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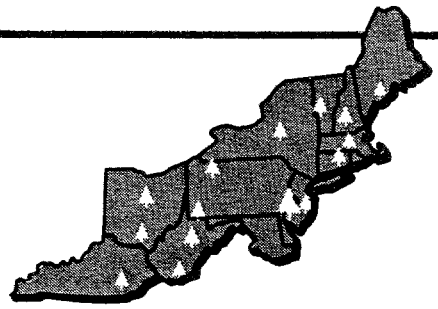

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NORTHEASTERN FOREST SURVEY BOARD-FOOT VOLUME EQUATIONS

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Abstract. International 1/4-inch board-foot volume equations are presented for the 17 species groups used in the forest survey of the 14 northeastern states. The volume equations are nonlinear in form.

As a result of the ever-increasing use of automated data processing equipment, the demand for volume tables in equation form has risen. Some find it more efficient to compute a volume for a given species, diameter, and bole length than to store and access the tables. Others may simply find the equation form more convenient to use than the tables. The board-foot volume equations presented were chosen for convenience and accuracy.

BACKGROUND

The development of the board-foot volume tables parallels that of the companion cubic-foot volume tables, as described by Barnard et al. (1973). The original data used to develop the tables are not available; thus only the individual cell values (means) were utilized in the development of the board-foot volume equations. The goal of the analysis was to find an equation form that would predict the individual cell values accurately and avoid trends in the residual errors (actual minus predicted values).

The volume tables are divided into 17 species groups. The resulting 17 tables give the International 1/4-inch board-foot volume by 2-inch diameter class and half-log height class (assumes a 1-foot stump). The minimum top diameter outside bark is 9 inches for hardwoods and 7 inches for softwoods. Diameters ranged from 10 to 40 inches for softwoods (species groups 1 to 6) and 12 to 40 inches for hardwoods (species groups 7 to 17). The number of half-logs ranged from 1 to 10. Thus the bole lengths in the equations ranged from 8 to 80 feet. The tables are applicable to the 14 northeastern states surveyed by the Resources Evaluation Unit of the Northeastern Forest Experiment Station.

EQUATION DEVELOPMENT

The recent rapid development of computer technologies has been paralleled by rapid developments in standard statistical programs. These packages have made the linear and nonlinear analyses of the board-foot volume tables used in

Table 1—Board-foot volume equation statistics

(International 1/4-inch log rule)

General equation form: $V = b_o + b_1 D^{b2} + b_3 D^{b4} H^{b5}$

Where:

V = gross board-foot volume

D = Diameter at breast height (dbh) in inches

H = bole length in feet

Group	Species	b_o	b_1	b_2	b_3	b_4	b_5	No. of cells	Average square error	Average relative error (%)	>5% <10%	>10%
Softwoods												
1	White, red pine	−12.25	−0.02418	2.6865	0.0961	2.2281	0.4222	147	29.0	1.3	8	2
2	Red, white, black spruce	−13.03	−0.05197	2.5248	0.1200	2.1999	0.4227	145	6.8	0.9	6	2
3	Balsam fir	−12.29	−0.08212	2.5641	0.1416	2.2657	0.3744	147	207.1	3.1	32	12
4	Hemlock	−8.36	−0.01433	2.7878	0.0771	2.2593	0.4202	137	29.2	1.2	7	1
5	Hard pines, tamarack, Norway spruce	−6.78	−0.00841	2.7001	0.0645	2.1938	0.4713	142	62.2	1.8	18	2
6	Cedar species	−8.89	−0.07324	2.4556	0.1216	2.2382	0.3249	116	4.1	0.6	2	0
Hardwoods												
7	Sugar maple	3.73	−0.00182	3.3766	0.0262	2.4291	0.6139	138	127.1	2.4	16	5
8	Soft maple, yellow-poplar	2.84	−0.00557	3.1808	0.0296	2.4606	0.5771	138	477.7	4.5	33	25
9	Ash species, aspen species	9.20	0.00052	3.0	0.0193	2.2165	0.8043	138	207.9	2.9	22	11
10	Black cherry	1.58	−0.00151	3.3878	0.0287	2.3875	0.6356	138	79.5	1.6	6	3
11	Birch species	8.23	0.00039	3.0	0.0206	2.2116	0.8019	138	203.4	3.1	23	11
12	Beech	−0.84	−0.01207	3.0043	0.0419	2.3951	0.5912	138	461.7	3.6	31	15
13	Basswood	2.66	−0.00313	3.2780	0.0282	2.4416	0.5940	138	232.4	3.0	30	7
14	Red oaks, sweetgum, blackgum	1.01	−0.00192	3.3188	0.0246	2.4268	0.6000	138	75.8	1.9	6	4
15	Chestnut oak	4.46	−0.00061	3.5972	0.0182	2.4804	0.5922	138	74.4	2.1	15	5
16	Hickory	−1.24	−0.00385	3.1648	0.0312	2.3888	0.6067	138	65.5	1.3	5	1
17	Other hardwoods	0.03	−0.00196	3.3236	0.0263	2.4162	0.6012	138	62.2	1.5	6	2

this study possible at a relatively low cost. Several simple and multiple linear equations, weighted and unweighted, as well as some logarithmic transformations of nonlinear equations, were tried. Finally, equations that are strictly nonlinear in their parameters were tried; they more adequately estimated the table values.

Nonlinear regression techniques have only recently come into widespread use.[1] With nonlinear regression the standard linear statistical tests cannot be applied with known reliability. Thus the equation forms were chosen for lack of trends in the residuals and minimum average relative error. The percent relative error is the absolute value of the difference between each table cell value and its predicted value, divided by the cell value. The average relative error is simply the average percent relative error over all cells in the table.

On this basis the equation form chosen was:

$$V = b_o + b_1 D^{b2} + b_3 D^{b4} H^{b5}$$

where:

$b_o . . . b_5$ = parameters to be estimated

For species groups 9 and 11, the nonlinear regression failed to converge because of the presence of the $b_1 D^{b2}$ term. With b_2 set equal to 3.0, the regression converged. The 3.0 value was chosen on the basis of the other hardwood species group values and on dimensional considerations.

BOARD-FOOT VOLUME EQUATIONS

The values of the parameters ($b_o . . . b_5$) and the number of "observations" (cells) for each species group are given in Table 1. Also shown are the average squared error (mean square error) and the average relative error. The number of values with relative errors greater than 5 percent but less than 10 percent, and the number greater than 10 percent are also given.

To use the equations, three items must be recorded for each tree; (1) species; (2) dbh; and (3) bole length (in feet) to the minimum diameter. As stated earlier, the minimum top diameter (outside bark) is 7 inches for softwoods and 9 inches for hardwoods. For example, the board-foot volume of a 22-inch dbh white pine, with a bole length of 56 feet, is 407 board feet, according to the table. [2] Using the equation yields a value of:

$$V = -12.25 - 0.02418(22)^{2.6865} + 0.0961(22)^{2.2281}(56)^{0.4222}$$
$$= 405.1 \text{ board feet}$$

The error in estimation is 1.9 board feet, or 0.46 percent. This equation form may be tedious to compute on a hand calculator but is easy to use on a computer.

The volume equations presented could not be chosen by rigorous statistical testing, because only the table cell means were available. The equations were developed to predict cell means as closely as possible with as few terms in the equations as possible. The use of the equation form is not necessarily advocated; the equations were developed only for convenience and efficiency.

REFERENCES

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[1] For a detailed treatment of the Gauss-Newton method of nonlinear regression, refer to sources such as Chapter 8 of Draper and Smith (1966).

[2] The board-foot volume tables used in the northeastern forest survey and in this study are available upon request from the Resources Evaluation Unit of the Northeastern Forest Experiment Station, 370 Reed Road, Broomall, PA. 19008.

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