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Geographically weighted regression model-assisted (GWRMA) estimation to improve precision of estimates combining remote sensing and ground plot data

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A B S T R A C T

Remotely sensed data play a vital role providing auxiliary variables that can improve precision of sample-based estimates without incurring the substantial cost of increasing sample size. In the setting of design-based inference, model-assisted estimators using generalized regression models provide a practical option to exploit such auxiliary variables to obtain estimates with smaller uncertainties than the Horvitz-Thompson (HT) estimator. Spatial heterogeneity is one of the main characteristics of spatial geographical populations and spatial nonstationarity in regression model parameters can greatly impact model predictions. However, model-assisted estimators typically do not incorporate spatial heterogeneity in the assisting models, and this may result in failing to gain the full precision improvement available from model-assisted estimation. Geographically Weighted Regression (GWR) offers a practical way to incorporate spatial heterogeneity of the data into the model. In this study, we substantiate the benefit of employing a GWR model-assisted estimator (GWRMA) in which GWR is used as a linking model between the response and auxiliary variables. The illustrative example we use to demonstrate the precision enhancing capacity of GWRMA is estimating tree volume in forest inventory monitoring. The performance of GWRMA was compared with the HT estimator and a linear regression model-assisted (LRMA) estimator when both the response and auxiliary variables are continuous. Several factors potentially impacting the estimators and their precision were investigated using Monte-Carlo simulation applied to populations constructed to represent different levels of spatial heterogeneity and correlation between the response and auxiliary variables. Because both LRMA and GWRMA estimators are asymptotically unbiased, variance is the key criterion for comparing the estimators. For the constructed populations with spatial heterogeneity, the GWRMA estimator achieved standard errors smaller than the HT and LRMA estimators, and the improvement in precision increased with increasing sample size. The precision advantage of the GWRMA estimator relative to the LRMA estimator was attributable to the capacity of the GWRMA estimator to adapt to spatial variation in the regression model leading to better local prediction of the target variable. The conventional model-assisted variance estimator substantially underestimated the variance of the GWRMA estimator. An alternative variance estimator based on averaging the GWRMA and LRMA variance estimates avoided the severe underestimation of the conventional model-assisted variance estimator. This average variance estimator yielded confidence intervals that exceeded the nominal 90% coverage but still had shorter length than the 90 % confidence intervals from LRMA estimator. Further exploration of variance estimators for the GWRMA estimator is a necessary next step to improve utility of the GWRMA estimator.

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1. Introduction

Remotely sensed data have demonstrated value for use in model-assisted estimation because they provide unique, wall-to-wall coverage of auxiliary information across a region of interest (Gallego, 2004; Stehman, 2009). With the rapid development of forest-related remote sensing products (e.g., maps of disturbance, canopy height, biomass, etc.), model-assisted estimators have been widely applied to improve the precision of forest inventory estimates (Breidt and Opsomer, 2017). Although the focus of our example applications is forestry, model-assisted estimators have been applied in other settings, with agriculture being a primary application area as well (e.g., Haack and Rafter, 2010; Ambrosio et al., 2023).

The United States Department of Agriculture's (USDA) Forest Inventory and Analysis (FIA) program monitors the status of forest characteristics (e.g., forest area, forest volume, and forest biomass) using a sample of permanent field plots. The FIA database was developed to record measurements of forest attributes of sample plots in a consistent format, spanning all states across the contiguous United States (CONUS). These sample data facilitate the use of estimators of parameters of forest characteristics for a region of interest (ROI). The Horvitz-Thompson (HT) estimator (Särndal et al., 1992; Cochran, 1977) is a general purpose design-unbiased estimator for probability sampling to estimate parameters of a ROI in forest inventory. The HT estimator uses only the target variable of interest to estimate population parameters. When one or more auxiliary variables correlated with the target variable are available, more precise estimates can be obtained to better support operational applications and decision making.

Model-assisted estimators combined with a probability sample of forest plots and remote sensing data have been increasingly used in the last two decades (Magnussen, 2015). Model-assisted estimators reduce the uncertainty of estimation compared to the HT estimator by taking advantage of the relationship between the response (target) variable and one or more auxiliary variables (Särndal et al., 1992). A regression model which depicts the relationship between the variable of interest (Y) and auxiliary variables (X) is used to assist the estimation of finite population parameters from a probability sample within the context of design-based inference. Efficiency of the survey estimates is gained by borrowing information from the auxiliary variables.

McConville et al. (2020) reviewed six commonly used generalized regression model-assisted estimators including ratio, post-stratification, regression, lasso, ridge, and elastic-net and their related variance estimators. Ene et al. (2016) and Ene et al. (2012) developed a design-based generalized linear regression model-assisted estimator (DBMA) where the target variable was plot-level Aboveground Biomass (AGB) and the auxiliary variables were airborne laser scanning data, land use, and forest types. For a set of constructed artificial populations and using Monte-Carlo simulation, Ene et al. (2012, 2016) compared the precision of the DBMA estimator to the HT estimator for both one-phase and two-phase simple random sampling and systematic sampling. In another application of model-assisted estimation in forestry, Hou et al. (2018) examined the impact of spatial resolution of remotely-sensed auxiliary variable on precision of estimates of firewood volume.

Spatial heterogeneity, also known as spatial nonstationarity, is defined as structural instability in the form of systematically varying model parameters or different response functions (Anselin and Griffith, 1988; Gelfand et al., 2003). Global regression models assume a single set of regression coefficients to represent the relationship between Y and X. However, because of spatial non-stationarity of geographically spatial populations, patterns of phenomena appear to vary as a function of spatial scale (Foody, 2004). Geographically weighted regression (GWR) (Fotheringham and Brunsdon, 1999) was proposed as a method for constructing local models to deal with the spatial heterogeneity inherent in geographic data. It has rapidly become a popular technique used in a wide number of fields including geography and ecology. GWR has been shown to be an efficient approach providing better prediction than global regression models (Zhang et al., 2009) when there is a spatially varying relationship between the target variable and auxiliary variables. In the case of forest-related variables, GWR has demonstrated utility to improve spatial predictions (e.g., Maselli et al., 2014, Chirici et al., 2020) and this improved spatial prediction is the key to more precise model-assisted estimation.

Here we use GWR to construct a model-assisted estimator that accounts for spatial heterogeneity to improve the precision of estimates. Liu et al. (2018) first proposed using GWR for model-assisted estimation. They provided the critical theory underlying the method, in particular establishing that the GWRMA estimator is asymptotically design unbiased. Liu et al. (2018) conducted a limited simulation study (four populations) evaluating the potential improvement in precision achieved by GWRMA for an unequal inclusion probability sampling design. One objective of our study is to apply and assess GWRMA in a simpler practical setting to demonstrate the utility of the approach for practitioners. That is, we apply the GWRMA estimator and compare its performance to the HT estimator and to a linear regression model-assisted estimator (LRMA) when the sampling design is simple random. Bias and variance of the estimators are assessed by simulation applied to a variety of populations constructed to represent different features and relationships between the response and auxiliary variables. We extend the work of Liu et al. (2018) in two major directions: 1) using the constructed populations, we assess how the precision advantage of GWRMA relative to LRMA varies as a function of sample size and degree of spatial heterogeneity in the population, and 2) we evaluate the performance of the generally recommended model-assisted variance estimator applied to the specific case of the GWRMA estimator.

2. Methods

2.1. Horvitz-Thompson estimator

The Horvitz-Thompson (HT) estimator (Särndal et al., 1992) is an unbiased estimator for probability sampling. We assume the population is fixed and the elements in this fixed population are countable (i.e., pixels, trees) with population size N and known population mean $\overline{Y}$. A probability sample s contains n elements randomly selected using a sampling design with known inclusion probability π_i for each sample element i. The measurements obtained on an element i, denoted as y_i, as well as the population parameters are treated as fixed values (Gregoire, 1998; Ståhl et al., 2024). An estimator is a statistic using measurements of the sampled elements to estimate the population parameters, and an estimate is a value obtained by applying the estimator to a particular probability sample. The HT estimator of the population mean is

$$\widehat{\overline{Y}}_{HT} = \frac{\sum_s y_i/\pi_i}{N} \tag{1}$$

For simple random sampling which has equal inclusion probabilities, the Horvitz-Thompson estimator of the sample mean is

$$\widehat{\overline{Y}}_{HT} = \frac{1}{n}\sum_{i\in s} y_i = \overline{y} \tag{2}$$

The variance of the HT estimator under simple random sampling is

$$V(\widehat{\overline{Y}}_{HT}) = \left(1 - \frac{n}{N}\right)\frac{1}{n}\frac{1}{N-1}\sum_{i=1}^{N}(y_i - \overline{Y})^2 \tag{3}$$

An unbiased variance estimator of the HT variance is

$$\widehat{V}(\widehat{\overline{Y}}_{HT}) = \left(1 - \frac{n}{N}\right)\frac{1}{n}\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \widehat{\overline{Y}}_{HT})^2 \tag{4}$$

and the estimated standard error is

$$\widehat{SE}_{HT} = \sqrt{\widehat{V}(\widehat{\overline{Y}}_{HT})} \tag{5}$$

2.2. Model-assisted estimation

2.2.1. Estimators

The model-assisted estimator for $\overline{Y}$ for simple random sampling is (Särndal et al., 1992)

$$\widehat{\overline{Y}}_{MA} = \frac{1}{N}\sum_{i=1}^{N}\widehat{y}_i + \frac{1}{n}\sum_{i\in s}\widehat{e}_i \tag{6}$$

where $\widehat{y}_i$ is the model prediction for population element i, and $\widehat{e}_i = y_i - \widehat{y}_i$ is the estimated residual for each sample element. The approximate variance of the MA estimator is

$$V(\widehat{\overline{Y}}_{MA}) = \left(1-\frac{n}{N}\right)\frac{1}{n}\frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \widetilde{y}_i\right)^2 \tag{7}$$

where y_i is the observed target variable and $\widetilde{y}_i$ is the population prediction of y_i based on a census level fit of the model (McConville et al., 2020, eqn. [4]). The variance of the model-assisted estimator depends on the magnitude of the squared residuals, $\left(y_i - \widetilde{y}_i\right)^2$, so a model that better predicts y_i will yield smaller variance. To estimate the variance, the population residuals are replaced by the sample residuals computed from the sample-based prediction $\widehat{y}_i$. For simple random sampling, Särndal et al. (1992, p. 235, Eq. 6.6.3 and Eq. 6.6.4) provide the approximate variance estimator

$$\widehat{V}(\widehat{\overline{Y}}_{MA}) = \left(1-\frac{n}{N}\right)\frac{1}{n}\frac{1}{n-1}\sum_{i\in s}(y_i - \widehat{y}_i)^2 \tag{8}$$

and the estimated standard error is then

$$\widehat{SE}(\widehat{\overline{Y}}_{MA}) = \sqrt{\widehat{V}(\widehat{\overline{Y}}_{MA})} \tag{9}$$

2.2.2. Assisting models

The linear regression model-assisted (LRMA) estimator uses a linear regression (LR) model as the assisting model to obtain the prediction of y_i. If p is the number of auxiliary X variables in the LR model, the prediction $\widehat{y}_i$ can be obained from the model

$$y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots \beta_p X_{pi} + \varepsilon_i \tag{10}$$

where $\beta_0, \beta_1, \cdots, \beta_p$ are regression coefficient parameters and ε_i is the model residual. The LR model is a global model and therefore cannot capture the spatial heterogeneity of random effects in the regression coefficients over the ROI.

Geographically Weighted Regression (GWR) is used to incorporate local non-stationarity by allowing the relationship between X and Y to vary across space and estimating locally weighted regression coefficients (Fotheringham et al., 2002). Suppose the n observations in sample s have location coordinates (u_i, v_i), $i \in s$. The general GWR model is defined as

$$y_i = \beta_0(u_i, v_i) + \sum_{k=1}^{p}\beta_k(u_i, v_i)X_{ik} + \varepsilon_i \tag{11}$$

where $\{\beta_0(u_i, v_i), \beta_1(u_i, v_i), \cdots, \beta_p(u_i, v_i)\}$ are $(p+1)$ spatial varying regression coefficients and $\varepsilon \overset{iid}{\sim} N(0, \sigma^2 I)$, and $\sigma^2 I$ is the $p \times p$ spatial variance matrix.

The regression coefficients for location i are estimated by $\widehat{\beta}_i = (X'W_iX)^{-1}X'W_iY$, where X is an $n \times p$ matrix with X'_{ik} at the i^{th} row, W_i is a diagonal weight matrix, and Y is the set of observed values of the sample. A kernel function must be defined for the diagonal of W_i. Fotheringham et al. (2002) summarized the commonly used kernel functions categorizing fixed spatial kernel functions and adaptive spatial kernel functions. In this study, we used a Gaussian adaptive spatial kernel with the bandwidth for the spatial kernel function determined by cross validation. The cross validation scores the root mean square prediction error for the geographically weighted regression and chooses the bandwidth that minimizes this quantity. The coordinate system used is North American Datum of 1983 (NAD83). The parameters for the GWR models were estimated using version 0.6.36 of the spgwr R package (Bivand and Yu, 2023). Other packages may implement different versions of GWR, but it was not within the scope of our study to evaluate sensitivity of the GWRMA estimator to these possible different implementations.

2.3. Simulation study using constructed populations

Monte-Carlo simulation (using R programming) was employed to evaluate the performance of the estimators applied to sets of constructed populations. Because performance of the estimators can vary under different conditions, we evaluated three critical factors: 1) degree of spatial heterogeneity of the population regression coefficients; 2) Pearson correlation between Y and X of the population; and 3) sample size, n.

2.3.1. Constructed populations

To generate a population of field plots, a spatial dataset containing N = 10,000 points with longitude and latitude coordinates was created. 90 m × 90 m grids were generated using ArcMap, and the centers with the geographical location of each grid were extracted and used to represent the centers of the population plots. To construct populations with spatial heterogeneity of the regression coefficients, the study area was divided into four equal area subregions so that each region had a population size of 2500 plots. The measurement y_i of plot i was generated from a different probability distribution in each subregion. Populations with three different levels of spatial heterogeneity of the regression coefficients between Y and X_1 were created – no spatial heterogeneity, moderate spatial heterogeneity, and strong spatial heterogeneity.

To create populations that were consistent with forest inventory tree volume estimation, the population mean of Y for the constructed populations was set to approximately 2500 ft^3/acre (175 m^3/ha). For the populations with no spatial heterogeneity, the target Y variable of all N = 10,000 plots was generated from the same normal distribution with population mean μ=2500 and standard deviation σ = 500, denoted Normal(2500, 500). The auxiliary X_1 variable for all N plots was generated from Normal(25, 4).

For the populations with moderate and strong spatial heterogeneity, the data from four equal area subregions were generated with different distributions in the subregions. In the moderate spatial heterogeneity populations, the four distributions were Normal(1500, 200), Normal (2500, 400), Normal(2500, 600), and Normal(3500, 800) for Y, and Normal(24, 5), Normal(25, 5), Normal(25, 5), and Normal(26, 5) for X_1. In the strong spatial heterogeneity populations, differences among the subregion means for Y were increased relative to the moderate populations but the standard deviation remained the same. The four distributions for Y were Normal(1000, 200), Normal(2000, 400), Normal (3000, 600), and Normal(4000, 800), and for X_1, the four distributions were Normal(23, 4), Normal(25, 5), Normal(25, 5), and Normal(27, 6). The populations created by this approach (Fig. 1) possess the intended spatial variation of the regression coefficients as the standard deviations of the y-intercept and slope increase with the degree of spatial heterogeneity (Table 1).

The models we examined have a target variable Y that is continuous and one or two continuous auxiliary variables, X_1 and X_2. These models would be consistent with applications such as the 11 examples described in McRoberts et al. (2022, Table 1) where Y was either volume or aboveground biomass and the auxiliary variables were created from remotely sensed data and an example reported by Francini et al. (2021) where Y was area of forest disturbance and the auxiliary variable was a mapped index of forest disturbance. We constructed both Y and X_1 variables to follow a normal distribution. Bivariate populations with defined Pearson correlations between Y and X_1 were generated using the

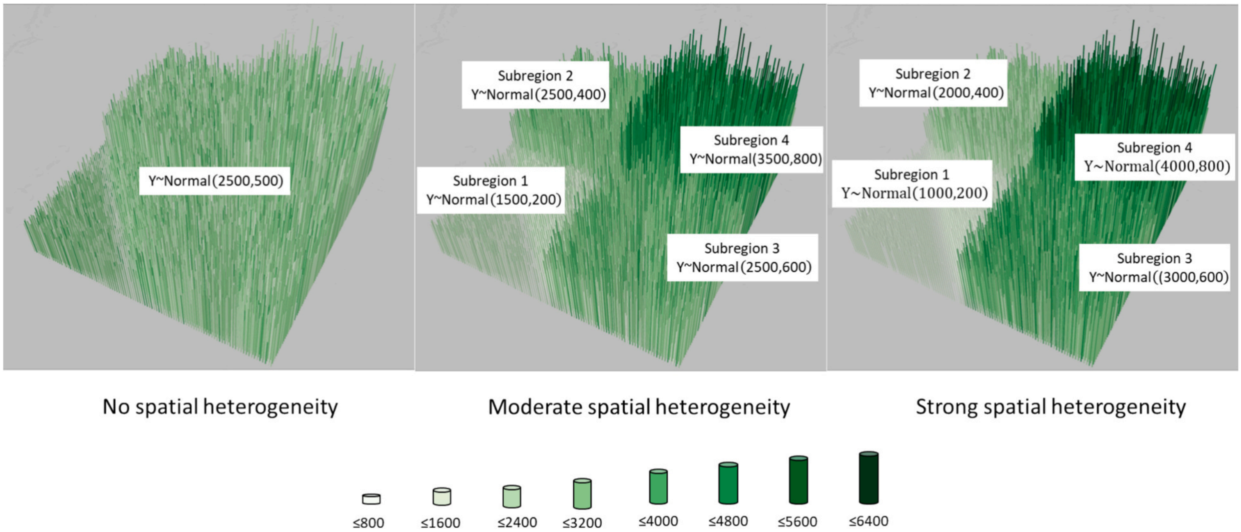

Fig. 1. Three populations (Y) with different levels of spatial heterogeneity but the same correlation between X and Y, the gradients of the color represent the different range of forest inventory tree volume in constructed populations.

Table 1
Mean and standard deviation (StDev) of y-intercepts and slopes for populations which include one auxiliary variable and have different correlations and spatial heterogeneity (SH). The means and standard deviations are estimates from a simple random sample of points (n = 1,000) selected from each of the populations where the local y-intercept and slope are estimated at each point based on the GWR kernel.

Correlation	Mean			StDev		
	No SH	Moderate SH	Strong SH	No SH	Moderate SH	Strong SH
	Intercept			Intercept		
0.1	2299	2054	1863	7	714	1030
0.3	1692	1605	2033	8	520	1090
0.5	1071	1095	1299	9	641	927
0.75	150	8	398	39	437	670
	Slope			Slope		
0.1	8	18	26	0.3	24	34
0.3	32	35	19	0.3	23	24
0.5	57	48	43	0.4	21	28
0.75	94	96	82	1.7	30	32

Copulas package in R (Hofert et al., 2025). Four sets of populations representing correlations between X_1 and Y of 0.1, 0.3, 0.5 and 0.75 were created. For all populations the mean of Y is close to 2500, the mean of X_1 is close to 25, and IQR of X_1 is in the range of 5–8. Where the populations differ (besides correlation of X_1 and Y) is the variability of Y which increases from the no spatial heterogeneity to strong spatial heterogeneity.

We also evaluated a limited set of populations with two auxiliary variables to further explore the properties of the GWRMA estimator (Table 2). The same motivation for the potential advantage of using GWRMA extends to the multiple auxiliary variable case. That is, if the regression coefficients vary spatially (i.e., the coefficients of each auxiliary variable), GWR should provide better local predictions of y_i than the global LRMA model and these better predictions will translate to smaller variance of the GWRMA estimator. In general, this same feature would extend to other more complex relationships of Y to the auxiliary variables such as non-linear. The structural form of the GWR model (Eq. (11) could be adapted to provide a better approximation to the presumed relationship as, for example, using a polynomial model to approximate a non-linear relationship.

For the two auxiliary variable populations, five sets of populations representing correlations between X_1 and Y and X_2 and Y of 0.1 and 0.1, 0.1 and 0.5, 0.1 and 0.75, 0.5 and 0.5, and 0.5 and 0.75 were created. For the populations with no spatial heterogeneity, the auxiliary X_2 variable for all N plots was generated from Normal(5, 1). For the populations with moderate and strong spatial heterogeneity, X_2 was generated for four equal area subregions with a different distribution in each subregion. In the moderate spatial heterogeneity populations, the four distributions for X_2 were Normal(1, 1), Normal(5, 1.2), Normal(5, 1.2), and Normal(6, 1.4). In the strong spatial heterogeneity populations, the four distributions for X_2 were Normal(4, 0.8), Normal(5, 1), Normal(5.5, 1.2), and Normal(6, 1.5). Similar to the one auxiliary variable populations, for all two auxiliary variable populations the mean of Y is close to 2500, the mean of X_1 is close to 25, the mean of X_2 is close to 5, the IQR of X_1 is in the range of 5–8, and the IQR of X_2 is in the range of 1–2.

Table 2
Mean and standard deviation (StDev) of y-intercepts and slopes for populations which include two auxiliary variables and have different combinations of correlations and spatial heterogeneity (SH). The means and standard deviations are estimates from a simple random sample of points (n = 1,000) selected from each of the populations where the local y-intercept and slope are estimated at each point based on the GWR kernel.

Correlation	Mean			StDev		
	No SH	Moderate SH	Strong SH	No SH	Moderate SH	Strong SH
	Intercept			Intercept		
0.1 & 0.1	2159	1567	1235	10	719	863
0.1 & 0.5	1228	1779	1907	8	803	1352
0.1 & 0.75	601	939	673	6	561	890
0.5 & 0.5	141	128	68	4	635	949
0.5 & 0.75	−665	−797	−904	3	844	511
	Slope X_1			Slope X_1		
0.1 & 0.1	7	17	25	0	22	30
0.1 & 0.5	6	0	−1	0	14	20
0.1 & 0.75	5	−3	−3	0	14	17
0.5 & 0.5	53	46	41	0	16	25
0.5 & 0.75	55	71	67	0	31	24
	Slope X_2			Slope X_2		
0.1 & 0.1	32	103	133	1	78	96
0.1 & 0.5	226	140	123	1	78	137
0.1 & 0.75	354	316	367	1	109	126
0.5 & 0.5	209	212	265	1	82	124
0.5 & 0.75	356	291	327	0.5	100	98

2.3.2. Monte-Carlo simulation

Because the population is known, Monte-Carlo simulation provides an efficient way to approximate the sample space properties of the estimators. Specifically, we use simulation to approximate the bias and variance of the HT and model-assisted estimators. The Monte-Carlo simulation generated 1000 simple random samples without replacement from each population. The expected value of each estimator is approximated by the mean of the estimates over the 1000 simulated samples. For example, the mean of the GWRMA estimates from the 1000 simulated samples would approximate the expected value of the GWRMA estimator, and the bias of the GWRMA estimator would be approximated by the difference between the expected value of the estimator and the known parameter (mean volume) from the population. The simulated variance of each MA estimator is the variance of the estimates over the 1000 samples, and the simulated standard error is the square root of the simulated variance. The simulated variance of the GWRMA estimator approximates the variance expressed by equation [7].

We also evaluated the bias of the standard error estimators for the LRMA and GWRMA estimators along with the average length and coverage of confidence intervals obtained from these variance estimators. For each simulated sample, the standard error of each of the three estimators (HT, LRMA, and GWRMA) was estimated and the mean of these estimated standard errors over the 1000 samples approximates the expected value of the standard error estimator. Comparison of this expected value to the simulated standard error quantifies the bias of the standard error estimator. Relative bias of a standard error estimator is the difference between the expected value of the estimator and the simulated standard error divided by the simulated standard error, where the simulated standard error is a close approximation to the square root of the model-assisted variance (equation [7]). We also evaluated coverage and average length of 90 % confidence intervals constructed using the variance estimators. Empirical coverage was calculated as the percent of the 1000 simulated samples that yielded a confidence interval that included the population parameter (total), and length was averaged over the 1000 confidence intervals estimated from the samples for each simulation run.

The design-unbiased properties of the model-assisted estimators and their variance approximations are large sample results (Särndal et al., 1992). Although it is not definitive what constitutes a "large sample," in the context of the LRMA estimator, Cochran (1977, p.195) stated that the terms ignored in the variance approximation are of order $1/\sqrt{n}$. In our study, sample sizes of n = 20, n = 100 and n = 250 were evaluated with the sample size of n = 20 included for the specific purpose of assessing performance for a sample size too small for the asymptotic properties to prevail. The sample sizes of n = 100 and n = 250 are assumed sufficiently large that the model-assisted estimators are approximately unbiased and that the large-sample variance approximations are unbiased. The simulation results for the estimators provide direct empirical evidence for whether the sample sizes are sufficiently large for the asymptotic properties to hold.

3. Results

3.1. Bias of estimators

For the larger sample sizes (n = 100 and n = 250), the asymptotic unbiased property of the model-assisted estimators was confirmed by the simulation results as the largest magnitude relative bias observed for LRMA and GWRMA was 0.14 % in the one auxiliary variable populations and −0.17 % in the two auxiliary variable populations (Tables 3 and 4). For the smallest sample size (n = 20), the relative biases for all three estimators were larger compared to the relative biases for n = 100 and n = 250. But even for n = 20, the absolute value of the relative bias was never larger than 0.40 % (Tables 3 and 4). The HT estimator is unbiased for all sample sizes, and the near zero relative bias observed in the simulation results confirms that feature.

3.2. Precision comparison of estimators of total volume

Having established that the GWRMA estimator does not have substantial bias, we then evaluated whether GWRMA improved precision relative to LRMA. This evaluation was based on the simulated standard errors for the different estimators. We define relative efficiency (RE) of the model-assisted estimators as the standard error of the model-assisted estimator divided by the standard error of the HT estimator. RE < 1 indicates the model-assisted estimator improved precision relative to the HT estimator. As expected, both LRMA and GWRMA estimators

Table 4
Relative bias (%) of the HT, LRMA and GWRMA estimators for simple random sampling (n = 100) from the constructed populations with different spatial heterogeneity and correlation (ρ) between Y and X_1, and Y and X_2.

	ρ = 0.1&0.1	ρ = 0.1&0.5	ρ = 0.1&0.75	ρ = 0.5&0.5	ρ = 0.5&0.75
No Spatial Heterogeneity					
HT	−0.08	−0.07	−0.09	−0.08	−0.10
LRMA	−0.07	−0.02	−0.01	−0.03	0.00
GWRMA	−0.05	0.00	0.00	−0.02	0.01
Moderate Spatial Heterogeneity					
HT	−0.04	−0.02	−0.03	−0.04	−0.08
LRMA	0.00	−0.03	−0.09	−0.03	−0.12
GWRMA	0.12	0.05	0.02	0.02	−0.06
Strong Spatial Heterogeneity					
HT	0.00	−0.01	−0.01	−0.02	−0.07
LRMA	0.04	−0.05	−0.12	−0.03	−0.14
GWRMA	0.16	−0.15	−0.17	−0.08	−0.29

Table 3
Relative bias (%) of the HT, LRMA and GWRMA estimators for simple random sampling (n = 20, n = 100 and n = 250) from the constructed populations with different spatial heterogeneity and correlation (ρ) between Y and one auxiliary variable.

	n = 20				n = 100				n = 250			
	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75
No Spatial Heterogeneity												
HT	−0.36	−0.36	−0.36	−0.40	−0.08	−0.04	−0.08	−0.11	−0.03	−0.03	−0.04	−0.05
LRMA	−0.32	−0.27	−0.24	−0.12	−0.07	−0.03	−0.06	−0.08	−0.02	−0.02	−0.02	−0.02
GWRMA	−0.32	−0.27	−0.20	−0.12	−0.05	−0.01	−0.05	−0.07	−0.02	−0.02	−0.01	−0.02
Moderate Spatial Heterogeneity												
HT	−0.20	−0.17	−0.16	−0.16	−0.04	−0.01	−0.04	−0.07	−0.01	−0.01	−0.02	0.00
LRMA	−0.08	−0.08	−0.12	−0.08	0.00	0.03	0.01	−0.03	0.00	0.00	−0.02	0.01
GWRMA	0.16	0.17	−0.20	−0.20	0.09	0.12	0.01	−0.03	0.09	0.08	0.00	−0.01
Strong Spatial Heterogeneity												
HT	−0.12	−0.14	−0.16	−0.16	0.00	−0.02	−0.02	−0.05	−0.01	−0.01	−0.01	0.00
LRMA	0.04	−0.38	−0.32	−0.20	0.05	0.02	0.01	0.00	0.01	−0.03	−0.01	0.00
GWRMA	0.28	−0.34	−0.24	−0.24	0.14	0.11	0.02	−0.07	0.12	0.04	0.04	−0.05

improved precision relative to the HT estimator and the improvement increased as the correlation between Y and X increased (Table 5). Also as expected, the RE of GWRMA was nearly the same as the RE of LRMA when there was no spatial heterogeneity.

For the moderate and strong spatial heterogeneity populations, GWRMA provided a reduction in standard error relative to LRMA (Table 5, Fig. 2). The reduction in standard error achieved by GWRMA was greater for the stronger spatial heterogeneity populations. Even for n = 20, GWRMA was advantageous to LRMA as the ratios of the GWRMA standard errors divided by the LRMA standard errors were between 0.8 and 0.9. These ratios decreased to a range of 0.5 to 0.7 for n = 250 indicating a substantial precision advantage accruing to GWRMA (Fig. 2). The precision advantage of GWRMA relative to LRMA diminished slightly as the population correlation increased as evidenced by the increasing slopes of the standard error ratios with increasing correlation (Fig. 2).

The advantage of GWRMA relative to LRMA was greater for the larger sample sizes. This result is consistent with the expectation that using GWR improves the prediction of y_i by allowing the parameters of the assisting regression model to vary spatially. For GWRMA, increasing the sample size has the usual benefit of reducing the standard error via the larger divisor (n) in the denominator of equation [7], but also gains an additional benefit of smaller squared residuals due to the better local prediction of y_i. This latter advantage is attributable to the larger sample size providing more sample data to better fit the local regressions of the GWRMA estimator.

The results for the two auxiliary variable populations further confirmed the finding that both LRMA and GWRMA improved precision relative to the HT estimator. Further, compared to LRMA, the GWRMA estimator achieved noticeably greater efficiency gains in populations exhibiting moderate and strong spatial heterogeneity (Table 6). The reduction in standard error achieved by GWRMA relative to LRMA was greater for the stronger spatial heterogeneity populations. The SE (GWRMA)/SE(LRMA) ratios for the strong heterogeneity populations were smaller than the ratios for the moderate spatial heterogeneity populations with the exception of equal ratios for the population with correlation of 0.5 & 0.75.

3.3. Properties of variance estimators

The relative bias of the standard error estimators was quantified by comparing the simulated expected values of the standard error estimators to the simulated standard errors (Table 7). The absolute value of the relative bias of the HT standard error estimator was less than 4 % for all sample sizes and populations, consistent with the theory that the HT variance estimator (equation [4]) is unbiased. The generally recommended variance estimator (equation [8]) for model-assisted estimation was used for the LRMA and GWRMA estimators. We denote this conventional variance estimator for the GWRMA estimator as vGWRMA because we later introduce two alternative variance estimators for use with the GWRMA estimator. The absolute value of the relative bias of the standard error estimator for LRMA was smaller than 5 % for the larger sample sizes (n = 100 and n = 250), but underestimated the standard error by as much as 8 % for the smaller sample size (n = 20). The bias of vGWRMA as an estimator of the variance of the GWRMA estimator was problematic as severe underestimation occured. The underestimation was particularly large for the smallest sample size (n = 20), but the relative bias was 15–26 % even for n = 100 and n = 250 (Table 7). The magnitude of the underestimation of vGWRMA was dramatically reduced by increasing the sample size from n = 20 to n = 100, but the marginal improvement in relative bias was much less when increasing the sample size from n = 100 to n = 250, ruling out the possibility that the underestimation would dissipate quickly with larger sample sizes.

To remedy the substantial underestimation of vGWRMA, we evaluated two alternative variance estimators. The first alternative was to use the LRMA variance estimator to estimate the variance of GWRMA. This variance estimator, denoted vGWRMA_LRMA, will overestimate the GWRMA variance because the LRMA variance always exceeded the GWRMA variance in populations with moderate or strong spatial heterogeneity (Table 5). The second alternative variance estimator, denoted vGWRMA_Avg, was the average of the GWRMA and LRMA variance estimators. The rationale for vGWRMA_Avg is that the variance of the GWRMA estimator will fall somewhere between the underestimate given by vGWRMA and the overestimate of vGWRMA_LRMA.

From the simulation study, the magnitude of the overestimation of the vGWRMA_LRMA variance estimator was so large for moderate and strong spatial heterogeneity (Table 7) that this estimator would not effectively reflect the improvement in precision gained by the GWRMA estimator relative to LRMA. The vGWRMA_Avg variance estimator was still biased, but the bias was much improved relative to the vGWRMA and vGWRMA_LRMA variance estimators. For the smallest sample size (n = 20), vGWRMA_Avg still underestimated the standard error of the GWRMA estimator, but the relative bias ranged from −6 % to −11 %. For the larger sample sizes, vGWRMA_Avg achieved the intended objective of converting the bias to overestimation of the GWRMA variance rather than the underestimation characteristic of the conventional vGWRMA variance estimator (equation [8]). The relative bias of vGWRMA_Avg ranged from 20–45 % for the strong spatial heterogeneity populations and from 10–20 % for the moderate spatial heterogeneity populations. The overestimation of vGWRMA_Avg was approximately half of the overestimation that occurred if vGWRMA_LRMA would be used to estimate the standard error of the GWRMA estimator. Consequently, the sample-based variance estimator vGWRMA_Avg was successful at reflecting some of the precision improvement gained by using the GWRMA estimator relative to the LRMA estimator.

The properties of the variance estimators for the two auxiliary variable populations (Table 8) were consistent with those of the one variable populations: 1) the variance estimator the LRMA estimator had smaller relative bias compared to the variance estimators of the GWRMA estimator; 2) vGWRMA severely underestimated the variance of the GWRMA estimator; 3) vGWRMA_LRMA severely overestimated the variance of the GWRMA estimator; and 4) vGWRMA_Avg was the best

Table 5
Relative efficiency (RE) of the LRMA and GWRMA estimators for the one auxiliary variable populations, where RE is the ratio of the standard error of the model-assisted estimator divided by the standard error of the HT estimator (RE < 1 indicates better precision of the model-assisted estimator).

	n = 20				n = 100				n = 250			
	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75
No Spatial Heterogeneity												
LRMA	1.02	0.99	0.91	0.72	0.99	0.94	0.84	0.61	0.99	0.95	0.86	0.65
GWRMA	1.04	1.00	0.92	0.72	0.99	0.94	0.85	0.62	1.00	0.95	0.86	0.65
Moderate Spatial Heterogeneity												
LRMA	1.02	0.98	0.90	0.68	1.00	0.97	0.89	0.66	1.00	0.96	0.88	0.66
GWRMA	0.89	0.85	0.80	0.60	0.71	0.69	0.66	0.49	0.66	0.64	0.61	0.46
Strong Spatial Heterogeneity												
LRMA	1.02	1.01	0.90	0.70	1.00	0.97	0.90	0.70	1.00	0.96	0.88	0.67
GWRMA	0.82	0.83	0.73	0.58	0.58	0.58	0.54	0.43	0.52	0.53	0.49	0.39

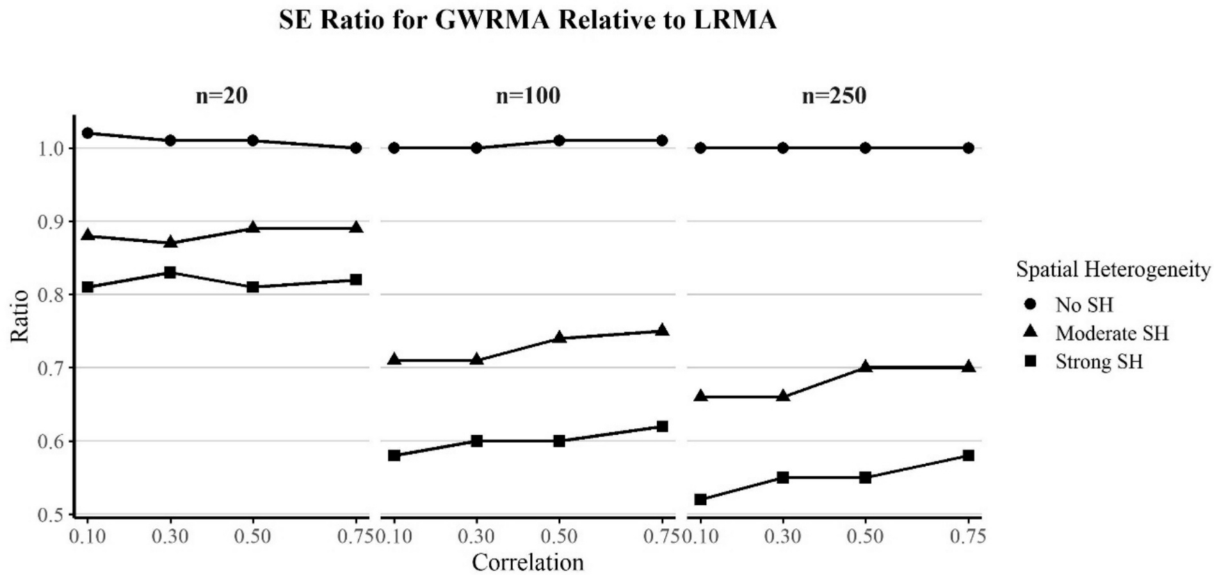

Fig. 2. Ratio of the standard error of GWRMA relative to the standard error of LRMA (ratios < 1 favor GWRMA) as a function of spatial heterogeneity (SH) and correlation between X and Y.

Table 6
Relative efficiency (RE) of the LRMA and GWRMA estimators (n = 100) for the two auxiliary variable populations, where RE is the ratio of the standard error of the model-assisted estimator divided by the standard error of the HT estimator. RE < 1 indicates better precision of the model-assisted estimator. The **bold** ratio is the standard error of the LRMA estimator relative to the GWRMA estimator, SE (GWRMA)/SE(LRMA), which is provided to directly compare the two model-assisted estimators. A ratio less than 1 indicates a precision advantage of GWRMA relative to LRMA.

	ρ = 0.1&0.1	ρ = 0.1&0.5	ρ = 0.1&0.75	ρ = 0.5&0.5	ρ = 0.5&0.75
No Spatial Heterogeneity					
LRMA	0.99	0.87	0.67	0.73	0.45
GWRMA	0.99	0.87	0.67	0.74	0.46
SE(GWRMA)/ SE(LRMA)	**1.00**	**1.00**	**1.00**	**1.01**	**1.02**
Moderate Spatial Heterogeneity					
LRMA	1.00	0.87	0.66	0.77	0.48
GWRMA	0.73	0.71	0.52	0.59	0.22
SE(GWRMA)/ SE(LRMA)	**0.73**	**0.82**	**0.79**	**0.77**	**0.46**
Strong Spatial Heterogeneity					
LRMA	1.00	0.85	0.66	0.77	0.54
GWRMA	0.59	0.57	0.42	0.49	0.25
SE(GWRMA)/ SE(LRMA)	**0.59**	**0.67**	**0.64**	**0.64**	**0.46**

performing variance estimator for the GWRMA estimator.

For the populations with no spatial heterogeneity, the variance estimators for both the LRMA and GWRMA estimators yielded confidence interval coverage just below the nominal 90 % except when the sample size was n = 20. For the populations with moderate and high spatial heterogeneity, observed coverage of intervals estimated using LRMA was generally below 90 %, but within 5 % of the nominal 90 %. The bias of vGWRMA impacted the observed coverage. The negative bias of vGWRMA translated to undercoverage, and this undercoverage was worse for the stronger spatial heterogeneity populations. The positive bias of vGWRMA_Avg for n = 100 and n = 250 translated to overcoverage of the confidence intervals constructed using this variance estimator. However, the observed coverage from vGWRMA_Avg did not exceed 95 % for the moderate spatial heterogeneity populations, and the maximum coverage was 98 % for one of the strong spatial heterogeneity populations (Table 9).

The LRMA and GWRMA estimators yielded shorter length intervals than the HT estimator due to the precision gained by the model-assisted estimators from incorporating auxiliary information (Appendix Table A1). Of particular interest is the comparison of the average length of intervals based on the LRMA estimator versus the GWRMA estimator where the latter intervals use the vGWRMA_Avg variance estimator. The confidence intervals estimated using the GWRMA estimator with vGWRMA_Avg yielded equal or shorter length confidence intervals than the LRMA estimator for all populations having a single auxiliary variable (Table 10). The advantage of GWRMA increased with increasing spatial heterogeneity, but only minor variation in the advantage occurred over increasing correlation between Y and X_1 and increasing sample size. The average lengths of the GWRMA confidence intervals were between 77 % and 85 % of the average length of the LRMA confidence intervals for the moderate and strong spatial heterogeneity populations (Table 10).

For the two auxiliary variable populations (Table 11), coverage was generally closer to the nominal 90 % than was the case for the one auxiliary variable populations. Average interval lengths were shorter for the two auxiliary variable populations (Appendix Table A2) relative to the one variable populations, reflecting the precision gain from incorporating an additional auxiliary variable. Severe undercoverage was again observed when the vGWRMA variance estimator was applied to the moderate and strong spatial heterogeneity populations. However, vGWRMA_Avg remedied this undercoverage and produced intervals of shorter average length (Table 12) than the intervals estimated using the LRMA estimator. Thus a strategy using the GWRMA estimator and vGWRMA_Avg outperformed the strategy using the LRMA estimator.

4. Discussion

Model-assisted estimators employing auxiliary variables constructed from remote sensing data have provided a successful strategy for improving precision of sample-based estimates in a wide variety of settings. Increasing the precision of forest attribute estimates via model-assisted estimation has been a particular focus of forest inventory monitoring (Westfall et al., 2022). Several model-assisted estimators, such as post-stratified estimators, ratio estimators, and generalized regression estimators, have been developed and implemented at different regional scales in the FIA program (McConville et al., 2020; Moisen et al., 2020). These estimators improve precision compared to

Table 7
Relative bias (%) of standard error estimators for HT, LRMA and GWRMA estimators of the constructed populations with one auxiliary variable. Relative bias is the difference between the simulated expected value of the standard error estimator and the simulated standard error divided by the simulated standard error (x100%). The standard error estimators based on vGWRMA, vGWRMA_Avg, and vGWRMA_LRMA are all options applied to the GWRMA estimator.

	n = 20				n = 100				n = 250			
	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75
No Spatial Heterogeneity												
HT	3	3	3	4	−4	−4	−4	−4	−3	−3	−3	−2
LRMA	−2	−3	−5	−8	−4	−3	−2	3	−3	−3	−3	−2
vGWRMA	−11	−11	−12	−14	−6	−5	−4	0	−4	−4	−3	−2
vGWRMA_Avg	−7	−8	−9	−11	−5	−4	−4	1	−4	−4	−3	−2
vGWRMA_LRMA	−4	−5	−6	−8	−4	−4	−3	2	−3	−3	−3	−2
Moderate Spatial Heterogeneity												
HT	1	1	1	0	1	1	1	1	−4	−4	−4	−2
LRMA	−4	−4	−5	−7	0	−1	−2	−1	−5	−5	−5	−4
vGWRMA	−31	−31	−30	−30	−18	−19	−20	−20	−15	−16	−16	−16
vGWRMA_Avg	−9	−8	−9	−11	15	14	10	9	18	17	13	14
vGWRMA_LRMA	10	10	8	5	41	39	33	32	43	43	37	38
Strong Spatial Heterogeneity												
HT	1	−1	1	0	2	2	2	1	−3	−3	−3	−2
LRMA	−4	−7	−5	−7	1	0	−2	−2	−4	−4	−4	−2
vGWRMA	−39	−40	−37	−37	−23	−22	−24	−26	−20	−20	−20	−22
vGWRMA_Avg	−6	−9	−6	−9	35	30	27	23	42	36	34	31
vGWRMA_LRMA	19	12	16	13	75	67	63	57	84	75	73	68

Table 8
Relative bias (%) of standard error estimators for the HT, LRMA and GWRMA estimators of the constructed populations with two auxiliary variables. Relative bias is the difference between the simulated expected value of the standard error estimator and the simulated standard error divided by the simulated standard error (x100%). The standard error estimators based on vGWRMA, vGWRMA_Avg, and vGWRMA_LRMA are all options applied to the GWRMA estimator. Sample size is n = 100.

	ρ = 0.1&0.1	ρ = 0.1&0.5	ρ = 0.1&0.75	ρ = 0.5&0.5	ρ = 0.5&0.75
No Spatial Heterogeneity					
HT	−4	−3	−3	−4	−4
LRMA	−5	−5	−5	−4	−4
vGWRMA	−7	−7	−7	−7	−6
vGWRMA_Avg	−6	−6	−6	−6	−5
vGWRMA_LRMA	−5	−5	−5	−5	−4
Moderate Spatial Heterogeneity					
HT	1	1	1	1	1
LRMA	−1	0	0	−1	0
vGWRMA	−21	−19	−19	−21	−40
vGWRMA_Avg	11	4	7	7	60
vGWRMA_LRMA	36	23	28	29	119
Strong Spatial Heterogeneity					
HT	2	2	1	2	1
LRMA	0	2	2	−1	−1
vGWRMA	−26	−22	−25	−25	−40
vGWRMA_Avg	31	20	25	23	57
vGWRMA_LRMA	70	51	60	57	113

the HT estimator.

Because of the spatially varying nature of the relationship between the response and auxiliary variables of geographical populations, further improvements in the precision of model-assisted estimators may be obtained if the assisting model linking the X and Y variables incorporates the spatial heterogeneity of the model regression coefficients. An assisting model such as one based on GWR (Liu et al., 2018) would improve prediction of y_i in such circumstances. Because the variance of a model-assisted estimator depends on the magnitude of the squared residuals, $\left(y_i - \widetilde{y}_i\right)^2$ (equation [7]), the crucial feature for the model is to generate a better prediction of y_i. The simulation results demonstrated that GWRMA was able to achieve this by taking into account variability of the coefficients of the assisting model over space. For populations with a single auxiliary variable, the GWRMA standard errors were 60–75 % of the magnitude of the LRMA standard errors for moderate and strong spatial heterogeneity when the sample size was n = 100, and 55–70 % of the magnitude of the LRMA standard errors when the sample size was n = 250 (Fig. 2). Similar substantial improvements relative to the precision of the LRMA estimator were obtained by applying the GWRMA estimator to populations with two auxiliary variables. Both the LRMA estimator and the GWRMA estimator can use an auxiliary variable to advantage to reduce the standard error, but the GWRMA estimator can extract greater benefit from the auxiliary variable than the LRMA estimator as the sample size increases (Fig. 2, Table 5).

The conventional model-assisted variance estimator (equation [8]) resulted in underestimation of the standard error of both LRMA and GWRMA estimators. McConville et al. (2020) reported similar underestimation of variance for the LRMA estimator as the assisting model increased in complexity (i.e., more predictors added to the model). GWRMA would qualify as a more complex model due to allowing the coefficients of the model to vary spatially. In our study, we found equation [8] underestimated the LRMA variance for the smaller sample size (n = 20) with larger underestimation of 20–40 % observed for the GWRMA estimator. The variance estimator averaging the estimated GWRMA and LRMA variances (vGWRMA_Avg) remedied the severe underestimation of vGWRMA. Although the overestimation of vGWRMA_Avg fails to reflect the full precision gain achieved by GWRMA, it is preferable to overestimate uncertainty than to under report it. Similarly, confidence interval coverage that exceeds the nominal confidence level is preferable to a confidence interval strategy that undercovers. The vGWRMA_Avg variance estimator combined with the GWRMA estimator resulted in confidence intervals with greater than the nominal 90 % coverage, but did so while achieving average confidence interval length shorter than the length of the confidence intervals from the LRMA estimator. Consequently, the precision advantage of the GWRMA estimator relative to LRMA estimator was captured by the shorter length confidence intervals estimated using vGWRMA_Avg.

The choice of a model in model-assisted estimators can be flexible depending on the response variable and auxiliary information. A better fitting model will improve the precision of the model-assisted estimator. However, design-based inference is not dependent on the quality of the fit of the regression model linking *X* to *Y* and model-assisted estimators are approximately design unbiased regardless of the model chosen (Särndal et al., 1992, Sec. 6.7). For example, if the actual relationship between Y and X is closely approximated by a quadratic polynomial, using a linear functional relationship in the model-assisted estimator (e.g., equation [10]) will yield less of a precision improvement than would

Table 9
Confidence interval coverage for nominal 90% intervals using the HT, LRMA, and GWRMA estimators for the two auxiliary variable populations. Two variance estimators were evaluated for use with the GWRMA estimator, vGWRMA and vGWRMA_Avg.

	n = 20				n = 100				n = 250			
	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75
No Spatial Heterogeneity												
HT	90	90	90	90	88	88	87	87	89	88	88	89
LRMA	88	87	87	85	88	88	89	91	89	88	88	90
vGWRMA	83	82	82	82	87	87	88	90	88	88	88	90
vGWRMA_Avg	85	85	84	83	87	88	88	90	88	88	88	90
Moderate Spatial Heterogeneity												
HT	88	89	89	89	90	90	91	90	89	89	89	90
LRMA	87	87	86	85	90	90	89	88	88	87	88	88
vGWRMA	70	69	72	70	82	81	83	80	83	84	83	82
vGWRMA_Avg	85	86	84	84	94	94	93	92	95	95	94	95
Strong Spatial Heterogeneity												
HT	89	89	89	89	90	90	90	89	88	88	88	89
LRMA	87	88	87	86	89	90	90	90	88	88	88	89
vGWRMA	63	65	66	66	79	80	78	77	80	81	80	79
vGWRMA_Avg	87	86	86	85	96	96	96	96	98	97	97	97

Table 10
Comparison of the average length of 90 % confidence intervals estimated using the GWRMA estimator and vGWRMA_Avg with intervals estimated using the LRMA estimator for the one auxiliary variable populations. Ratios shown are average length estimated using GWRMA versus average length estimated using LRMA. Ratios < 1 favor the GWRMA estimator (shorter average length). SH = spatial heterogeneity.

	n = 20				n = 100				n = 250			
	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75
No SH	0.97	0.96	0.97	0.97	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00
Moderate SH	0.83	0.83	0.84	0.85	0.82	0.82	0.82	0.83	0.82	0.82	0.83	0.82
Strong SH	0.79	0.81	0.80	0.81	0.77	0.78	0.78	0.78	0.77	0.77	0.78	0.77

Table 11
Confidence interval coverage for nominal 90 % intervals using the HT, LRMA, and GWRMA estimators for the two auxiliary variable populations (n = 100). Two variance estimators were evaluated for use with the GWRMA estimator, vGWRMA and vGWRMA_Avg.

	ρ = 0.1&0.1	ρ = 0.1&0.5	ρ = 0.1&0.75	ρ = 0.5&0.5	ρ = 0.5&0.75
No Spatial Heterogeneity					
HT	88	89	90	87	88
LRMA	88	88	88	88	88
vGWRMA	87	87	87	87	87
vGWRMA_Avg	87	87	87	87	88
Moderate Spatial Heterogeneity					
HT	90	90	90	91	90
LRMA	90	89	90	89	89
vGWRMA	79	81	81	79	60
vGWRMA_Avg	93	91	92	92	99
Strong Spatial Heterogeneity					
HT	90	90	90	91	90
LRMA	89	91	91	89	89
vGWRMA	78	79	79	79	61
vGWRMA_Avg	96	95	96	92	99

be achieved by a quadratic model, but model-assisted estimator using the linear model would still be design-unbiased. The assisting model may include an assumed variance structure, but Särndal et al. (1992, p. 239) emphasize that the importance of the variance structure "lies above all in the planning of the survey" (i.e., optimal choice of sampling design).

A consequence of the role of the model in design-based inference is that assumptions that are critical to evaluate when the objective is to apply GWR to model spatial nonstationarity (e.g., diagnostics related to residual spatial autocorrelation, local multicollinearity, and effective number of neighbors) have minor impact when using GWR in the framework of model-assisted estimation and design-based inference. The key feature of the GWR model used to reduce variance in design-based inference (see eqn. [7]) is how well GWR predicts y_i for each element i of the population. This raises another direction of future research. We did not evaluate other spatially explicit methods (e.g., Generalized Additive Models with spatial splines, multiscale GWR, and Bayesian spatially varying coefficient models) or other prediction methods such as random forest (Dagdoug et al., 2023, Esteban et al., 2019). It would be interesting to compare these methods with the simple GWRMA estimator we examined and to assess the variance estimators for these other strategies in terms of their bias and confidence interval properties.

In general, model-assisted estimation always involves a choice regarding how the prediction $\widetilde{y}_i$ is obtained. For example, Massey and Mandallaz (2015b) included a comparison of regression models versus k nearest neighbors (kNN) for estimating net change in forest inventories and concluded that both kNN and linear models offered similar

Table 12
Comparison of the average length of 90 % confidence intervals estimated using the GWRMA estimator and vGWRMA_Avg with intervals estimated using the LRMA estimator for the two auxiliary variable populations. Ratios shown are average length estimated using GWRMA versus average length estimated using LRMA. Ratios < 1 favor the GWRMA estimator (shorter average length). SH = spatial heterogeneity, and n = 100.

	ρ=0.1&0.1	ρ=0.1&0.5	ρ=0.1&0.75	ρ=0.5&0.5	ρ=0.5&0.75
No SH	0.99	0.99	0.99	0.99	0.99
Moderate SH	0.81	0.85	0.84	0.83	0.74
Strong SH	0.77	0.80	0.78	0.78	0.73

magnitudes of improvement in precision. Massey and Mandallaz (2015b) noted that an advantage of the classical regression estimators was that the analytical variance estimates were found to be valid in their study and these variance estimates are exactly reproducible, whereas bootstrap variance estimators do not possess this feature. Comparisons of the type carried out by Massey and Mandallaz (2015b) but extended to the spatial realm would be needed to confirm if other prediction methods would improve precision relative to GWRMA. Model-assisted estimation continues to be an active area of research. For example, Hou et al. (2023) extended univariate model-assisted estimation to a multivariate approach in which multiple forest attributes are predicted simultaneously for remotely sensed auxiliary variables. As another example, Andersen et al. (2024) compared an assortment of model-assisted estimators and their associated variance estimators for a two-stage sampling design. This work addressed the problem of estimating domain parameters such as total biomass of trees in forestland and average tree biomass per hectare of forestland using airborne remote sensing data as the auxiliary variables.

Magnussen and Nord-Larsen (2021) proposed an interesting alternative strategy to account for spatial heterogeneity in regression models by incorporating spatial model strata. This approach requires dividing the population into strata and applying a model-assisted estimator within each stratum (i.e., latent model subgroup) thus allowing the regression coefficients to vary spatially. GWR offers an alternative practical and efficient way to provide better prediction than a non-spatially varying model (Charlton and Fotheringham, 2009; Zhang et al., 2009) because of its ability to capture the potential space-varying relationships between Y and X variables across the domain of interest and correctly partitioning residual uncertainty in the model. Further, GWR has been in use for more than two decades and implementing the estimator is feasible for users through GWR software, R or Python packages and ArcGIS. Therefore, choosing this model as an assisting model in design-based, model-assisted estimation becomes a viable practical option for incorporating spatial heterogeneity.

Simple random sampling combined with a model-assisted estimator represents a common strategy in practice. More complex strategies, for example two-phase sampling, are sometimes implemented. For two-phase systematic sampling, Opsomer et al. (2007) developed model-assisted estimators that use nonparametric regression. In their approach, an initial sample of units is selected and the auxiliary information is available for all first-phase sample units. Then a phase-two sample is randomly selected from the phase-one sample. The GWRMA estimator would be applicable to two-phase sampling by using the GWR model to obtain the model predictions of the target variable for all phase-one sample units. The corresponding variance estimator in two-phase sampling will be more complex than one-phase (McRoberts et al., 2024). Paul et al. (2024) evaluated the GWRMA estimator under a two-phase sampling design and presented the approximate variance and the estimate of variance of GWRMA estimator. The precision of GWRMA was compared to LRMA through a simulation study, but Paul et al. (2024) did not evaluate the bias of their proposed variance estimator. Development of an unbiased variance estimator of GWRMA for two-phase sampling is another important extension of the methodology.

The GWRMA estimator, which takes into account spatial heterogeneity of the relationship between the target variable and auxiliary information in the assisting model, offers a simple, practical way to improve precision relative to the HT estimator and the LRMA estimator. This simplicity extends to variance estimation as more complex models may require more complex variance estimators (Massey and Mandallaz, 2015a). Although GWR has previously been proposed (Liu et al., 2018), it remains an underutilized option for more fully extracting information from remote sensing auxiliary data to decrease standard errors of sample-based estimates. In contrast to the more complex sampling designs used in previous studies of GWR in model-assisted estimation (Liu et al., 2018, Paul et al., 2024), we focused on simple random sampling. Practitioners may more confidently implement the GWRMA estimator when working with a simpler, commonly used sampling design. Relative to previous studies, our simulation study expanded the range of population conditions evaluated when comparing the precision of GWRMA to LRMA estimators.

5. Conclusion

From our results spanning a variety of constructed populations and range of sample sizes, we can conclude the following: (1) the GWRMA estimator, similar to the LRMA estimator, is asymptotically design-unbiased; (2) the GWRMA estimator yields better precision than the LRMA estimator when the population has spatial heterogeneity; (3) the improvement of the precision of the GWRMA estimator relative to the LRMA estimator is greater in more spatially heterogeneous populations; (4) the precision improvement achieved by the GWRMA estimator relative to the LMRA estimator increases with increasing sample size; (5) the conventional variance estimator suggested for model-assisted estimators tends to severely underestimate the variance of the GWRMA estimator, especially for small sample size ($n = 20$); (6) the GWRMA_Avg variance estimator resolved the severe underestimation of the conventional model-assisted variance estimator, and coverage and length of confidence intervals constructed using GWRMA_Avg would yield confidence intervals with moderate overcoverage but shorter average length than confidence intervals obtained using the LRMA estimator; and (7) although the confidence interval coverage and average length indicate that GWRMA_Avg variance is a serviceable option, development of a less biased variance estimator with a more rigorous theoretical basis is needed to ensure the full utility of the GWRMA estimator.

CRediT authorship contribution statement

Dingfan Xing: Writing – original draft, Validation, Software, Methodology, Formal analysis, Conceptualization. **Nicholas N. Nagle:** Methodology, Funding acquisition. **Stephen V. Stehman:** Writing – review & editing, Validation, Supervision, Methodology, Conceptualization. **Todd A. Schroeder:** Resources, Project administration, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix Table A1. **Average length of nominal 90 % confidence intervals estimated using the HT, LRMA, and GWRMA estimators for the one auxiliary variable populations. Two variance estimators are evaluated for use with the GWRMA estimator, vGWRMA and vGWRMA_Avg.**

	n = 20				n = 100				n = 250			
	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75	ρ = 0.1	ρ = 0.3	ρ = 0.5	ρ = 0.75
No Spatial Heterogeneity												
HT	364	365	365	389	162	162	162	172	102	102	102	108
LRMA	353	339	306	249	160	154	140	113	101	97	88	71
vGWRMA	328	315	286	232	158	151	137	111	101	96	87	70
vGWRMA_Avg	341	327	297	241	159	153	139	112	101	97	88	71
Moderate Spatial Heterogeneity												
HT	651	651	651	654	290	290	290	291	182	182	182	183
LRMA	632	605	553	414	287	275	251	189	181	174	158	119
vGWRMA	395	376	359	275	168	160	152	114	107	102	97	72
vGWRMA_Avg	526	504	466	352	235	225	207	156	149	142	131	98
Strong Spatial Heterogeneity												
HT	911	903	911	921	406	406	405	410	255	255	255	258
LRMA	882	840	773	606	402	385	351	275	253	243	221	173
vGWRMA	451	450	421	339	177	179	164	130	110	111	102	80
vGWRMA_Avg	700	678	622	491	310	300	274	215	195	188	172	134

Appendix Table A2. **Average length of nominal 90 % confidence intervals estimated using the HT, LRMA, and GWRMA estimators for the two auxiliary variable populations. Two variance estimators are evaluated for use with the GWRMA estimator, vGWRMA and vGWRMA_Avg. Sample size is n = 100.**

	ρ = 0.1 & 0.1	ρ = 0.1 & 0.5	ρ = 0.1 & 0.75	ρ = 0.5 & 0.5	ρ = 0.5 & 0.75
No Spatial Heterogeneity					
HT	162	162	162	162	162
LRMA	159	139	106	119	74
vGWRMA	156	136	104	117	72
vGWRMA_Avg	157	137	105	118	73
Moderate Spatial Heterogeneity					
HT	290	290	290	290	291
LRMA	285	249	190	219	140
vGWRMA	164	165	120	134	39
vGWRMA_Avg	232	211	159	182	103
Strong Spatial Heterogeneity					
HT	406	406	406	405	407
LRMA	398	345	268	305	218
vGWRMA	175	178	126	146	61
vGWRMA_Avg	307	275	209	239	160

Data availability

Data will be made available on request.

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