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Northeastern Forest Survey Revised Cubic-Foot Volume Equations

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Abstract. Cubic-foot volume equations are presented for the 17 species groups used in the forest survey of the 14 northeastern states. The previous cubic-foot volume equations were simple linear in form; the revised cubic-foot volume equations are nonlinear.

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As the value of timber increases, the importance of accurately estimating tree volume increases. In an effort to improve the accuracy of volume estimation, I revised the cubic-foot volume equations used in the Northeastern Forest Survey. No new data were collected; only the equation form was changed.

Background

In the late 1940's, a set of cubic-foot volume tables was developed and used by the Northeastern Forest Survey. In the 1950's, the cubic-foot volume tables were converted to equation form by using simple linear regression and were published later by Barnard, Bickford, and Mayer (1973). They described the background of the equation development. Seventeen cubic-foot volume equations were formed. Species were grouped based on similarities in tree form. The resulting 17 tables give the gross cubic-foot volume of wood by 2-inch diameter class and by number of 8-foot bolts (assumes a 1-foot stump). The minimum top diameter (outside bark) is 4 inches. Diameters ranged from 5 to 41 inches. The number of bolts ranged from 1 to 10. Thus the bole lengths used in the equations ranged from 8 to 80 feet. The tables are applicable to the 14 northeastern states surveyed by the Resources Evaluation Unit of the Northeastern Forest Experiment Station.

The equations in this report were developed directly from the tables rather than from individual tree data. The same approach was used to develop the northeastern board-foot volume equations (Scott 1979). Rather than using simple linear regression, I used nonlinear regression techniques. The same equation form and regression techniques were used to develop regression coefficients for the cubic-foot equations presented here. Because the equation form is a generalization of the simple linear equation, the nonlinear equations should be more accurate in estimating the table values.

Equation Development

Nonlinear regression techniques only recently came into widespread use. This was possible because of advances in computer technology and statistical software. The nonlinear regression technique used here is the Gauss-Newton method.[1] With nonlinear regression, the usual linear statistical tests cannot be applied with known reliability. Thus the equations were chosen for lack of trends in the residuals and minimum average relative error. The percent relative error is the absolute value of the difference between the table value and predicted value, divided by the table value. The average relative error is simply the percent relative error averaged over the number of table values.

The same equation form used for the board-foot volume equations was chosen for the cubic-foot volume equations.

$$V = b_0 + b_1 D^{b_2} + b_3 D^{b_4} H^{b_5}$$

where:

$b_0 \ldots b_5$ = parameters to be estimated

If parameter b_1 equaled 0, b_4 equaled 2, and b_5 equaled 1, this equation would reduce the simple linear equation form used previously.

The equation form given above failed to converge for chestnut oak (species group 15). As a result, the b_2 term was set equal to 2.0 on the basis of the parameter's value for other species groups.

[1] For detailed treatment of the Gauss-Newton method of nonlinear regression, refer to sources such as Chapter 8 of Draper and Smith (1966).

Cubic-Foot Volume Equations

The estimated values of the parameters ($b_0 \ldots b_5$) and the number of table values (cells) for each species group are given in Table 1. Also shown are the average squared error (mean square error) and the average relative error. The average squared errors were reduced from a range of 138.2 to 1078.0 for the linear equations to 0.2 to 7.1 for these nonlinear equations. The number of values (cells) with relative errors greater than 5 percent but less than 10 percent, and the number greater than 10 percent are also given. A difference of less than 5 percent was considered desirable.

To use the equations, three items must be recorded for each tree: (1) species, (2) dbh, and (3) bole length (in feet). The bole length is taken from a 1-foot stump to a flexible top diameter (outside bark) of 4 inches or more when knots or other limits to merchantibility are below the 4-inch point. The estimated gross cubic-foot volume (excluding bark) of an 18-inch dbh beech, with a bole length of 48 feet, is 54 cubic feet according to the table.[2] The equation yields a value of:

$$V = -0.60 - 0.00711\,(18)^{2.2693} + 0.01399\,(18)^{2.0190}\,(48)^{0.6518}$$

$$= 54.1 \text{ cubic feet}$$

The error in estimation is 0.1 cubic feet, or 0.18 percent. This equation form may be tedious to compute on a pocket calculator, but it is easy to use on a computer.

[2] The cubic-foot volume tables used in the Northeastern Forest Survey and in this study are available upon request from the Resources Evaluation Unit, Northeastern Forest Experiment Station, 370 Reed Road, Broomall, PA 19008.

The cubic-foot volume equations presented here are an improvement on the previously published equations. The nonlinear form more closely approximates the table values, particularly for the smallest diameter classes. Because only the table values were available, the equations presented could not be chosen by rigorous statistical testing. The equations were developed to predict table values as closely as possible with as few parameters as possible. The use of the equation form is not necessarily advocated; the equations were developed only for convenience and improved accuracy.

Literature Cited

Barnard, Joseph E,; Bickford, C, Allen; Mayer, Carl E. Forest survey cubic-foot volume equations. 1973; USDA For. Serv. Res. Note NE-166. 2 p.

Draper, N.R.; Smith, H. Applied regression analysis. New York: John Wiley and Sons, Inc.; 1966.

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Table 1.—Cubic-foot volume equation statistics

General equation form: $V = b_0 + b_1 D^{b_2} + b_3 D^{b_4} H^{b_5}$

Where:

V = gross cubic-foot volume
D = Diameter at breast height (dbh) in inches
H = bole length in feet

Group	Species	b_0	b_1	b_2	b_3	b_4	b_5	No. of cells	Average squared error	Average relative error (%)	No. of cells	
											> 5% <10%	>10%
Softwoods												
1	White, red pine	0.11	-0.05977	2.0498	0.04965	2.0198	0.3468	171	0.2	1.5	9	4
2	Red, white, black spruce	0.17	-0.06315	2.0654	0.05122	2.0264	0.3508	170	0.3	1.7	13	3
3	Balsam fir	-0.10	-0.05444	2.1194	0.04821	2.0427	0.3579	171	1.1	2.2	21	4
4	Hemlock	0.24	-0.05895	2.0362	0.04947	2.0172	0.3366	149	0.2	1.6	10	4
5	Hard pines, tamarack, Norway spruce	-0.03	-0.05604	2.0473	0.05022	2.0198	0.3242	162	0.2	1.6	10	5
6	Cedar species	0.19	-0.05904	1.9935	0.04981	2.0027	0.3214	136	0.2	1.6	14	2
Hardwoods												
7	Sugar maple	-0.19	-0.01171	1.8949	0.01340	1.9928	0.6471	174	0.4	1.9	3	6
8	Soft maple, yellow-poplar	-0.45	-0.00523	2.2323	0.01338	2.0093	0.6384	174	1.0	2.6	16	7
9	Ash species, aspen species	0.06	-0.02437	1.5419	0.01299	1.9885	0.6453	174	0.4	1.8	3	6
10	Black cherry	-0.04	-0.01783	1.8109	0.01358	1.9905	0.6553	174	0.4	1.7	1	6
11	Birch species	-0.27	-0.00675	1.9738	0.01327	1.9967	0.6407	174	0.4	1.9	5	7
12	Beech	-0.60	-0.00711	2.2693	0.01399	2.0190	0.6518	174	1.4	3.0	18	9
13	Basswood	-0.39	-0.00622	2.0066	0.01310	1.9939	0.6494	174	2.9	2.6	13	7
14	Red oaks, sweetgum, blackgum	-0.13	-0.00536	~~1.1972~~ 1.9172	0.01131	1.9975	0.6549	174	0.3	2.0	8	6
15	Chestnut oak	-0.26	0.00038	2.0	0.01068	1.9980	0.6438	174	0.7	2.1	6	7
16	Hickory	-0.27	-0.00466	2.1575	0.01174	2.0035	0.6640	174	0.4	2.0	11	6
17	Other hardwoods	0.13	-0.00183	2.3600	0.00944	2.0608	0.6516	174	7.1	3.1	4	14

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