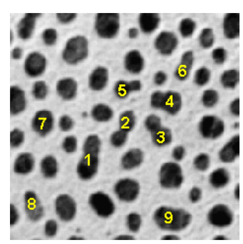


(d) (e) (f)

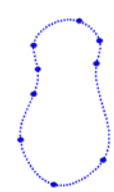


(g) Knot correspondence

Figure 4. Alignment and point correspondence.



(a) Blob image



(b) A blob model

	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9
ξ_d	0.32	0.38	0.22	0.23	0.07	0.13	1.45	1.92	0.96
ξ_s	0.21	0.43	0.44	0.17	0.11	0.06	0.59	0.13	0.32
ξ	0.54	0.81	0.66	0.40	0.18	0.19	2.04	2.05	1.28

(c) Matching cost

Figure 5. Model-based shape detection.

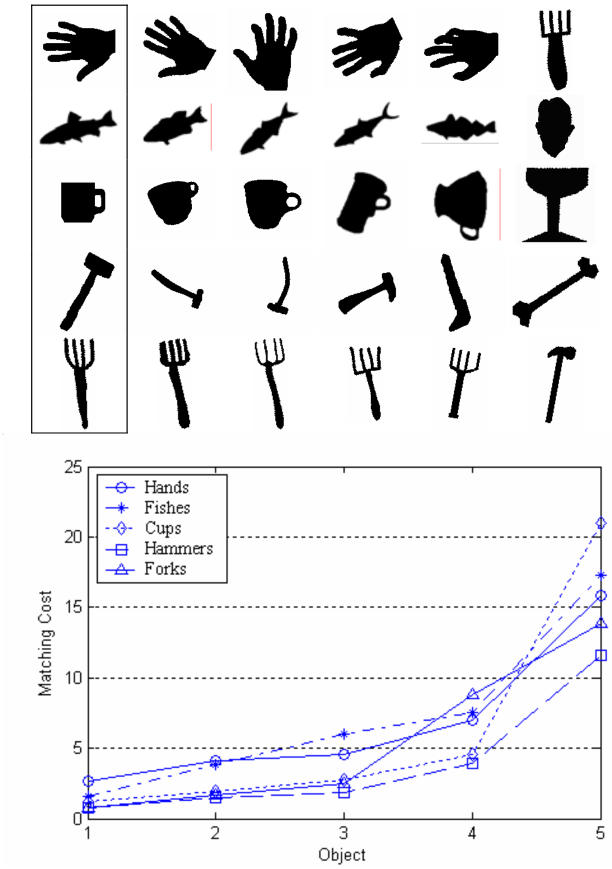


Figure 6. Database retrieval.

The algorithm is implemented using Matlab™ on 1.8GHz Intel Pentium PC with 256MB main memory. Although matching time is dependent on the size and shape of object, an average matching time is 0.15 second. This can be much improved when C or C++ is used.

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