



Biomass inventory optimization for willow-based pellet production integrating seasonal supply and stochastic demand

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ABSTRACT

Biomass supply chains face both internal challenges (low energy density, bulkiness, seasonality) and external challenges (market uncertainty). We developed a two-stage stochastic mixed-integer linear programming (MILP) model that configures an (s, S) inventory control policy for a willow-based pellet production facility. This model accounts for seasonal willow harvesting capacity and stochastic daily wood pellet demand. Using a case study of a proposed wood pellet production facility in Schenectady County, New York, USA, we evaluated model-suggested inventory control policies using multiple out-of-sample pellet demand scenarios by assuming a moderate demand uncertainty with a 10% coefficient of variation. Inventory policies acquired from modeling stochastic demands resulted in a 1% mean cost reduction compared to the deterministic approach. More importantly, these policies reduced cost variability by 44% under moderate demand uncertainty. We expected the performance advantage of the stochastic model to increase as uncertainty levels rise. Test cases show that frequent inventory policy adjustments provide additional cost savings. The model successfully accounted for seasonal supply constraints and stochastic market demand to facilitate a multi-feedstock strategy that offers additional supply chain resilience and associated cost reduction. Overall, the stochastic modeling framework provides facility managers with more robust inventory planning under real-world constraints. The framework's computational efficiency and broad applicability make it suitable for adoption by diverse biomass industries with uncertainties in their supply chains, particularly those facing seasonal feedstock supply constraints and stochastic end product demands.

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Introduction

Biomass, defined as organic material derived from plants through photosynthesis (McKendry 2002), plays a vital role in the global transition to sustainable energy systems and greenhouse gas reduction (Demirbas 2005). Dedicated woody biomass energy crops, particularly short-rotation woody crops (SRWCs) including willow (*Salix* spp.), and herbaceous crops like switchgrass (*Panicum virgatum*) and miscanthus (*Miscanthus* spp.), have been widely studied in the United States as renewable feedstocks for bioenergy, biofuels, and bioproducts (Zalesny et al. 2011).

The North American wood pellet sector has expanded alongside the global demand for renewable energy. Exports are mainly for Europe and Asia, and the United States has a substantial production share of these exports (Atasoy et al. 2023; Guo et al. 2023). Policy drivers on the demand side, such as the European Union's (EU) Renewable Energy Directive, have incentivized biomass to displace coal. On the supply side, pellet industries rely on varied raw materials, including sawmill residues, logging residues, and low-grade pulpwood (Dale et al. 2017; Brandeis and Abt 2019). Logistical challenges arise from the low energy density of raw biomass and the fragile nature of finished pellets, which can lead to increased handling costs and safety risks (Dafnomilis et al. 2018; Gilvari et al. 2021). The industry's pursuit of consistent pellet quality has triggered

interest in dedicated woody energy crops such as willow (*Salix* spp.) and poplar (*Populus* spp.), which provide more uniform feedstock characteristics compared to the conventional raw materials described above (Lee et al. 2024).

Willow (*Salix* spp.) is a fast-growing SRWC typically harvested on a three- to four-year coppice rotation, yielding approximately eight to ten oven-dry tonnes per hectare per year under low-input cultivation on marginal soils (Stolarski et al. 2011; Zalesny et al. 2011). Beyond bioenergy, willow has been studied and deployed across a range of applications, including biofuels, bioproducts, and land reclamation, and is being actively commercialized in the northeastern United States, where both marginal land availability and biomass demand are substantial (Zalesny et al. 2011; Langholtz et al. 2016). While wood pellets are conventionally produced from either mill residues or processed roundwood, willow offers several advantages for pellet production, for example, high-density growth and efficient harvesting, processing characteristics similar to forest-derived woody biomass, and the potential to utilize marginal agricultural land without competing with food production (Stolarski et al. 2011; Stachowicz and Stolarski 2023). These attributes make willow a promising, dedicated feedstock for consistent-quality pellet manufacture, supporting its growing commercial adoption as a bioenergy crop (Langholtz et al. 2016).

Biomass supply chains often face challenges due to raw material characteristics, such as their bulkiness, high moisture content, and seasonality in supply. They also experience uncertainties, including market volatility, policy shifts, and social factors (Rentizelas et al. 2009; Rentizelas 2013; Gautam et al. 2017). For pellet facilities, inventory control is critical to maintain optimal stock levels to meet production requirements and customer demand while minimizing costs (Chen and Simchi-Levi 2004).

The Economic Order Quantity (EOQ) model is the first formal inventory model and provides the foundation for many other models (Harris 1990). Extensions of EOQ include other deterministic models, followed by stochastic inventory theory that integrates uncertainties in product demand (Arrow et al. 1951; Dvoretzky et al. 1952a, 1952b). Stochastic inventory theory has adopted policy-based inventory control, such as the (s, S) policy, whose optimality and practical efficiency have been established under broad cost structures and review settings (Axsäter 2015; Perera and Sethi 2023). Under an (s, S) policy, the inventory position is reviewed at the start of each period and a replenishment order is placed only when inventory falls at or below the reorder level (s), with the order quantity raising the inventory position up to the order-up-to level (S) (Axsäter 2015). The variable-quantity replenishment (s, S) policy often outperforms fixed-lot policies when demand is stochastic and fixed order costs exist (Huh and Janakiraman 2008; Akhtari et al. 2019). Despite extensive application of stochastic inventory control (Kim et al. 2011; Kostin et al. 2012; Giarola et al. 2013; Osmani and Zhang 2013, 2014; Shabani and Sowlati 2016; Pham et al. 2022; Fukkwarmdee 2023), its application in biomass supply chains remains limited.

To address these inventory challenges in willow-based pellet systems, we propose a two-stage stochastic mixed integer linear programming (MILP) model framework to optimize inventory control of a hypothetically proposed willow-based wood pellet facility in Schenectady County, New York State, USA. The

model provides optimal inventory control across multiple simulated stochastic pellet demand scenarios using the (s, S) inventory control policy.

This study responds to a clear research gap: while stochastic optimization has been applied to biofuel and biomass logistics design, willow-based pellet facilities' inventory management under seasonal supply and stochastic demand has not been studied using a two-stage stochastic (s, S) framework (Shabani and Sowlati 2016; Nguyen and Chen 2018). We contribute by: (1) developing a comprehensive MILP framework integrating seasonal supply constraints with stochastic pellet demand, (2) demonstrating and comparing different inventory policy adjustment frequencies for practical implementation, and (3) evaluating multi-feedstock strategies for enhanced supply chain resilience. Collectively, the framework delivers insights for pellet manufacturers on strategic inventory targets and procurement timing under operational constraints, while offering researchers a generalizable approach for seasonal biomass systems grounded in modern stochastic inventory theory. To our knowledge, this is the first application of a two-stage stochastic (s, S) MILP framework to willow-based pellet inventory management. Our objective is to demonstrate that stochastic optimization can reduce overall inventory management cost and uncertainties compared to deterministic approaches.

Materials and methods

As illustrated in Figure 1, a pellet production facility operates as a central node in a biomass energy supply chain. Raw willow biomass is harvested from several supply zones and transported to the facility, where it goes through processing procedures to produce wood pellets to meet the demand of energy markets. The supply of willow raw materials fluctuates seasonally; the demand for wood pellets displays both seasonal trends and random fluctuations.

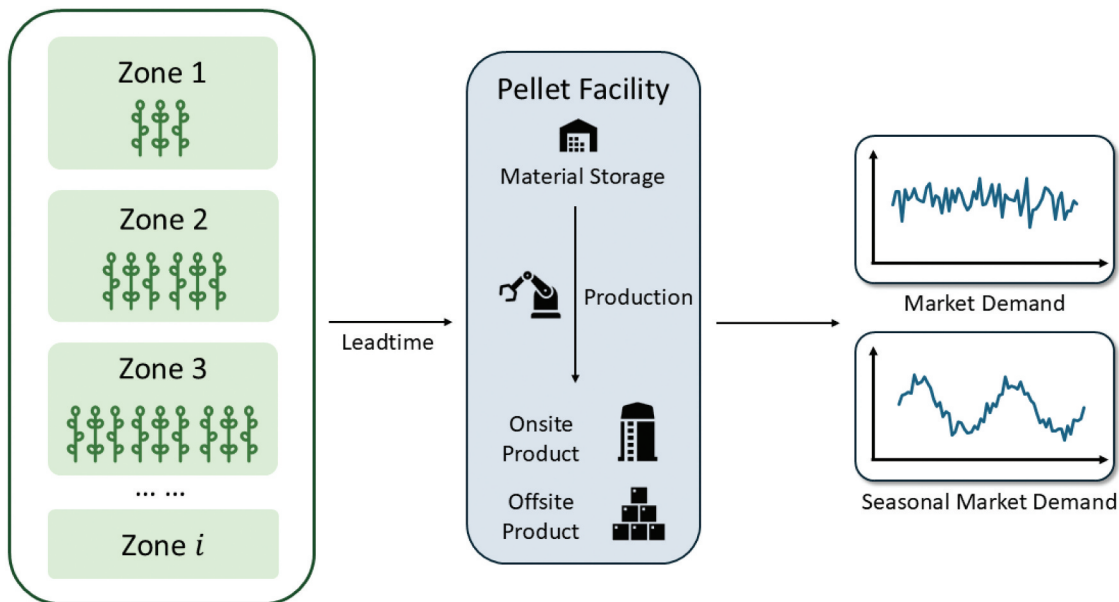


Figure 1. An illustration of a hypothetical willow-based wood pellet supply chain structure. In our tests, we assume that there are three distinct willow raw materials supply zones, one wood pellet facility, and fluctuation of pellet demand between seasons and days across one year.

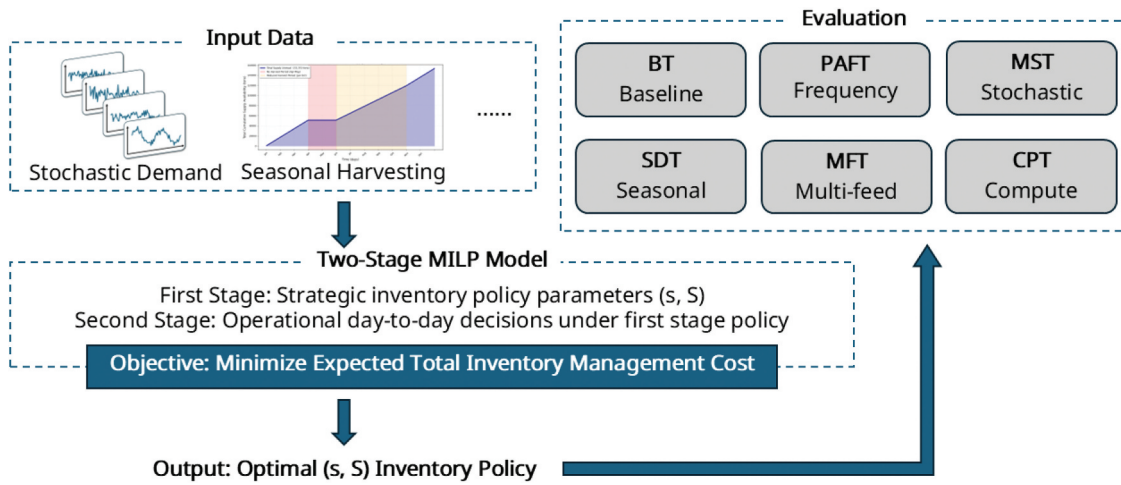


Figure 2. Overview of the two-stage stochastic MILP modeling framework. The framework takes seasonal willow biomass supply and stochastic pellet demand scenarios as inputs, optimizes inventory policy parameters in stage 1, and simulates daily facility operations in stage 2. Model performance is evaluated through six test cases.

Figure 2 provides an overview of the proposed two-stage stochastic MILP modeling framework, illustrating the relationships among supply inputs, stochastic demand scenarios, policy optimization, and evaluation.

Mathematical model

Two-stage stochastic programming and (s, S) inventory control

The proposed model uses a two-stage stochastic programming framework (Birge and Louveaux 2011) that separates the general inventory policy decisions from tactical facilities operational decisions. To be specific, the first stage determines strategic inventory control parameters (s, S), while the second stage handles daily ordering, transportation, production, and storage after stochastic pellets demand scenarios are observed.

The proposed model aims to identify cost-minimal inventory control parameters that remain robust across a range of demand realizations, while explicitly accounting for the seasonal constraints of willow biomass supply. All monetary values are reported in United States Dollars (USD). By separating the policy-setting stage from the operational simulation stage, the framework enables facility managers to pre-commit to inventory targets that perform well on average across uncertain future demand scenarios. The model outputs period-specific (s, S) thresholds that guide daily procurement decisions throughout the operating year.

We assume managers will review the willow raw material inventory at the facility daily. At the start of each day, the current inventory position is compared against the model-derived s and S that have been pre-calculated for the current planning period (e.g., a day, week, or month). A replenishment order is triggered only if the inventory falls at or below s . Material will be reordered if the facility's willow raw material inventory is below s ; the order quantity is calculated by subtracting the current inventory from S . Inventory policy parameters are allowed to be adjusted at different pre-defined frequencies (e.g. daily, weekly, or monthly). The inventory policy could also be built to address seasonal biomass supply

constraints that will be detailed in the mathematical formulation below.

Model notation

The mathematical model uses the notation summarized in Tables 1 and 2. Table 1 presents the indices and parameters that define the problem structure, while Table 2 lists the decision variables organized by their roles in the two-stage stochastic programming framework. Unless otherwise specified, all biomass and pellet quantities are reported in oven-dry tonnes (1 tonne = 1,000 kg).

Mathematical formulation for single feedstock (willow-only model)

The decisions and constraints are organized into two stages: the first-stage policy constraints that determine the inventory control parameters US_m and LS_m , and the second-stage constraints that govern daily inventory management operations under each stochastic pellet demand scenario. The numbering of the second stage constraints uses a format of (*index1.index2*) where *index1* identifies the constraint group and *index2* identifies the sequence of constraints within each group.

Objective function. The objective function (Equation OF) minimizes the expected total annual inventory management cost of the willow-based pellet supply chain to meet the demands across all modeled scenarios ω . It aggregates, for every day t and supply zone i , for each scenario ω , the variable procurement and transport expense $c_i Y_{\omega,t,i}$, the fixed ordering charge $f_i R_{\omega,t,i}$ that is incurred only when the binary variable $R_{\omega,t,i} = 1$, the daily cost of storing raw willow materials within the facility $sH_{\omega,t}$, the costs of storing finished pellets on-site $w_{inner} L_{\omega,t}^{inner}$ and off-site $w_{outer} L_{\omega,t}^{outer}$, and finally the penalty $oB_{\omega,t}$ associated with any lost sales $B_{\omega,t}$ that arise when daily demand exceeds available pellet inventory. Because of the summation of cost and penalty spans across all demand scenarios, we selected a formulation reflecting the risk-neutral decisions and seeking the minimum expected total cost plus penalty. In the further

Table 1. Model indices and parameters.

Category	Symbol	Description
Indices	m	Index of inventory policy adjustment periods
	t or τ	Index of days within a year
	ω	Index of modeled demand scenarios; Ω is the total number of scenarios modeled
	i	Index of willow raw material supply zones
	k, k_{max}	Remaining days for material arrival since the ordering day; k_{max} denotes the total days needed
	u, u_{max}	Remaining days for production completion since the beginning day of production; u_{max} denotes the total days needed
Cost Parameters	c_i	Variable cost of ordering each tonne of willow from zone i
	f_i	Fixed cost each time ordering willow from zone i
	s	Raw material daily storage cost measured by dollar per tonne
	w_{inner}	On-site pellets daily storage cost by dollar per tonne
	w_{outer}	Off-site pellets daily storage cost by dollar per tonne
	o	Lost sales penalty by dollar per tonne of pellets
Capacity Parameters	$capStore$	Raw material storage capacity in the facility in tonnes
	$capInter$	Processing capacity by the facility in tonnes
	$capInner$	On-site finished product storage capacity in tonnes
	$capProc$	Daily procurement capacity in tonnes
Supply Parameters	$yield_i$	Annual supply capacity from zone i measured in tonnes
	$a_{t,i}$	Cumulative raw material harvesting capacity in zone i up to day t since the beginning of each year, derived from annual supply $yield_i$ by applying seasonal harvesting capacity factors
	r	Biomass-to-pellet weight-based conversion ratio, applies to both willow and forest residue
Demand Parameters	$d_{\omega,t}$	Pellet demand (scenario ω , day t)
Other	h_0	A facility's raw material inventory level at the beginning of each year
	l_0	A facility's Initial pellet inventory level at the beginning of each year
	M	A large positive number

Table 2. Decision variables.

Stage	Symbol	Type	Description
First Stage	US_m	Continuous	Upper inventory threshold (period m)
	LS_m	Continuous	Lower inventory threshold (period m)
Second Stage	$N_{\omega,t}$	Binary	Daily raw material reorder decision (scenario ω , day t)
	$R_{\omega,t,i}$	Binary	Daily raw material reorder decision from zone i (scenario ω , day t)
	$Y_{\omega,t,i}$	Continuous	Daily order quantity from zone i (scenario ω , day t)
	$H_{\omega,t}$	Continuous	Daily raw material inventory in a facility (scenario ω , day t)
	$I_{\omega,t}^{inner}$	Continuous	Daily on-site finished product inventory (scenario ω , day t)
	$I_{\omega,t}^{outer}$	Continuous	Daily off-site finished product inventory (scenario ω , day t)
	$L_{\omega,t}$	Continuous	Daily total finished product inventory (scenario ω , day t)
	$B_{\omega,t}$	Continuous	Daily lost sales of pellet (scenario ω , day t)
	$l_{\omega,t,k}$	Continuous	Daily in-transit raw material amount inventory before reaching the facility (scenario ω , day t , k days remaining)
	$P_{\omega,t}$	Continuous	Total daily raw material procurement (scenario ω , day t) from all supply zones
	$Q_{\omega,t,u}$	Continuous	Daily in-process raw material amount before final product (scenario ω , day t , u days remaining)

explanations of this model, unless explicitly stated, we may omit the ω for simplification.

$$\min z = \frac{1}{\Omega} \sum_{\omega} \left\{ \sum_t \left[\sum_i (c_i Y_{\omega,t,i} + f_i R_{\omega,t,i}) + s H_{\omega,t} + w_{inner} I_{\omega,t}^{inner} + w_{outer} I_{\omega,t}^{outer} + o B_{\omega,t} \right] \right\} \quad (OF)$$

Subject to:

First-stage decisions and constraints. The first-stage constraints (1) and (2) enforce $US_m \geq LS_m$, and limit the inventory

policy upper bound level by the facility's total raw material storage capacity $capStore$.

$$US_m \geq LS_m \quad \forall m \quad (1)$$

$$US_m \leq capStore \quad \forall m \quad (2)$$

Second stage decisions and constraints.

Group 1: Daily procurement. Constraints (1.1) to (1.2) describe the daily procurement logic for each scenario. A capital M in (1.1) sets up the logic between the continuous order quantity $Y_{\omega,t,i}$ and the binary decision $R_{\omega,t,i}$; any order amount greater than zero will set the value of $R_{\omega,t,i}$ to be one.

Constraint (1.2) tracks the total willow raw material order quantity on day t from all zones as $P_{\omega,t}$.

$$M \times R_{\omega,t,i} - Y_{\omega,t,i} \geq 0 \quad \forall \omega, t, i \quad (1.1)$$

$$\sum_i Y_{\omega,t,i} - P_{\omega,t} = 0 \quad \forall \omega, t \quad (1.2)$$

Group 2: Seasonal supply. Constraint (2.1) makes sure the accumulative biomass ordered up to day t from supply zone i is no more than the accumulative willow raw material production up to day t from that same zone. To test the impact of willow supply seasonality, we created constraint (2.2) to make sure willow biomass is not harvested beyond the viable harvesting season (e.g. during the critical growth period). Here, $a_{t,i}$ denotes the cumulative raw material harvesting capacity in supply zone i up to day t , as defined in Table 1.

$$\sum_{\tau=0}^t Y_{\omega,\tau,i} \leq a_{t,i} \quad \forall \omega, t, i \quad (2.1)$$

$$Y_{\omega,t',i} = 0 \quad \forall \omega, t' \in \text{during off season}, i \quad (2.2)$$

Group 3: (s, S) inventory implementation. The (s, S) inventory control policy is executed through constraints (3.1) to (3.5). When on-hand raw material stock $H_{\omega,t}$ falls below the lower bound LS_m , binary variable $N_{\omega,t}$ is set to be one by (3.1) and (3.2) triggers reordering. If $N_{\omega,t} = 1$, constraints (3.3) and (3.4) set the replenishment $P_{\omega,t}$ amount to raise the raw material inventory level exactly to the upper bound US_m ; otherwise, constraint (3.5) sets the order amount to be zero when $N_{\omega,t} = 0$. Constraint (3.6) ensures the daily raw material purchase amount does not surpass the daily procurement capacity (due to accessible transportation, etc.) by the facility.

$$M \times N_{\omega,t} \geq LS_m - H_{\omega,t} \quad \forall \omega, t, m \quad (3.1)$$

$$M(1 - N_{\omega,t}) \geq H_{\omega,t} - LS_m \quad \forall \omega, t, m \quad (3.2)$$

$$P_{\omega,t} \geq US_m - H_{\omega,t} - M(1 - N_{\omega,t}) \quad \forall \omega, t, m \quad (3.3)$$

$$P_{\omega,t} \leq US_m - H_{\omega,t} + M(1 - N_{\omega,t}) \quad \forall \omega, t, m \quad (3.4)$$

$$P_{\omega,t} \leq M \times N_{\omega,t} \quad \forall \omega, t \quad (3.5)$$

$$P_{\omega,t} \leq capProc \quad \forall \omega, t \quad (3.6)$$

Group 4: Transportation lead time. Transportation lead time is modeled through constraints (4.1) to (4.3). Constraint (4.1) inserts the freshly ordered material into the k_{max} day queue, and (4.3) advances each shipment one day closer to the arrival date at the facility. Constraint (4.2) makes sure raw material inventory in transit can only come from the initial order amount.

$$I_{\omega,t,k_{max}} = P_{\omega,t} \quad \forall \omega, t \quad (4.1)$$

$$I_{\omega,t,k} = 0 \quad \forall \omega, t \leq k_{max} - 1, k \leq k_{max} - t \quad (4.2)$$

$$I_{\omega,t,k} = I_{\omega,t-1,k+1} \quad \forall \omega, t \geq 2, k \leq k_{max} - 1 \quad (4.3)$$

Group 5: Raw material inventory dynamic. Constraint (5.1) initializes the willow raw material stock at the beginning of each year. Once material reaches the facility, constraint (5.2) updates $H_{\omega,t}$ each day by subtracting the amount of raw material transferred to production from storage and adding the raw materials arrivals to the facility due to reordering.

$$H_{\omega,1} = h_0 - Q_{\omega,1,u_{max}} \quad \forall \omega \quad (5.1)$$

$$H_{\omega,t} = H_{\omega,t-1} - Q_{\omega,t,u_{max}} + I_{\omega,t,1} \quad \forall \omega, t \geq 2 \quad (5.2)$$

Group 6: Production pipeline. The chipping and pelletizing production line imposes a u_{max} day delay that is modeled by $Q_{\omega,t,u}$ and governed by constraints (6.1) and (6.2).

$$Q_{\omega,t,u} = 0 \quad \forall \omega, t \leq u_{max} - 1, u \leq u_{max} - t \quad (6.1)$$

$$Q_{\omega,t,u} = Q_{\omega,t-1,u+1} \quad \forall \omega, t \geq 2, u \leq u_{max} - t \quad (6.2)$$

Group 7: Finished product inventory. Finished pellet product inventory balance at the start of the year is calculated by constraint (7.1). Constraint (7.2) tracks final product inventory at the start of day $t - 1$ plus newly produced pellets minus demand $d_{\omega,t}$ equals the inventory stock at the start of the next day t , with any shortfall captured as lost sales $B_{\omega,t}$. Constraint (7.3) partitions that final product inventory between on-site storage $L_{\omega,t}^{inner}$ and off-site storage $L_{\omega,t}^{outer}$.

$$L_{\omega,1} = l_0 + B_{\omega,1} - d_{\omega,1} \quad \forall \omega \quad (7.1)$$

$$L_{\omega,t} = L_{\omega,t-1} + B_{\omega,t} + rQ_{\omega,t,1} - d_{\omega,t} \quad \forall \omega, t \geq 2 \quad (7.2)$$

$$L_{\omega,t} = L_{\omega,t}^{inner} + L_{\omega,t}^{outer} \quad \forall \omega, t \quad (7.3)$$

Group 8: Capacity constraints. Constraint (8.1) limits the total in-process raw material quantity to be no more than the total facility processing capability $capInter$. Constraint (8.2) ensures that the onsite finished product inventory never exceeds the onsite product storage capacity $capInner$. Constraint (8.3) calculates the needed external storage when the onsite finished product storage capacity is filled.

$$\sum_u Q_{\omega,t,u} \leq capInter \quad \forall \omega, t \quad (8.1)$$

$$L_{\omega,t}^{inner} \leq capInner \quad \forall \omega, t \quad (8.2)$$

$$L_{\omega,t}^{outer} \geq L_{\omega,t} - capInner \quad \forall \omega, t \quad (8.3)$$

Group 9: Sustainable operation between years. Sustainable operation (defined here as maintaining end-of-year raw material and finished product inventory levels no lower than their respective start-of-year levels, ensuring production continuity across consecutive operating years) is enforced by constraints (9.1) to (9.2): raw material and finished product inventories on the last day of a year must at least equal to their opening levels h_0 and l_0 at the start of the year.

$$H_{\omega,t_{\max}} \geq h_0 \quad \forall \omega \quad (9.1)$$

$$L_{\omega,t_{\max}} \geq l_0 \quad \forall \omega \quad (9.2)$$

Group 10: Variable types. Finally, constraints (10.1) and (10.2) declare binary variable $R_{\omega,t,i}$ and non-negativity for all decision variables.

$$\text{All decision variables} \geq 0 \quad (10.1)$$

$$R_{\omega,t,i} \in \{0, 1\} \quad (10.2)$$

Collectively, these constraints ensure that the model delivers a cost-minimal and operationally sustainable inventory policy that is robust across a range of stochastic demand scenarios considered.

In addition, we also extend the base model with additional decision variables and constraints to incorporate forest residues as an alternative feedstock, while maintaining the core (s, S) inventory policy framework. The extended model, hereafter referred to as the multi-feedstock model, incorporates three willow supply zones and one additional forest residue supply zone (see Table 3), while retaining the (s, S) policy structure of the base willow-only model.

Case study context

We selected one of the proposed facility locations from a previous study (Wang et al. 2020) as a hypothetical wood pellet facility location. This site is in Schenectady County,

New York, and is surrounded by abundant existing and potential willow biomass cultivation sites. County-level estimates of annual willow biomass availability were obtained from the U.S. DOE Billion-Ton Report (Langholtz et al. 2016). All biomass and pellet quantities in this study are reported in oven dry tonnes (1 tonne = 1,000 kg; hereafter ‘tonnes’). All distances are reported in kilometers (km). Source data originally expressed in U.S. short tons or miles have been converted using 1 U.S. short ton = 0.907 tonnes and 1 mile = 1.609 km; distances are rounded to the nearest kilometer, and model parameters originally set as round numbers in English units are retained as approximate metric equivalents. All monetary values in this study are reported in U.S. dollars (USD).

To demonstrate the model’s ability to capture the economic trade-offs between transportation costs and supply availability, we delineated three concentric willow raw material supply zones around the facility (Figure 3): Zone 1 (0 to <32 km from the facility), Zone 2 (32 to <80 km), and Zone 3 (80 to 161 km). Other delineation methods could be implemented depending on landscape characteristics, such as zoning based on different travel distances along road-networks, or zoning based on land ownership and administrative boundaries. A simple radius-based zone method was implemented for this supply area based on the available production data. The annual willow availability was estimated at 6,773 tonnes in Zone 1, 38,629 tonnes in Zone 2, and 93,678 tonnes in Zone 3 by prorating willow supplies from each county and the proportion of each county within each supply zone.

Table 3. Additional variables and parameters for multi-feedstock extension.

Category	Symbol	Description	Range/Values
Extended Indices	num_willow_zones	Number of willow supply zones	3
	num_forest_zones	Number of forest residue supply zones	1
	i	Total number of supply zones	Total 4 zones; $i = 1, 2, 3$ for willow; $i = 4$ for forest residue
Extended Cost Parameters	c_i (for $i = 4$)	Variable ordering cost for forest residue zones	\$70/tonne
	f_i (for $i = 4$)	Fixed ordering cost for forest residue	\$100/order

Table 4. Operational parameters used for the willow-based pellet production facility inventory management model.

Symbol	Parameter	Value	Unit
$k_{\max}$	Transportation lead time	2	days
$u_{\max}$	Production process time	2	days
r	Conversion ratio	0.8	pellet/biomass weight ratio
s	On-site raw material storage cost	1.00	\$/tonne/day
w_{inner}	On-site product storage cost	0.50	\$/tonne/day
w_{outer}	Off-site product storage cost	1.50	\$/tonne/day
$capStore$	Raw material storage capacity	2000	tonnes
$capInter$	Processing capacity	1000	tonnes
$capInner$	On-site product storage capacity	2000	tonnes
$capProc$	Daily raw material procurement capacity	800	tonnes/day
o	Lost sales penalty	235	\$/tonne
f_i	Fixed ordering cost	100	\$/order
c_i	Variable ordering costs (Zone 1–3)	43.6, 46, 50	\$/tonne

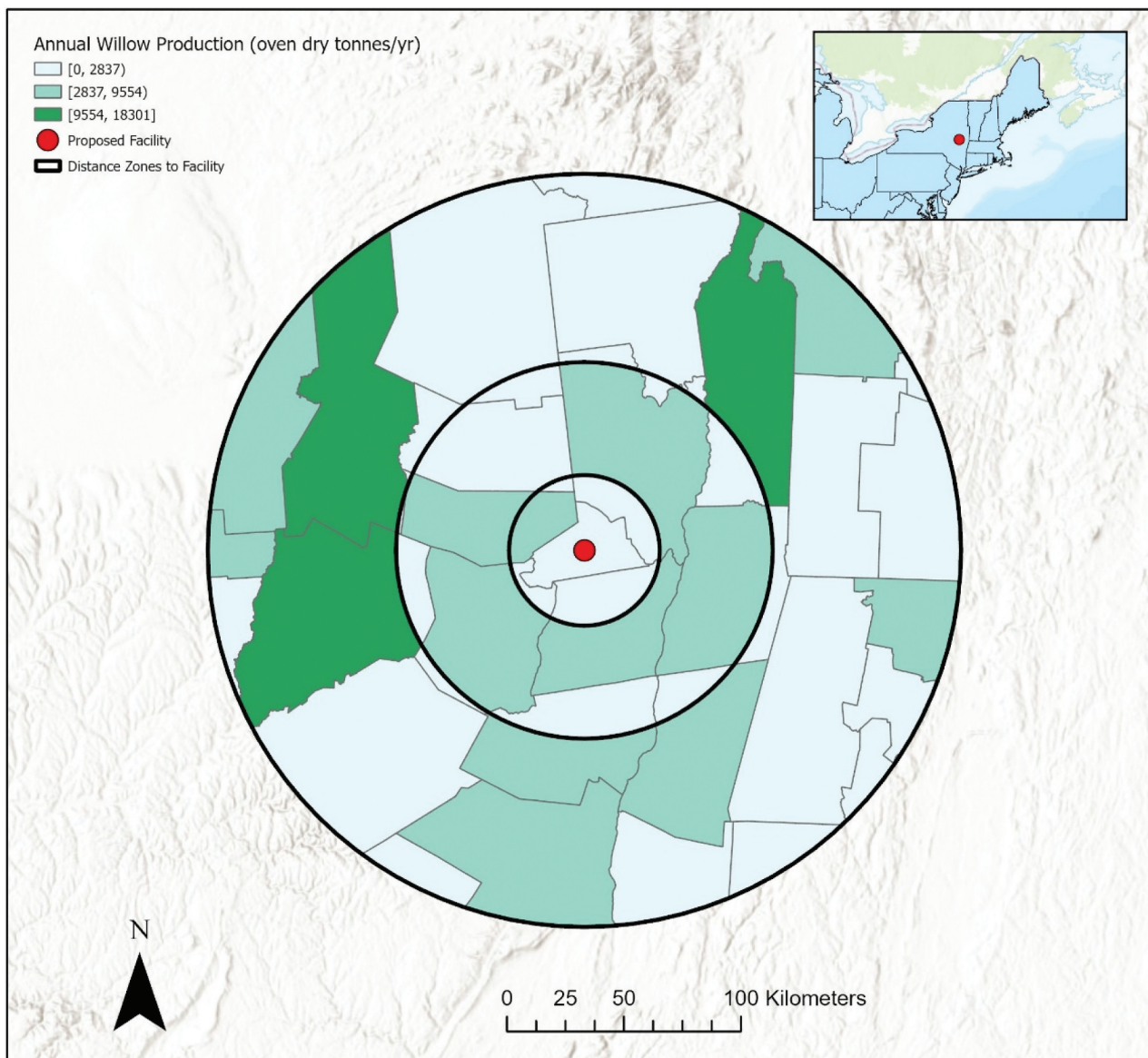


Figure 3. We calculated willow biomass availability by distance zone from the proposed Schenectady County, New York willow pellet production facility based on the report of Langholtz et al. (2016).

Assumptions and parameter estimation

Supply and demand parameters

We assumed the pellet production facility operates 360 days per year as an operation cycle. The use of a 360-day annual cycle instead of 365 days is also a modeling convenience that facilitates clean division of the cycle into different inventory policy adjustment periods for fair comparison (e.g. daily, six-day approximating weekly, or 30-day approximating monthly) and simplifies computational implementation.

We modeled willow's harvesting seasonality and used certain predefined operational capacity limitations. Willow biomass supply could be constrained by harvesting capacity, e.g. labor availability, equipment limitations, and logistics coordination, among other variables. Ideal willow harvesting should be planned during the dormant season when leaves are off to minimize disruption to the natural coppicing cycle and reduce biomass quality degradation. Some research

suggests that harvesting willow during the leaf-on season may result in biomass loss. However, if sustained production is preferred by a pellet facility, there might be benefits to harvesting a certain amount of willow during leaf-on seasons. To reflect this assumption, we allowed willow harvesting in suboptimal months, but we adjusted the harvesting capacity rate to account for the decrease in willow production when harvested during sub-optimal leaf-on months. More specifically, we assumed different willow harvesting capacities at different months throughout the year: full capacity during the dormant season (November to March), zero capacity during the critical growth period (April to May), and reduced 80% of full capacity during the late growing season (June to October). We calculated a daily maximum accumulative willow raw material supply up to 139,071 oven-dry tonnes in total across a year and used it to set the maximum available amount of willow up to each day, as shown in Figure 4.

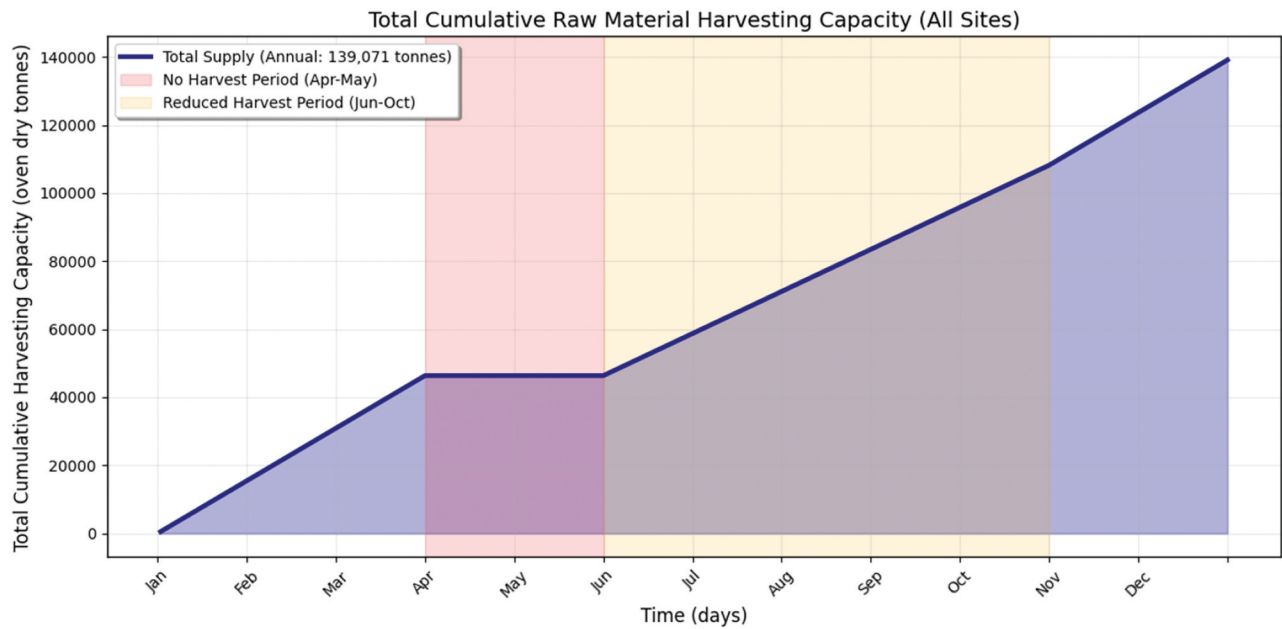


Figure 4. Cumulative willow harvesting capacity through a year in the study site. It represents the maximum achievable cumulative harvest given operational constraints (e.g. labor, equipment, logistics).

We also tested model runs under the assumption of stochastic daily demand for wood pellets. Inventory management literature often assumes a normal distribution of end-product demand with known mean and variance, though its appropriateness depends on demand characteristics (Ramaekers and Janssens 2008). For our base case, we assumed daily demand follows a normal distribution with a mean of 100 tonnes/day and a standard deviation of 10 tonnes/day, using a coefficient of variation (CV) of 10%. We generated multiple random demand scenarios using this distribution as input to incorporate stochasticity and improve the robustness of the facility's willow raw material inventory control policies (see Appendix A, Figure A1 for three randomly generated scenarios from the assumed normal distribution of pellet demand). Empirical sales data from the U.S. Energy Information Administration (EIA) for the eastern U.S. region (U.S. Energy Information Administration 2025) reveal a seasonal demand pattern with elevated consumption in summer and pre-winter months. This pattern is consistent with the distinct seasonal cycles of industrial and non-industrial pellet markets documented in the literature (Flinkman et al. 2018). Rather than asserting specific causal mechanisms for the observed peaks, we incorporated these empirical patterns directly into the SDT test case via the seasonal demand factors described below. Note that the true market demand for a specific facility's pellet products was not available for this study; therefore, we used sales data as a proxy for demand in some of the other test cases for this study.

To empirically validate the use of a normal distribution for our application, we analyzed nine years (2016–2024) of monthly densified biomass fuel sales data from the U.S. Energy Information Administration (U.S. Energy Information Administration 2025). Testing within-month demand variability, 92% of observations in the East region (where our case study is located) passed Shapiro–Wilk normality tests ($p > .05$), with a 100% pass rate when considering the

U.S. total. Based on those data, we calculated seasonal demand factors by dividing each month's sales volume by the annual average monthly sales. To convert these 12 monthly factors into continuous daily values for a 360-day model, we applied cubic interpolation to these monthly factors, using mid-month points (days 15, 45, etc.) as reference nodes to generate daily seasonal factors for all 360 days. These interpolated daily seasonal factors were then multiplied with the base stochastic demand distribution (mean = 100 tonnes/day, standard deviation = 10 tonnes/day) to generate seasonally adjusted daily pellet demand scenarios that preserve both empirical market trends and stochastic variability.

Parameter estimation and justification

Due to limited commercial-scale willow-based wood pellet production data, some parameters used in this study were adopted from conventional wood pellet operations with adjustments. The values and units of these operational parameters are summarized in Table 4. Willow biomass and conventional woody biomass share similar physical and thermochemical properties relevant to pellet production, like comparable bulk density, lignin content, and pelletizing behavior. Those properties support the adaptation of some conventional operational parameters to a willow-based system, subject to the adjustments noted below (Stachowicz and Stolarski 2023).

Transportation and processing parameters. Two days of transportation lead time from supply sites to the wood pellet facility was set, considering the maximum 161 km radius transportation distance in our case study and corresponding administrative activities for each raw material order. Two days of processing time from raw material to end product was set, considering pre-treatment, pelleting, cooling, and packaging (Póliska et al. 2014). The biomass-to-pellet conversion ratio varies, due to several affecting factors, including feedstock moisture content, preprocessing requirements, and dry matter

Table 5. Design of test cases used in testing of the inventory management model.

Category	Test Case	Objective	Key Parameters	Figure Reference
Baseline	Base Test (BT)	Establish baseline performance	Seasonal supply; Integrating three demand scenarios; Monthly inventory policy adjustment.	Figure 6
Operational Analysis	Policy Adjustment Frequency Test (PAFT)	Test various adjustment frequency	Daily/Weekly/Monthly; Out-of-sample method; 50 runs per frequency.	Figure 7
Method Validation	Multiple Scenario Test (MST)	Stochastic vs. deterministic	One to five demand scenarios; Out-of-sample method; 30 runs per scenario setting.	Figures 8 and 9
Application	Seasonal Demand Test (SDT)	Market seasonality integration	Adjusted demand scenarios	Figure 10
	Multi-Feedstock Test (MFT)	Multi-feedstock benefits	Willow and forest residue supply	Figures 11 and 12
	Computational Performance Test (CPT)	Computational efficiency	Integrating two demand scenarios; MIP gaps 0.11%–1%.	Figure 13

Note. Other tests will use same parameters with BT unless otherwise specified. MIP = mixed-integer programming.

losses. For example, dry matter losses of willow biomass in storage can range from 6.3% to 33.6% (Therasme et al. 2020). For model simplification, we adopted a uniform conversion ratio of 0.8 (meaning 1 oven-dry tonne of biomass yields 0.8 oven-dry tonnes) for both willow and forest residue feedstocks, acknowledging that previous studies and actual conversion rates may vary between feedstock types.

Cost parameters. Lost sales penalty was considered as the price of purchasing wood pellets at market price to fill lost sales, and \$235 per tonne was used as the penalty because it was the average densified biomass fuel price for the East U.S. region in 2024 (U.S. Energy Information Administration 2025). A base willow feedstock price of \$42 per tonne was set according to EIA 2024 average feedstock costs (U.S. Energy Information Administration 2025). Transportation costs were estimated using a rate of \$0.10 per tonne-km, with average Euclidean distances of 16, 40, and 80 km from supply zones 1, 2, and 3 to the facility. This results in transportation costs of \$1.60, \$4.00, and \$8.00 (rounded) per tonne for the three zones, yielding total variable ordering costs of \$43.60, \$46.00, and \$50.00 per tonne, respectively. These delivered prices are consistent with EIA 2024 average feedstock costs for the eastern U.S. region (U.S. Energy Information Administration 2025), with zone-specific premiums reflecting increased transportation distances. Fixed ordering costs of \$100 per order apply across all supply zones, representing administrative and coordination expenses of procurement activities. For the multi-feedstock extension, forest residue variable cost was set at \$70/tonne, with \$100 fixed cost per order. This is higher than the primary willow variable cost, reflecting the additional procurement costs associated with spot contracting and less optimized logistics typical of secondary feedstock sourcing outside the primary willow supply chain infrastructure. The \$70/tonne figure represents a conservative estimate based on the assumption that forest residue procurement costs, including collection, processing, and transportation, exceed those of contracted willow supply.

Capacity parameters. The processing capacity was set at 1,000 tonnes, representing the maximum amount of raw material that can be simultaneously processed, excluding storage. This value was back-calculated as: (transport lead time +

processing time) $\times$ expected daily demand with variability buffer $\times$ raw material input ratio $\times$ redundancy factor = $(2 + 2)$ days $\times$ 100 tonnes/day $\times$ 1.2 (accounting for demand variability at 2σ level with 10% coefficient of variation) $\times$ 1.25 (inverse of 0.8 conversion ratio, representing raw material input per unit finished product) $\times$ 1.67 (redundancy factor, equivalent to 5/3). On-site raw material and product storage capacity were both set to 2,000 tonnes, providing a ten-fold buffer relative to the eastern U.S. regional average facility production capacity of 172 tonnes per day in 2024 (U.S. Energy Information Administration 2025). Off-site end-product storage capacity has no limit, representing external rental facilities. The daily procurement limit is 800 tonnes based on the assumed maximum truck loading capacity per facility per day. Storage costs were established considering their proportion within the total wood pellet supply chain costs reported in previous studies (Visser et al. 2020). Raw material storage cost was included at \$1.00/tonne/day. The on-site finished product storage cost of \$0.50/tonne/day is lower due to the higher density and stability of pelletized material. Off-site storage cost was \$1.50/tonne/day, accounting for additional handling and transportation to external facilities, and premium rental rates for third-party storage.

Test cases design and implementation

To comprehensively evaluate our stochastic optimization model, we designed six test cases that progressively examine different aspects of the model's performance (Table 5). These tests are structured to: (1) establish baseline performance metrics, (2) evaluate key operational parameters, (3) validate the stochastic approach against deterministic methods, and (4) assess computation time requirements.

To assess the quality of optimization solutions, we implemented an out-of-sample evaluation framework. After obtaining optimal inventory policies from our two-stage stochastic programming model using the predefined parameters, we extracted these first-stage decisions and applied them to independently generated demand scenarios that were not used during optimization. These out-of-sample scenarios follow the same statistical properties but represent previously unseen

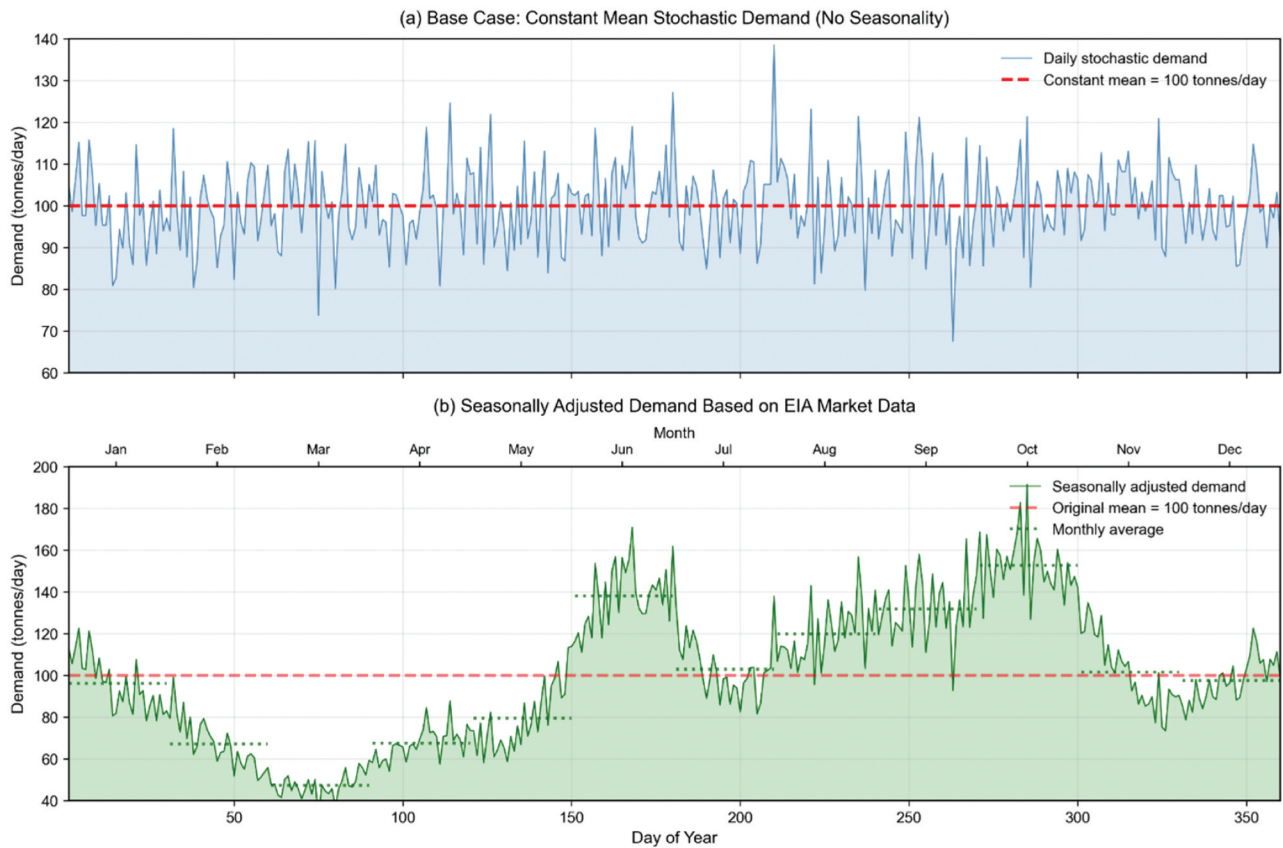


Figure 5. Comparison of a random scenario generated based on constant-mean stochastic demand (panel a) and seasonally adjusted demand based on 2024 EIA market data (panel b).

demand realizations. We then simulated the operational performance under these new samples and compared key performance metrics. This out-of-sample evaluation approach ensures unbiased performance assessment across different parameters.

Base test (BT) is for establishing baseline performance. Seasonal willow raw material supply, an integration of three demand scenarios, and monthly inventory policy adjustment frequency are used as parameters.

Policy adjustment frequency test (PAFT) compares the solution quality of daily, weekly, and monthly adjustment frequencies using the out-of-sample evaluation approach. For each policy adjustment frequency, 50 independent evaluating runs are conducted, with each run using one randomly generated out-of-sample demand scenario. Objective values from the 50 evaluating runs are aggregated to establish the statistical significance of cost differences across inventory policies generated from different adjustment frequencies.

The multiple scenario test (MST) employs the same out-of-sample approach to evaluate stochastic optimization quality. Separate optimization runs are conducted using one to five randomly generated demand scenarios to derive different inventory policies. The single-scenario case ($\omega=1$) represents the deterministic approach. For each inventory policy obtained, we conducted 30 independent single-demand-scenarios to generate statistical distributions of objective

values. This design enables fair comparison between deterministic (optimized for a single demand scenario) and stochastic (optimized for two to five demand scenarios) approaches by testing all policies under identical evaluation conditions.

Seasonal demand test (SDT) evaluates model performance under realistic market conditions by incorporating empirical seasonal patterns (see an example in Figure 5). This preserves both seasonal market trends and demand uncertainty, enabling assessment of inventory optimization under real-world seasonal consumption patterns. Wood pellet demand can be classified into industrial (mainly electricity generation) and non-industrial (mainly residential heating) markets (Flinkman et al. 2018; U.S. Energy Information Administration 2025), which exhibit different demand patterns. We used monthly densified biomass fuel sales data from the EIA as a proxy for true demand, given that wood pellets constitute the primary component of densified biomass fuel products (U.S. Energy Information Administration 2025).

Multi-feedstock test (MFT) extends the baseline willow-only model (BT) to evaluate raw biomass material supply diversification benefits using the multi-feedstock model extension described in the previous section, which can substitute forest residues for willow. Forest residues are incorporated as an alternative feedstock with year-round availability, priced at \$70/tonne, with \$100 fixed ordering cost. This enables optimization across two feedstock types with different cost structures

Table 6. Summary of key performance metrics across six test cases.

Test Case	Purpose	Main Finding
BT	Baseline	IM policy adapts to seasonal willow supply with zero stockouts; the upper-level (US) peaked at 750 tonnes in month 3; the lower level (LS) was around 250 tonnes.
PAFT	Policy adjustment frequency	More frequent policy adjustment lowers inventory management cost: • Daily adjustment: \$2.419 M/yr • Weekly adjustment: \$2.424 M/yr • Monthly adjustment: \$2.441 M/yr
MST	Stochastic vs. deterministic	Considering stochasticity in demand results in ~1% average cost reduction and 44% CV reduction.
SDT	Seasonal demand	IM policy adapts to empirical demand seasonality with zero stockouts; the upper-level (US) peaked at 800 tonnes in month 3, and was elevated in months 8 to 10 between 319 and 350 tonnes.
MFT	Multi-feedstock	Supply diversification reduces peak pre-season raw material inventory requirements. Due to higher cost, forest residue was used only during months 4 to 5 outside of the Willow harvesting season.
CPT	Computation performance	Using 0.15% MIP gap requires 260s running time; using MIP gap < 0.15% requires > 30,000s running time.

and availability patterns, examining whether multi-feedstock flexibility can reduce costs and improve supply chain resilience.

For the computational performance test (CPT), convergence times are recorded for each mixed-integer programming (MIP) gap tolerance (explained later with examples) level run, while all other parameters remain controlled to be the same as the BT cases.

All MILP models in our study were implemented and solved using CPLEX optimization solver (version 22.1.0: IBM Corp. 2022) within an Anaconda environment. All tests were conducted on a Windows 11 Enterprise system with Intel Xeon Gold 5217 CPU and 64GB RAM. Default CPLEX parameters were used except for the MIP gap tolerance parameter.

Results

Table 6 summarizes the key performance metrics across all six test cases. Detailed results for each test case are presented in the following subsections.

Base test performance (BT results)

Following the BT parameters, the model-reported optimal monthly inventory policy demonstrated distinct seasonal patterns. The upper stock level (US) peaked at 750 tonnes in month 3 of each year, with the corresponding lower stock level (LS) at 250 tonnes, representing strategic preparation for the April–May no-harvesting period. During the other stable willow supply months (1, 4–11), the policy maintained moderate levels with US around 135 tonnes and LS around 30 tonnes, indicating efficient and close to just-in-time procurement operations.

The raw material inventory buildup in month 3 reflects the model's anticipation of a seasonal willow supply disruption. Increasing the daily raw material storage level to 750 tonnes (Figure 6) also helped increase pellet production and storage to meet the demand from months 4 and 5 when willow could not be harvested. The convergence of US and LS levels to 234 tonnes in month 12 resulted from the sustainable operation constraint (Constraints 9.1 and 9.2), which requires the end-of-year willow inventory level and pellet storage level to be no less

than the initial willow inventory level and pellet storage level at the beginning of a year to ensure the pellet mill can operate sustainably between two consecutive years.

The finished wood pellet product inventory also followed a clear annual cycle, peaking at approximately 5,000 tonnes by the end of the spring accumulation period in both internal and external storages (Appendix A, Figure A2). When entering the no-harvesting season in months 4 and 5, the pellet mill will utilize both the raw willow material storage and the final pellet product storage to meet the pellet demand before it can acquire new raw willow materials at month 6. Benefiting from those extra accumulated storage, we found there is no stockout across all pellet demand scenarios when the mill needs to meet the modeled pellet demands.

Inventory policy adjustment frequency analysis (PAFT results)

Results of the inventory PAFT (Figure 7), aggregated across 50 independent runs, reveal cost variations across different inventory policy adjustment frequencies. Allowing the most frequent daily adjustments achieves the lowest mean total inventory management costs at \$2.419 million per year, while monthly adjustments result in the highest costs at \$2.441 million. Weekly adjustments provide an intermediate total annual cost of \$2.424 million.

Stochastic vs. deterministic comparison (MST results)

We tested and compared the performance of stochastic optimization approaches versus the deterministic approach in configuring monthly inventory policies (Figure 8). The box plot distribution compares the total annual costs by implementing inventory policies derived from different-sized training samples that reflect the stochastic fluctuation of pellet demands on top of the assumed 100 tonnes/day constant demand. Those policies were evaluated and compared on independent testing scenarios that were not used during the policy training and optimization process.

The deterministic approach ($\omega=1$) shows the highest mean objective value (costs) at approximately 2.32 million USD with considerable variance, indicating higher total cost and poorer robustness to demand uncertainty. As the number of training

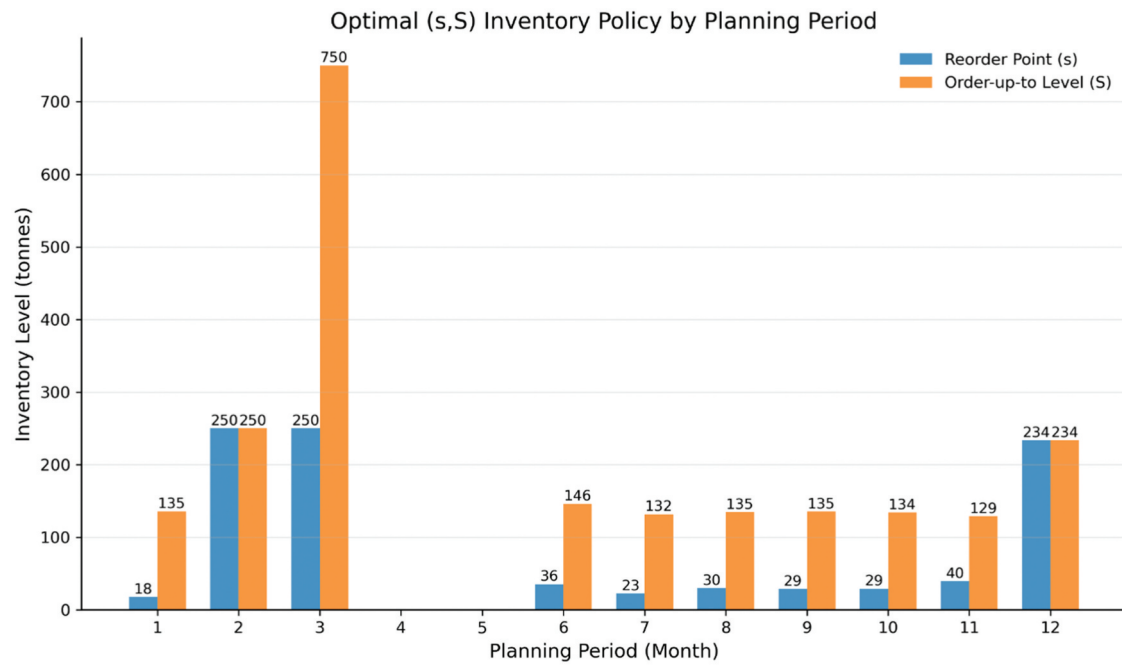


Figure 6. The optimal monthly willow raw material inventory policy for the base test.

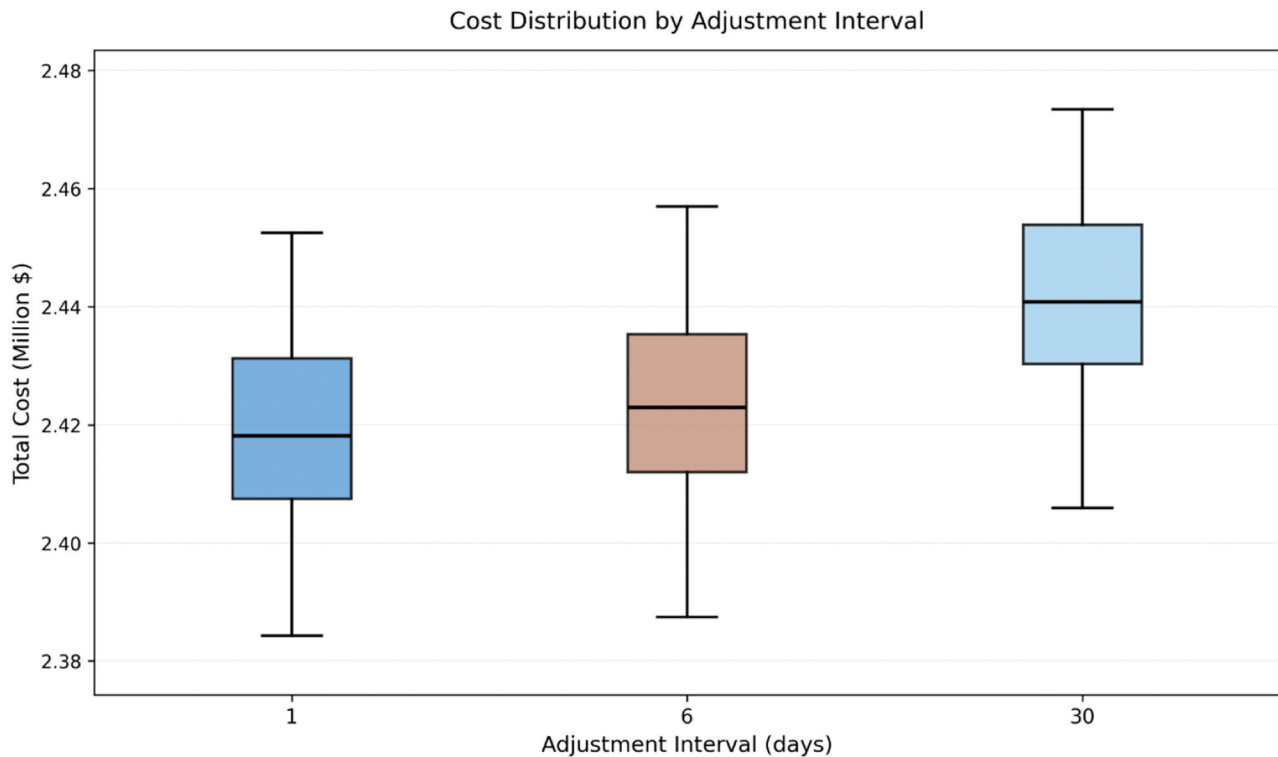


Figure 7. Objective value distribution across different inventory adjustment intervals.

scenarios increased from 2 to 5, the mean costs decreased. Policies trained on three to five scenarios achieved mean costs around 2.29–2.30 million USD, representing an approximately 1% cost decrease through a stochastic modeling approach over the deterministic approach.

Beyond mean cost comparison, the stochastic approach demonstrates substantial advantages in operational stability (Figure 9). The deterministic policy ($\omega=1$) exhibited a standard

deviation of \$23,044, while stochastic policies ($\omega=3-5$) maintained standard deviations around \$12,900, representing approximately 44% reduction in coefficient of variation. In absolute terms, this corresponds to a narrowing of the observed cost range (maximum minus minimum across 30 out-of-sample evaluation runs) from \$76,897 (deterministic) to approximately \$50,000 (stochastic, $\omega=3-5$), providing facility managers with substantially more predictable annual cost outcomes.

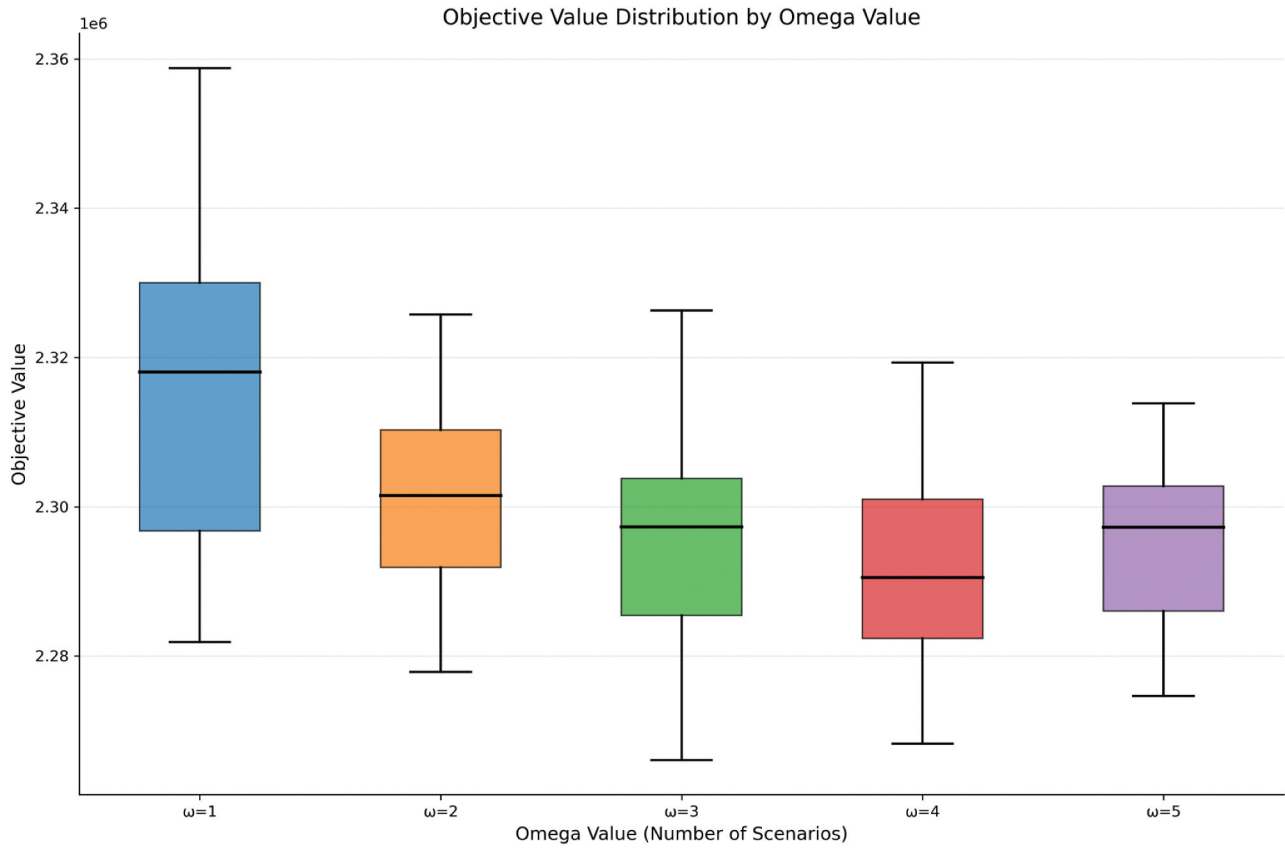


Figure 8. Objective value distribution across different demand scenarios.

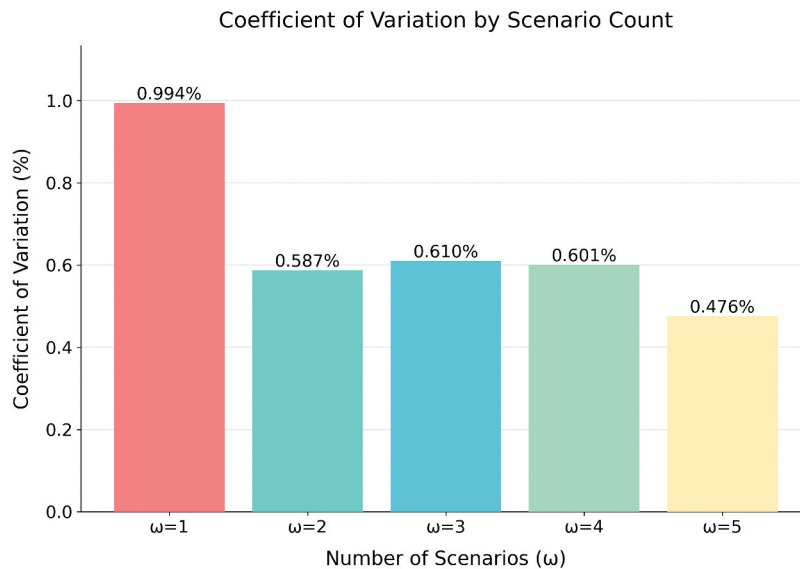


Figure 9. The coefficient of variation of total inventory management cost by scenario count integrated into the modeling.

Seasonal market data integration (SDT results)

Under the empirical market conditions with seasonal demand variations shown in Figure 5b, the optimal inventory policy (Figure 10) demonstrated adaptive behavior throughout the year. The policy showed high inventory targets during month 3 (US = 800 tonnes) in preparation for the April–May harvesting

restriction, similar to the base test. However, months 6–11 exhibited more dynamic adjustments compared to the base test, with inventory levels responding to seasonal demand patterns. Notably, months 8–10 showed elevated inventory levels (US ranging from 319–350 tonnes) corresponding to the pre-winter pellet demand surge identified in the market data.

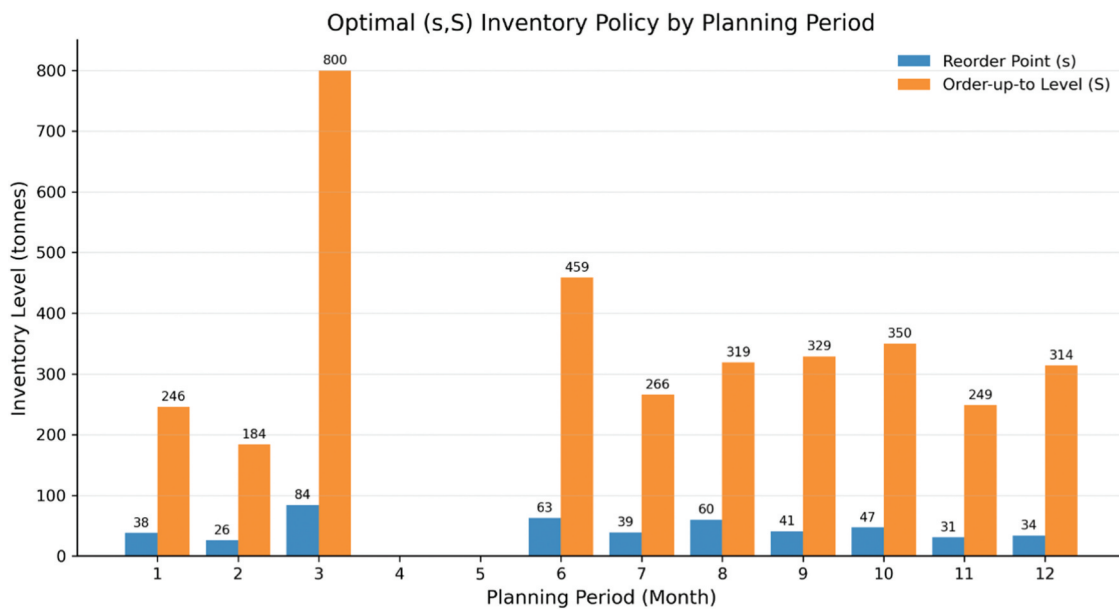


Figure 10. Optimal monthly willow inventory policy under assumed seasonal pellet demand and price trend.

The finished product inventory (Appendix A, Figure A3) fluctuated much more than the base test because of the fluctuation in demand. Despite these fluctuations, it maintained a similar annual cycle as the base test, achieving zero stockouts across all 12 months of production, demonstrating the robustness of the optimized inventory policy under more realistic market conditions.

Multi feedstock benefits (MFT results)

By using the MFT parameters, we conducted multi-feedstock analysis, which demonstrates certain benefit in biomass supply diversification. Figure 11 shows an optimal procurement strategy utilizing both willow and forest residue, with forest residue (priced at \$70/tonne, with \$100 fixed cost per order) serving as a supplementary source during willow's no-harvesting periods (months 4–5). Because the forest residue price was higher than the willow raw material price in our tests, we observed that the mill will only use forest residues for pellet production during the non-willow-harvesting season.

The corresponding optimal inventory policy (Figure 12) exhibits smaller seasonal variations compared to single willow-based feedstock operations, as supply diversification reduces extreme inventory requirements before willow's non-harvesting season.

Computational performance analysis (CPT results)

We tested the trade-off between the MILP's solution accuracy reported by computing time and the relative gap between the best solution found and the best possible solution (Figure 13) using the CPT settings. The analysis demonstrated a substantial increase in computational time as the MIP gap tolerance tightens. If we set the MIP gaps at 0.20% or higher, solution times remain at approximately 100–200 seconds by

integrating two stochastic demand scenarios into the second stage decision. An inflection point can be found at 0.15% MIP gap, where the solution time is approximately 260 seconds. However, tightening the gap further to 0.11% results in solution times exceeding 30,000 seconds, representing more than a 100-fold increase for minor MIP accuracy improvements. Based on the CPT analysis, a MIP gap tolerance of 0.15% for our model with two stochastic demand scenarios provides a good balance for practical implementation, enabling reasonable solution times while maintaining adequate precision for operational decision-making in facility management.

Discussion

This study aimed to demonstrate that stochastic optimization approaches such as two-stage MILP models can better address the unique challenges faced by pellet producers due to seasonal fluctuations and uncertainty of raw material supplies and pellet demand through proactive biomass raw material inventory management. We created multiple test cases by referencing published willow harvesting and supply research. Our test cases approximated pellet facilities' storage and production processes and utilized the best available pellet market demand and price data published for the eastern United States. Instead of modeling the entire woody biomass pellet supply chain, we focused on raw material acquisition, transportation, storage, pellet production, and product storage by a single facility to test a novel biomass inventory management model. It is worth noting that our model assumes sustained market demand for pellets, with fluctuation based on historical trends, though the developing North American wood pellet market is substantially influenced by export-related policies, such as EU renewable energy directives.

Inventory management is one of many important decisions a pellet production facility needs to make to support

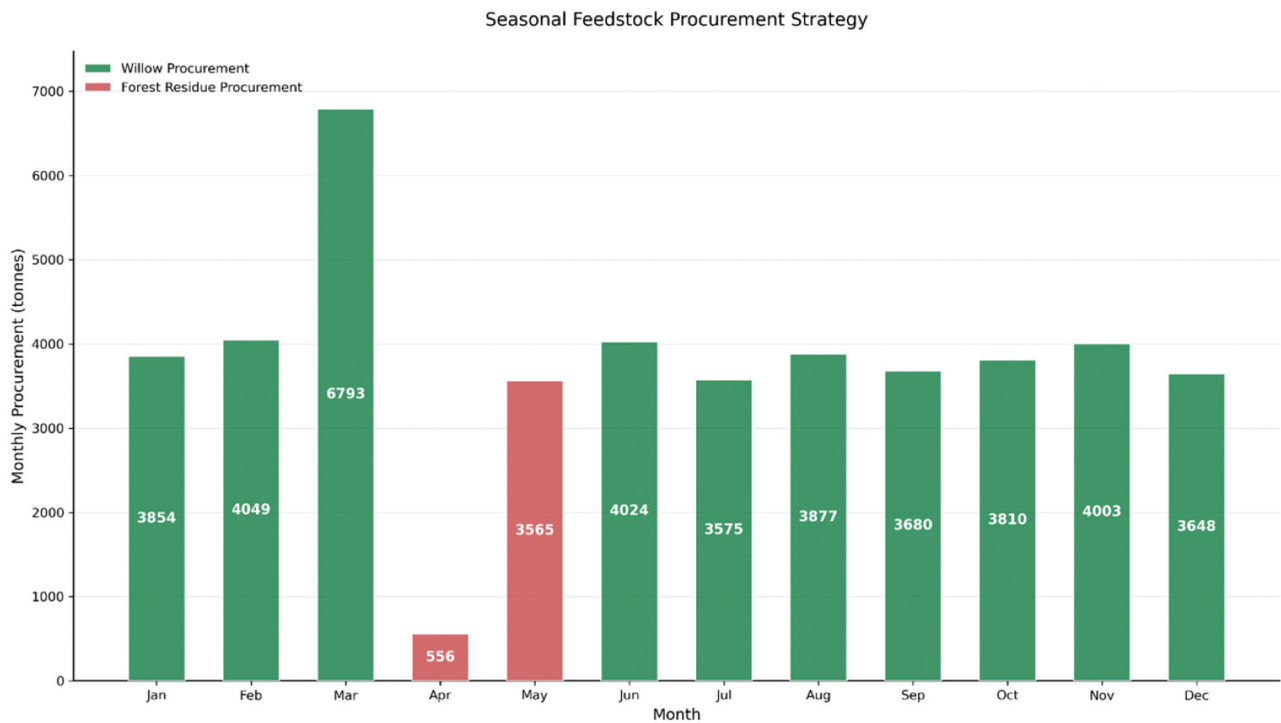


Figure 11. Monthly biomass procurement for multi-feedstock test cases.

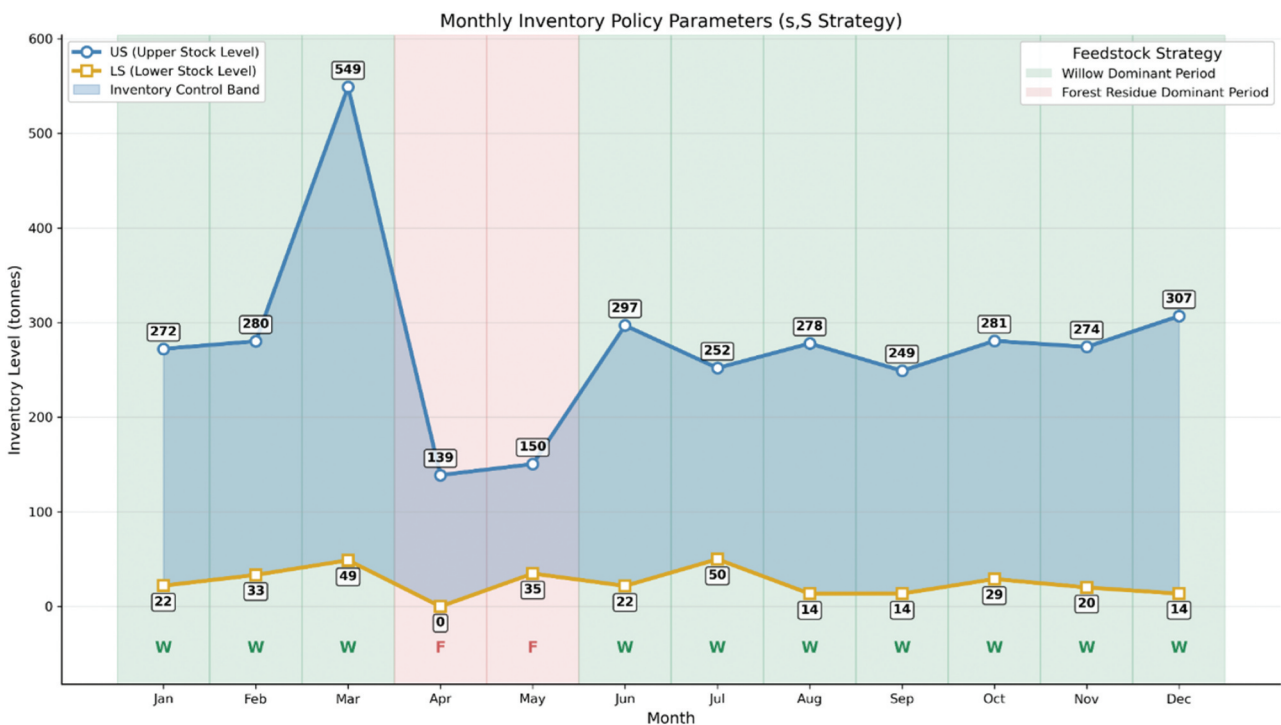


Figure 12. The optimal inventory policy for multi-feedstock. Note: the upper stock level and lower stock level were calculated based on the total biomass amount of willow and forest residual.

continuous facility operation and meet pellet demand. In general, keeping a higher level of raw material supply would help a facility better adapt to seasonal raw material supply and the fluctuation of pellet demand and pellet market price; however, this would also require more storage space and increased storage costs, along with the other risk of raw material quality

degrading with longer storage time. Following the commonly used (s, S) inventory management policy, our model results configured a set of upper and lower bound willow raw material inventory levels for each predefined adjustment period (day, week, month), accounting for storage costs. Those results not only are consistent with intuitive and logical

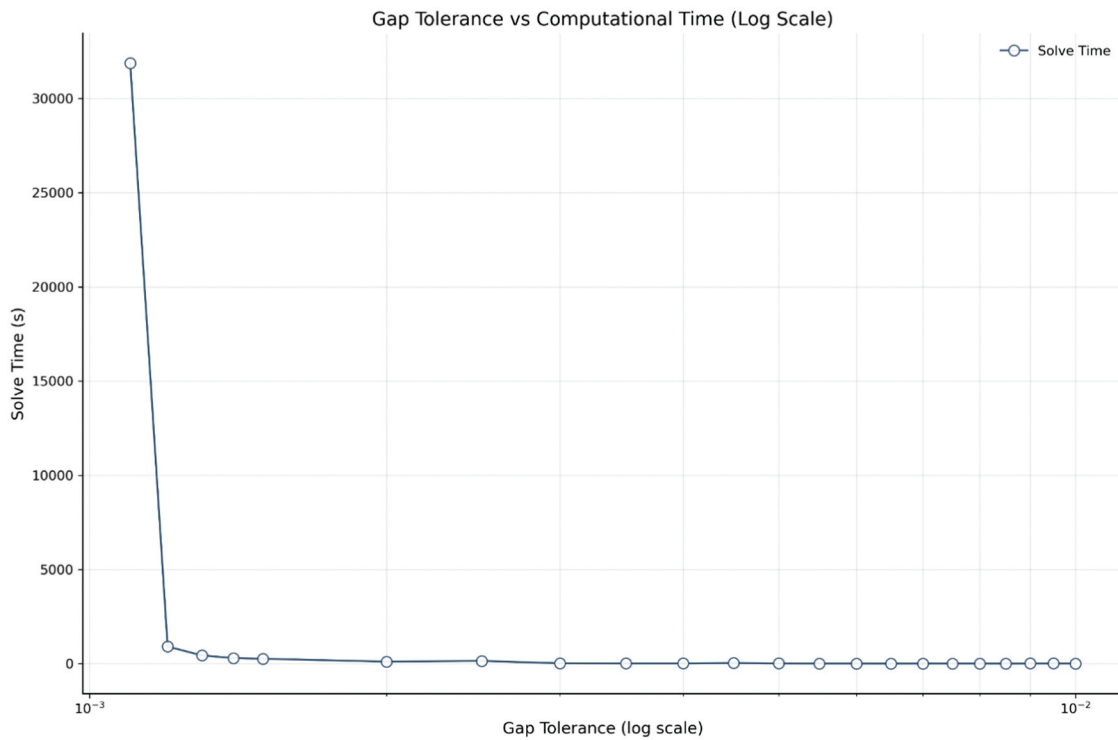


Figure 13. Solve time across different MIP gap tolerances.

solutions but also provide quantified and time-specific management suggestions.

The choice of inventory policy adjustment frequency involves practical trade-offs beyond pure cost optimization. While the more frequent daily policy adjustment achieves the lower inventory management costs in our model, the practical challenge of implementing daily policy adjustments may outweigh theoretical cost savings compared to longer adjustment periods. Real-world implementation must also account for supplier coordination costs, management capacity, and transaction overhead not captured in our current optimization framework. Facility managers may need to balance tradeoffs between cost efficiency and operational complexity when determining the best inventory policy adjustment frequency.

Deterministic models are ideal for optimal inventory management when a supply chain (or a part of a supply chain) possesses a high level of certainty. However, this is rare for most biomass supply chains. Stochastic models are necessary in helping managers gain insights and confidence in dealing with uncertainties. Our stochastic modeling framework can lead to inventory management policies that prevent pellet stockout while reducing overall annual inventory management costs and the variability of cost outcomes. Under real-world management conditions, we anticipate that there will be many different types of interacting uncertainties influencing the facility's inventory management and other operations, including machine maintenance, breakdown and repair, raw material price or transportation cost variations, pellet price change, the variation of the products' quality, competition, and other sources of uncertainty. Similar modeling methods could also be

developed to incorporate the influence of such uncertainties into the decision-making process.

Compared to implementing an (s, S) policy optimized for a deterministic, average end product demand, the stochastic framework delivers measurable advantages without prohibitive computational cost. At a recommended MIP gap of 0.15%, the model solves in approximately 260 seconds by simultaneously considering two possible stochastic pellet demand scenarios and yields a robust monthly inventory policy. For facility managers, the practical implication is that a small additional computational investment delivers more predictable cost outcomes and more adaptive plans for raw material acquisition, storage, and finished product inventory, particularly as real-world demand uncertainty increases.

Quantifying the joint distributions of those uncertainties and integrating them into a stochastic programming model is a challenging task and requires future studies. It is important to note that our observed benefits (cost improvement and variance reduction) are based on moderate demand uncertainty conditions. Real-world pellet markets may encounter higher volatility driven by weather extremes affecting heating demand, customer variability, market dynamics (e.g. level of competition), currency fluctuations, and regulatory changes. Stochastic optimization demonstrates that performance benefits would likely scale with uncertainty levels, suggesting that facilities facing greater real-world volatility would likely experience larger advantages from adopting stochastic inventory optimization frameworks. Testing model performance under higher volatility conditions, e.g. with a demand coefficient of variation of 20% or higher, will further quantify these scaling

benefits and is identified as a priority direction for future research.

Seasonality is a common characteristic of many biomass sources, including willow, switchgrass, and many agricultural products. Different types of storage facilities could be built or rented to store raw materials to prepare for the non-harvesting season. For demonstration purposes, this study assumed in-facility storage for willow biomass and both in- and out-of-facility storage for pellet products. Inventory management policy is only implemented for in-facility raw material storage in this model. The seasonal inventory patterns revealed by the model provide insights for facility managers facing similar operational constraints and designing optimal storage solutions based on local conditions. Specifically, the model can help explore the impact of various in-field, in-facility, and off-site storage options for raw materials based on variable storage costs.

Because of the ecosystem services and other positive externalities associated with the willow biomass supply chain, such as carbon sequestration, improved soil productivity, and the restoration of abandoned sites, government incentives are commonly allocated to this industry through tax incentives, renewable energy incentives, and other mechanisms. Our model can help identify potential financial bottlenecks within the willow biomass supply chain and guide the more effective allocation of these incentives. Targeted policy incentives addressing specific constraints, such as storage infrastructure financing, transportation costs, or harvest season labor support, may be more effective than broad-based subsidies in supporting biomass industry development. However, the real-world situation is more complicated than our model currently considers. Expanding our model to cover longer supply chains with more “nodes” and “arcs” would be important for future modeling research in this field. To guide the consideration of specific policy incentives, greenhouse gas benefits and other ecosystem services could be incorporated using the results of life-cycle assessments for these supply chains.

The willow supply availability patterns used in this study employ county-level historical data, while more detailed research of New York State land availability and biomass production potential for willow has been conducted recently (Hossain and Volk 2024). Integration with such spatially explicit data could enable more granular inventory management optimization that accounts for detailed feedstock land availability. However, expanding from our current model framework to account for hundreds of individual supply locations would significantly increase model complexity, likely requiring decomposition algorithms or heuristic approaches to maintain computational tractability. Another limitation of the current model from the willow supply side is that it does not explicitly consider supply agreements between growers and producers or between producers and consumers, which could significantly alter risk distribution and optimal inventory strategies in practice.

The multi-feedstock tests, using both willow and forest residues, demonstrate the benefits of using forest residues to substitute for willow during the non-harvesting season of willow, which includes lowering the cost of storing a large amount of willow raw material before the non-harvesting season, and

potentially decreasing total operational cost to the mill. This multi-feedstock strategy also enhances supply chain resilience by diversifying biomass sources and buffering against supply disruptions or demand volatility. However, multi-feedstock operation is more complicated and may create more challenges in the mill’s quality control, which could translate to added costs. For example, different feedstock types may require separate processing procedures to meet industry standards such as ENplus certification requirements. Additionally, multi-feedstock operations may require more sophisticated processing equipment capable of handling varying feedstocks, with different moisture contents, particle sizes, and material flow characteristics, potentially increasing both capital investment and maintenance costs compared to single-feedstock operations. The relative benefits of a feedstock-switching strategy to target the lowest cost biomass type at any given point in time must be balanced with these types of operational considerations.

Conclusions

This research contributes a stochastic MILP framework for optimizing inventory management in willow-based pellet production facilities, addressing the gap in current biomass supply chain literature. The two-stage stochastic programming approach integrates seasonal supply constraints with demand uncertainty, providing inventory policies that outperform traditional deterministic methods. More specifically, the tested stochastic framework reduces both expected inventory management cost and its variability. These findings are consistent with broader evidence that stochastic optimization approaches outperform deterministic methods in biomass supply chain contexts (Shabani and Sowlati 2016; Akhtari et al. 2019), and extend this evidence to the specific setting of willow-based pellet inventory management under seasonal supply constraints. Test cases demonstrate that the added complexity of the stochastic approach yields measurable improvements over the simpler deterministic alternatives.

The strategic inventory accumulation patterns for offsetting seasonal supply gaps highlight the challenge of willow-based pellet production, while the multi-feedstock analysis demonstrates the advantage and necessity of supply diversification, as it reduces both costs and stockout risks. Key practical insights also include adjusting the frequency of inventory policy to balance efficiency and complexity. For our test cases, the total annual willow availability across all three supply zones (139,071 tonnes; Langholtz et al. 2016) substantially exceeds the modeled annual raw material requirement. However, in regions with lower willow availability, year-round multi-feedstock operation may be more important.

For businesses in the pellet industry, this research provides insights into inventory policy design and strategic planning. For researchers, it establishes the foundation of using stochastic optimization methods in a willow-based pellet production system that can be extended to other seasonal biomass systems. As the biomass energy sector continues to grow in importance for renewable energy transitions, optimization frameworks that address the unique challenges of its supply chain could become more valuable. Future research

directions include real-world implementation through industry partnerships, integration of better demand forecasting and dynamic pricing, and development of user-friendly decision support tools.

Disclosure statement

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Appendix

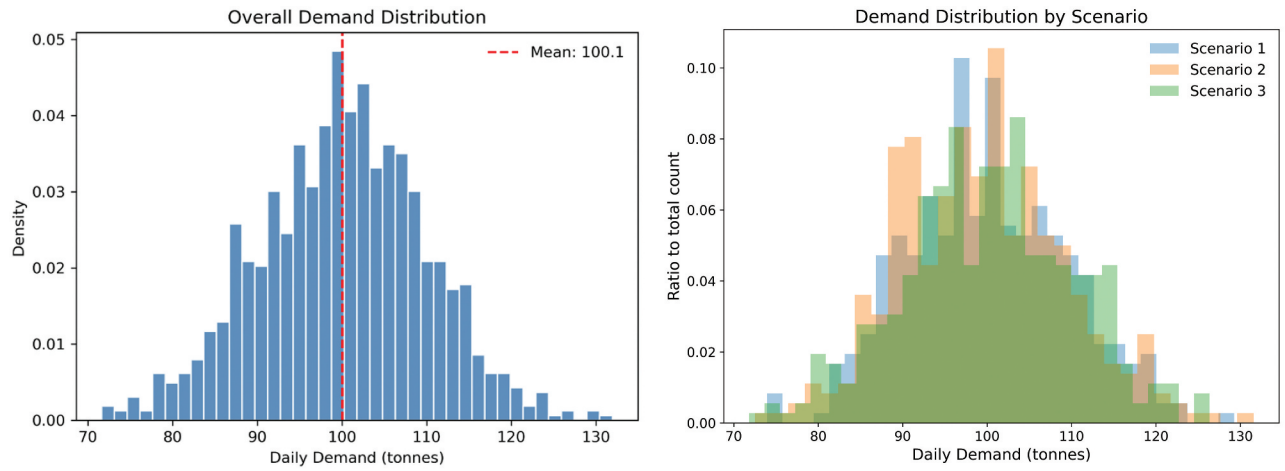


Figure A1. Three independently generated pellet demand distributions (Scenarios 1, 2 and 3) across the modeled year, each representing a random draw from the same normal distribution. Note: these scenarios reflect base case demand with a constant mean of 100 tonnes/day and CV = 10%, without seasonal adjustment. Seasonal demand variation patterns are addressed separately in the seasonal demand test (SDT) and illustrated in [Figure 5b](#).

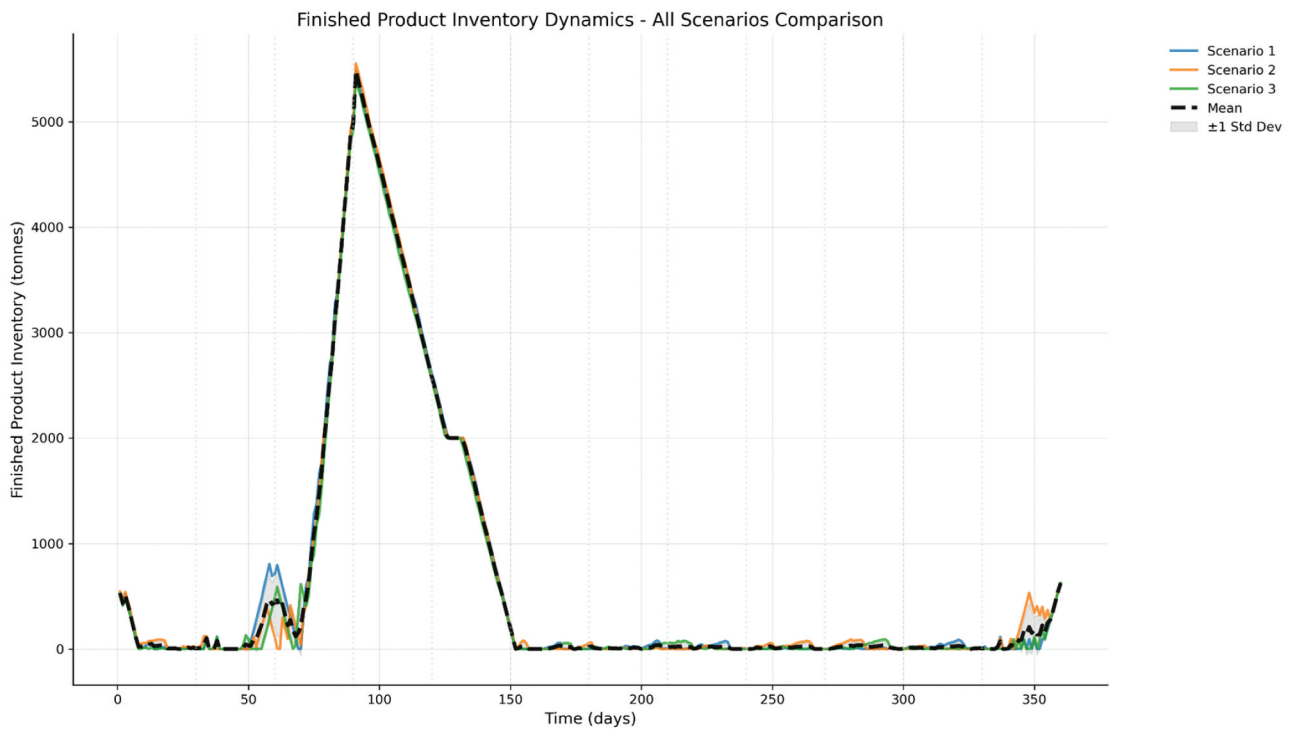


Figure A2. The daily finished product (pellets) inventory for base test.

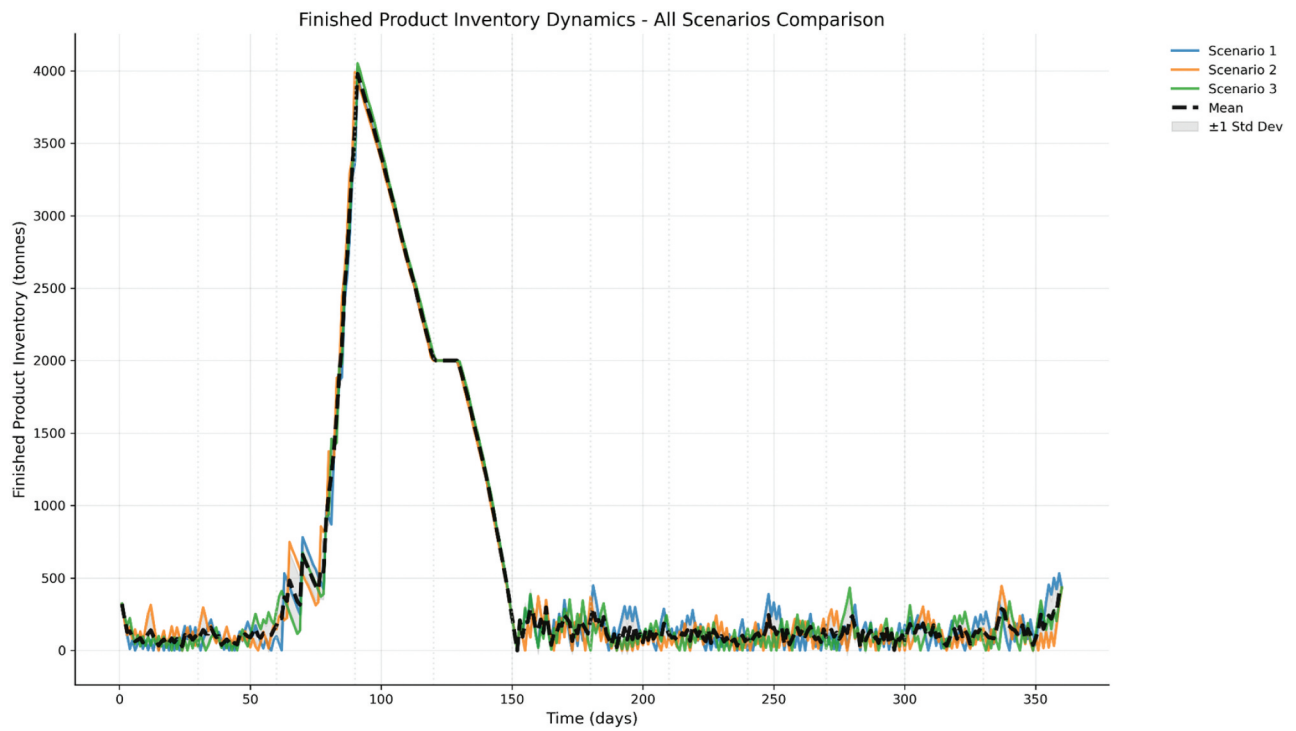


Figure A3. The daily finished product (pellets) inventory under real world demand and price trend.