

Modeling the migration of tight bends: a comparison between analytical and numerical approaches for simulating the hydrodynamics and considerations about migration

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Abstract: *Currently, only two-dimensional (2D) linear analytical models can be practically used to predict meander migration of rivers at reach-scale. Such models assume that the ratio of channel width to radius of curvature is much smaller than one. Unfortunately, sharp bends are commonly found in nature and in constructed re-meandered channels. Moreover, linear models assume or compute bed topography with a certain degree of idealization. The hydrodynamic fields obtained with 2D linear and fully non-linear models are compared for a restored, highly meandering reach on Trout Creek in the Tahoe Basin, California, USA. Analysis of observed localized bank retreat suggests that a fully non-linear model is required for meander-migration simulations in tight bends, while the overall migration patterns and rates can still be captured fairly well by the linear model.*

Keywords: *Linear hydrodynamic model, non-linear hydrodynamic model, meander migration, bathymetry, topographic steering, secondary flow.*

1. INTRODUCTION

During the last decade the United States has seen a rapid increase in spending to restore ecological functions and processes of degraded streams and their floodplain (Bernhardt et al., 2005). Currently, a similar effort to re-naturalize streams is taking place within the European Union under the European Water Framework Directive (Kaika, 2003). Stream restoration projects at corridor scale often require the establishment of a stable planform configuration. Since the late 1970s much research, both theoretical, experimental, and in the field, has been carried out to better understand and quantify the migration of meandering streams. This has yielded analytical 2D models with different degrees of sophistication, fully numerical 2D models, and 3D models of flow, sediment transport, and channel migration. As of today, only the analytical 2D models can be practically used to predict migration at reach scale. The general approach, based on the governing equations of water continuity and momentum balance, either solves for sediment conservation or makes an assumption for the bed transverse profile in bends, and it is limited to the case of mildly curved channels, for which a linearization approach can be adopted. Linear models were developed, among others, by Ikeda et al. (1981), Blondeaux and Seminara (1985), Johannesson and Parker (1989a), Johannesson and Parker (1989b), Sun et al. (2001), and Zolezzi and Seminara (2001). These process-based models provide a good approximation of the fluvial processes in bends and give formulations for velocity and depth that are used to compute the rate and direction of migration (Johannesson and Parker, 1985; Garcia et al., 1994; Abad and Garcia, 2006; Camporeale et al., 2007). On the other hand, they present some shortcomings. In particular, they strictly could not be applied to cases characterized by high local curvature. Unfortunately, sharp bends are not uncommon in nature, and are also frequently observed

in man-made channels like those re-meandered for restoration purposes. Moreover, linear models reproduce bed topographies which present a certain degree of idealization: for instance, only point bars are captured in the case of Ikeda et al. (1981)'s model, and point and alternate bars in the case of the models by Johannesson and Parker (1989b) and Zolezzi and Seminara (2001). Natural rivers present in general more arbitrarily complex bathymetries, and man-made channels are characterized by further morphological forcing, represented by the typical sequence of riffles and pools, implemented from design, associated to dramatic changes in depth and roughness.

The impact of both linear models' limitations – validity for low curvature and idealized bathymetry – is analyzed in this paper, which, for the case of the highly-sinuuous man-made Trout Creek in Tahoe Basin (California, USA), aims to assess the performance of a linear analytical model in terms of hydrodynamic description and migration-prediction capabilities.

2. LINEAR AND NON-LINEAR MODEL

The linear and non-linear 2D models herein compared are implemented respectively in the RVR Meander and River2D platforms. The RVR Meander platform (Langendoen et al., 2010; Motta et al., 2011) merges and extends the functionalities of the first version of RVR Meander (Abad and Garcia, 2006) and CONCEPTS (Langendoen and Alonso, 2008; Langendoen and Simon, 2008; Langendoen et al., 2009), and is composed by a hydrodynamics/bed morphodynamics module, a bank erosion module, and a migration module. Hydrodynamics and bed morphodynamics are computed according to the linear model illustrated by Garcia et al. (1994), which is a slightly modified version of Ikeda et al. (1981)'s and Johannesson and Parker (1985)'s models. It provides an analytical solution of the linearized 2D St. Venant Equation, written in streamwise and transverse coordinates:

$$\frac{1}{1+n^*C^*}U^*\frac{\partial U^*}{\partial s^*}+V^*\frac{\partial U^*}{\partial n^*}+\frac{C^*}{1+n^*C^*}U^*V^*=-\frac{g}{1+n^*C^*}\frac{\partial H^*}{\partial s^*}-\frac{\tau_s^*}{\rho D^*} \quad (1)$$

$$\frac{1}{1+n^*C^*}U^*\frac{\partial V^*}{\partial s^*}+V^*\frac{\partial V^*}{\partial n^*}-\frac{C^*}{1+n^*C^*}U^{*2}=-g\frac{\partial H^*}{\partial n^*}-\frac{\tau_n^*}{\rho D^*} \quad (2)$$

$$\frac{1}{1+n^*C^*}\frac{\partial(U^*D^*)}{\partial s^*}+\frac{\partial(V^*D^*)}{\partial n^*}+\frac{C^*}{1+n^*C^*}V^*D^*=0 \quad (3)$$

Superscript star indicates values with dimensions, s^* and n^* are streamwise and transverse coordinates, U^* and V^* are streamwise and transverse depth-averaged velocities, g is the acceleration of gravity, H^* is water stage, τ_s^* and τ_n^* are streamwise and transverse bed shear stresses, ρ is water density, D^* is water depth, and C^* is the local curvature. The bed morphology is described assuming that the transversal bed slope is proportional to the local curvature through a constant of proportionality A named scour factor (Ikeda et al. (1981)'s assumption)

$$\frac{\eta_1^*}{D_{ch}^*}=-AC^*n^* \quad (4)$$

where η_1^* is the bed elevation deviation from the bed elevation of the channel centerline at a particular cross section and D_{ch}^* is the reach-averaged water depth for uniform flow. An analytical solution, for low curvature and constant discharge and channel width, is obtained for streamwise and transverse velocity and water depth, expressed as sum of their uniform-flow reach-averaged value plus a perturbation, which varies in both streamwise and transverse directions. The solution depends on upstream and local curvature, sinuosity, width-to-depth ratio, Froude number, and reach-averaged friction coefficient; it is not reported here for brevity (see Garcia et al. (1994) and Motta et al. (2011)). Two alternative methods are used for computing bank erosion and centerline migration: with the first method (migration-coefficient method) the outer bank displacement rate, equal to the centerline displacement rate (so that the channel width keeps constant), is proportional to the excess velocity at the outer bank through a dimensionless migration coefficient:

$$R^*=E_0(U_b^*-U_A^*) \quad (5)$$

R^* is the migration rate (with dimensions of length over time), U_A^* is the cross-sectionally-averaged velocity, and U_b^* is the depth-averaged near-bank velocity. The dimensionless migration coefficient E_0 is usually obtained via calibration against historic channel centerlines. With the second method (physically-based method), the centerline displacement rate is based on the modeling of the physical processes responsible for bank retreat (hydraulic erosion, cantilever failure, and planar failure). Here just hydraulic erosion is considered, which is expressed as

$$R^* = M^* \left(\tau^* / \tau_c^* - 1 \right) = k^* \left(\tau^* - \tau_c^* \right) \quad (6)$$

where M^* is the erosion-rate coefficient (with dimensions of length over time), τ_c^* is the critical shear stress, $k^* = M^* / \tau_c^*$ is the erodibility, and τ^* is the shear stress acting on the bank (here taken as equal to that calculated at the boundary bed/bank by the analytical flow model). M^* , τ_c^* and k^* are all site-specific.

The River2D model is a 2D Finite Element hydrodynamic model (Steffler and Blackburn, 2002). It is basically a transient model but also provides for an accelerated convergence to steady-state conditions, which solves for the two horizontal components of flow velocity and for the depth the 2D complete St. Venant equations in Cartesian coordinates:

$$\frac{\partial U^* q_x^*}{\partial x^*} + \frac{\partial V^* q_x^*}{\partial y^*} + \frac{g}{2} \frac{\partial D^{*2}}{\partial x^*} = gD^* (S_{0x} - S_{fx}) + \frac{1}{\rho} \left(\frac{\partial D^* \tau_{xx}^*}{\partial x^*} + \frac{\partial D^* \tau_{xy}^*}{\partial y^*} \right) \quad (7)$$

$$\frac{\partial U^* q_y^*}{\partial x^*} + \frac{\partial V^* q_y^*}{\partial y^*} + \frac{g}{2} \frac{\partial D^{*2}}{\partial y^*} = gD^* (S_{0y} - S_{fy}) + \frac{1}{\rho} \left(\frac{\partial D^* \tau_{yx}^*}{\partial x^*} + \frac{\partial D^* \tau_{yy}^*}{\partial y^*} \right) \quad (8)$$

$$\frac{\partial q_x^*}{\partial x^*} + \frac{\partial q_y^*}{\partial y^*} = 0 \quad (9)$$

Most of the notations have been introduced earlier, $q_x^* = U^* H$ and $q_y^* = V^* H$ are the discharge intensities, S_{0x} and S_{0y} are the bed slopes, S_{fx} and S_{fy} are the friction slopes, and τ_{xx}^* , τ_{xy}^* , τ_{yx}^* , and τ_{yy}^* are the components of turbulent stress tensor.

3. APPLICATION

3.1. Case study

Simulations are presented for the case of Trout Creek, which is a restored, highly-meandering reach in the Tahoe Basin (California, USA). It was re-meandered in 2000, and its highly sinuous configuration has shown bank instability during the past ten years. Figure 1a shows the old ("Pre-design") and new ("Design") alignments, on an aerial picture taken in 2005. The re-meandered reach goes from the Pioneer Trail gage (upstream end) to the gage near Tahoe Valley at Martin Avenue (downstream end). The "Design" alignment was implemented in stages through 2000-2001, and water was first introduced into the channel in October 2001. The "Pre-design" sinuosity is 1.43 (reach length of about 2700 m), while, for the "Design" alignment, it is 1.79 (about 3400 m). The bankfull discharge is 2.21 m³/s and 2.97 m³/s respectively upstream and downstream of the confluence of the tributary Cold Creek highlighted in Figure 1a (Swanson Hydrology + Geomorphology, 2004). As regards the section properties, in the portion upstream (downstream) of the Cold Creek confluence, the average bottom width is 2.12 m (2.74 m), the average top width is 5.15 m (6.71 m), and the average channel depth is 0.85 m (1.22 m). The geometric mean size of the sediments in the bed is 1.023 mm (range 0.012 - 6.429 mm). The valley slope is about 0.34%. The channel is characterized by very low width-to-depth ratio and very high local curvatures and closely consecutive bends. The bed topography is composed by a sequence of riffle and pool-point bar sections (Figure 1b). Flow goes overbank at least once per year, reshaping the floodplain. The most observed bank failure mechanism is cantilever (Figure 1c),

caused by fluvial erosion at the toe, and in some cases the failed material remains at the toe of the channel and protects the bank. A field campaign was conducted in Trout Creek in the summer of 2008. Several instruments were used for bank-properties characterization: among them, estimation of critical shear stress and erosion-rate coefficient for hydraulic erosion (Eqn. 6) was carried out with either a Jet-Testing Apparatus (Hanson and Cook, 2004; Figure 2a) or a Cohesive Strength Meter (Paterson, 1989; Figure 2b).

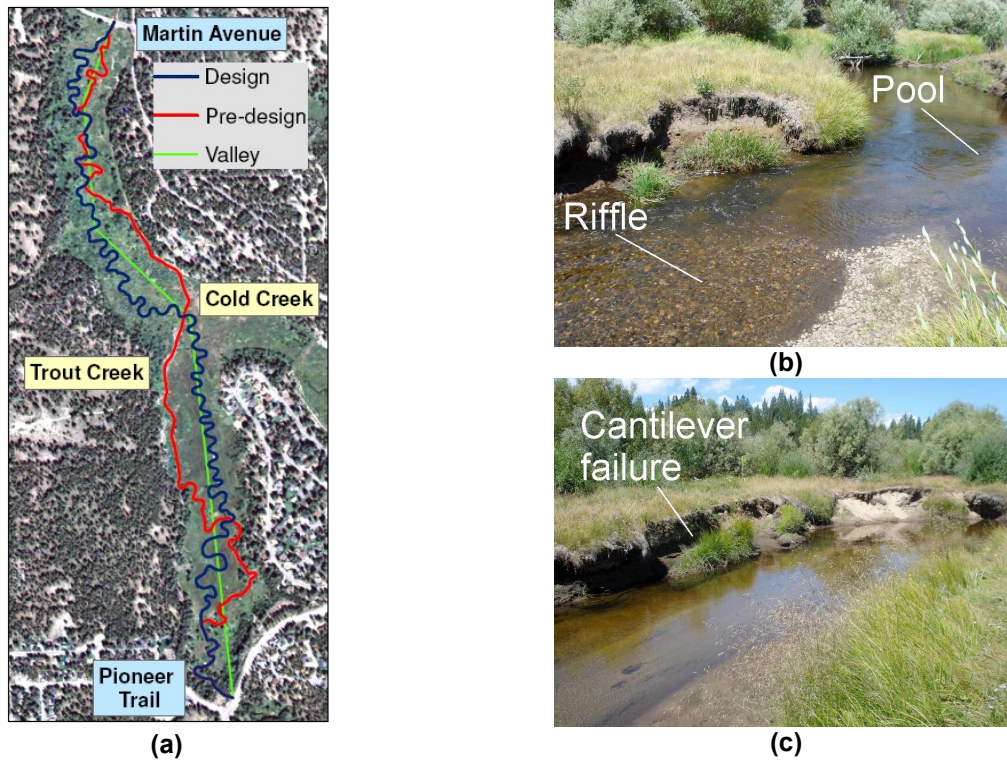


Figure 1 (a) Alignments for Trout Creek in the Tahoe Basin (California, USA), flow is northward; (b) sequence riffle-pool; (c) cantilever failure at the outer bank.



Figure 2 Field instrumentation used in the field campaign of summer 2008 for bank erosion characterization: (a) jet-testing apparatus; (b) Cohesive Strength Meter.

3.2. Simulations

Two sets of simulations are presented. The first set is aimed to compare the performance of linear analytical model (implemented in RVR Meander) and fully non-linear numerical model (implemented in River2D) for the same highly-sinuous planform configuration ("Design" alignment in Figure 1a) and same bed topography (Ikeda et al. (1981)-type bed, i.e. curvature-induced topography, see Eqn. 4), in order to investigate the extent to which using a linear model is still reasonable for modeling

hydrodynamics and making meander-migration considerations in case of planform configurations characterized by very high curvatures. The goal of the second set of simulations is to identify the error made when applying a linear model for centerline-migration prediction in relatively short-term scenarios and especially in presence of a bed topography and roughness field which are substantially different from what assumed by the linear model.

It is worth discussing here the assumption of constant channel width (in time and space) required by the analytical model adopted. Observations show that many meandering alluvial streams, in the long term, maintain a roughly constant width even while actively migrating (Ikeda et al., 1981), and aerial images generally confirm this for Trout Creek. Therefore, making hydrodynamic computations for constant channel width is reasonable. As regards migration, localized and temporary widening in meandering channels may happen (Nanson and Hickin, 1983). For the purpose of this study, we recognize and accept the approximation introduced, in light of others made in the modeling, e.g. the simplified bed bathymetry.

3.3. Results

3.3.1. Comparison of linear and-linear models for highly sinuous configuration

A reach of Trout Creek that has shown clear and significant bank retreat at the outer bank is considered (Figure 3a). Bankfull discharge of $2.21 \text{ m}^3/\text{s}$ and constant channel width of 3.63 m, equal to the average between top and bottom width, are used. Bankfull depth is on average 0.85 m. A Ikeda et al. (1981)-type bed topography can be generated by estimating the scour factor A , which characterizes the transverse bed slope (Eqn. 4), from the cross section profiles at sections 1-B and 1-D (Figure 3b,c), knowing local transverse bed slope, curvature, and depth. The resulting values are respectively 1.64 and 2.75. The friction coefficient is estimated such that bankfull depth is obtained for bankfull flow, and corresponds to Manning's roughness coefficient of 0.05.

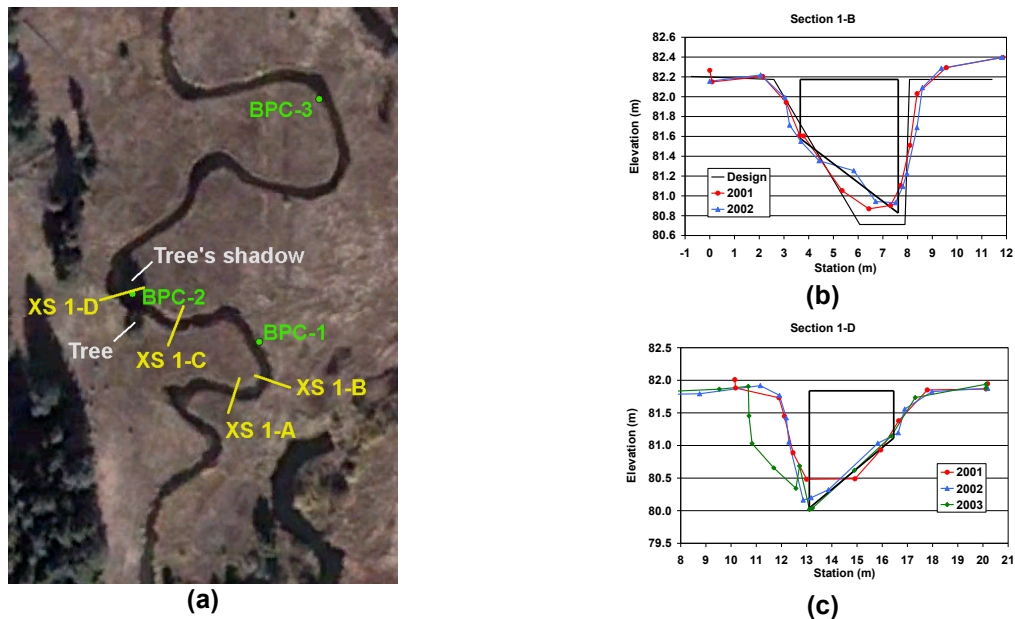


Figure 3 (a) Trout Creek reach considered for simulations (XS = surveyed cross section, BPC = bank-properties-characterization location), flow is northward; estimation of the scour factor for (b) Section 1-B and (c) Section 1-D.

Figure 4 and Figure 5 show the comparison between the flow fields predicted by linear model (RVR Meander) and fully non-linear model (River2D), in terms of flow velocity (Figure 4) and bed shear stress magnitudes (Figure 5), for curvature-induced bed topography with scour factor of 2 (based on the estimated values at sections 1-B and 1-D). Both models do not consider any secondary flow correction for lateral redistribution of primary flow, and in both cases the shear stress field is calculated assuming a spatially constant friction coefficient. It can be observed that the linear model provides

fairly similar results to those obtained with the complete non-linear model, both in terms of magnitude and distribution of the flow variables, which therefore justify the use of linear models outside of their theoretical mild-curvature validity range. On the other hand, especially for the tight bends like those shown in detail in Figures 4-5, we notice an earlier transition of the core of high velocity and bed shear stress from consecutive and opposite outer banks (for effect of “topographic steering”) in the case of the non-linear model, as also observed by Camporeale et al. (2007, their Figure 8) and Bolla et al. (2009, their Figure 19) for simpler planform configurations. This, as shown later, might have a consequence on channel migration patterns. Finally, the turbulent stresses in the RHS of Eqns. 7-8 have a negligible effect on the flow distribution in Figures 4-5 for River2D.

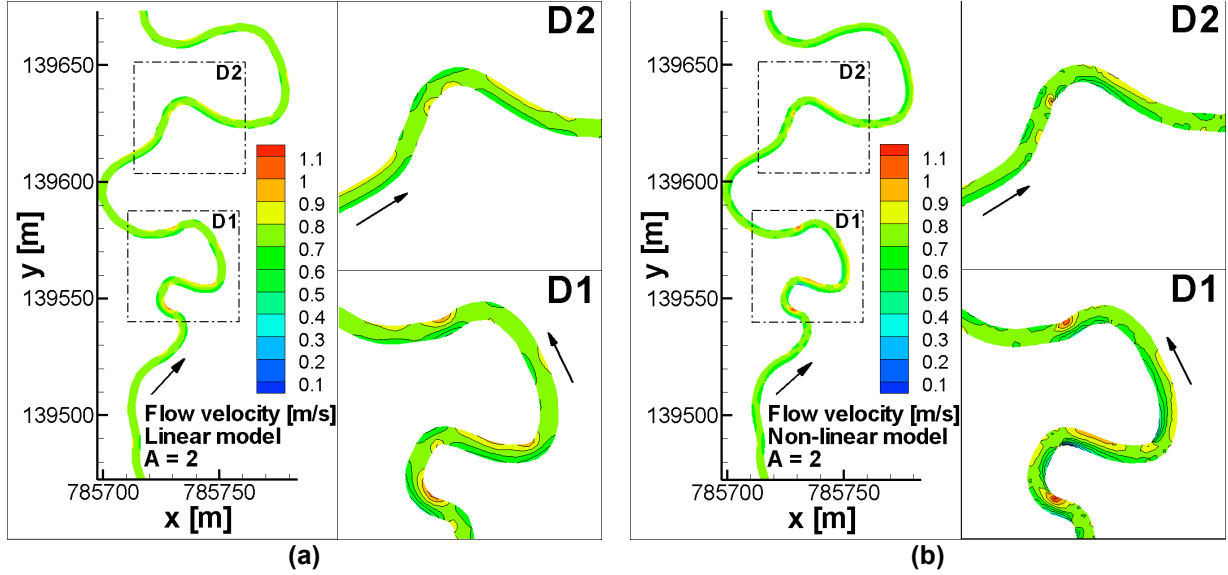


Figure 4 Comparison between linear model (RVR Meander) and non-linear model (River2D) for flow velocity magnitude (scour factor of 2).

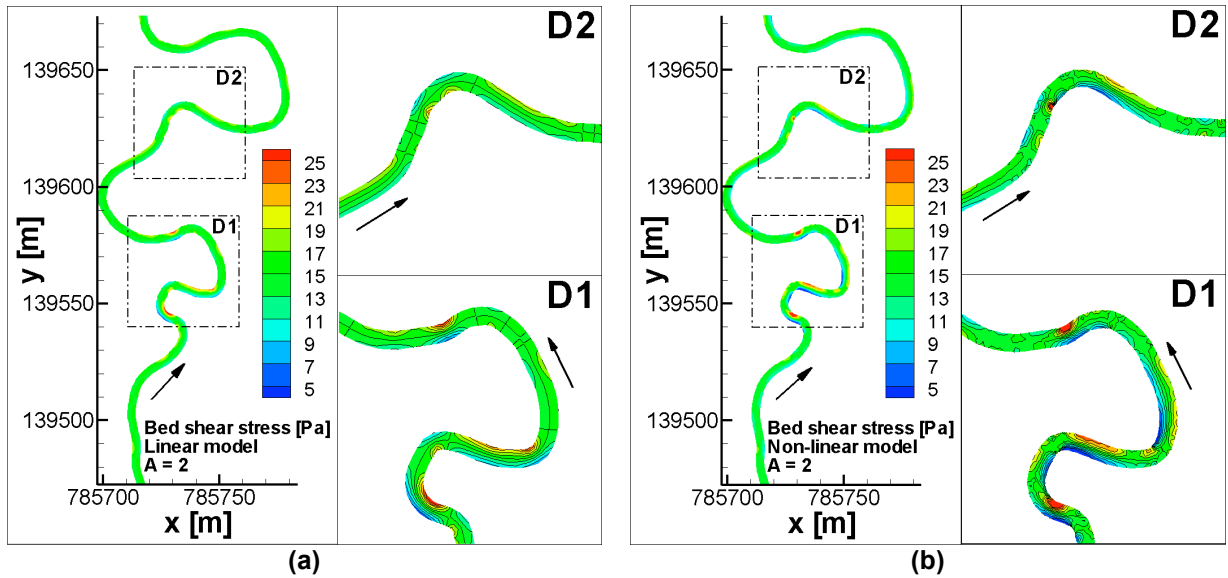


Figure 5 Comparison between linear model (RVR Meander) and non-linear model (River2D) for bed shear stress magnitude (scour factor of 2).

3.3.2. Migration-prediction performance of the linear model

The simulations presented earlier refer to a curvature-induced bathymetry which dramatically differs from that actually present in the creek, implemented by design and composed by a sequence of riffle

sections (in the straight reaches between bends) and pool-point bar sections (at the bends). This configuration represents further morphological forcing on the flow, both in terms of depth and roughness (riffles sections are rougher than pool-point bar sections). Figure 6 shows a comparison between the longitudinal bed profile predicted by the analytical model (Ikeda et al. (1981)-type bed bathymetry with scour factor of 2) and the thalweg of the implemented design: the difference is significant, and this may translate into differences between the migration patterns observed and those simulated with the linear model considering the idealized bed bathymetry.

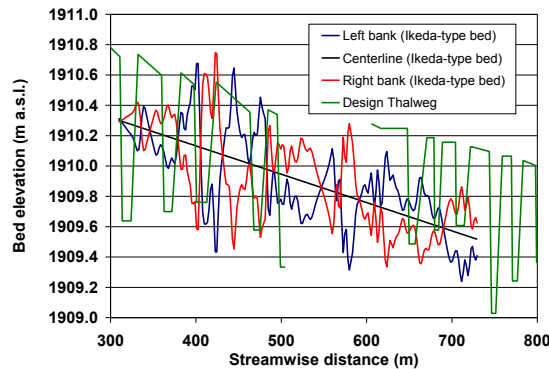


Figure 6 Comparison between the bed bathymetry assumed by the analytical model (Ikeda et al. (1981)-type bed) and that implemented by design in Trout Creek.

RVR Meander is here used to predict the creek migration in the period from 2001 to 2007, adopting either the migration-coefficient approach (MC, Eqn. 5, with calibrated migration coefficient of $5 \cdot 10^{-8}$) or the physically-based approach (PB, Eqn. 6, with measured average critical shear stress of 2.12 Pa and a calibrated erosion-rate coefficient of $8 \cdot 10^{-10}$ m/s to match the observed migration rates and indirectly account for flow intermittency and bank erosion protection by slump blocks at the toe). Figure 7 shows the comparison of observed and simulated centerlines for the year 2007, using two alternative values of scour factor A : 2, which is the one actually extracted from the surveyed cross sections, and 4.5, value which was calibrated to indirectly account for the effect of secondary flow on the lateral redistribution of streamwise momentum towards the outer bank, following the approach developed by Johannesson and Parker (1989a). In the case $A = 2$, the general tendency of the simulated planform configuration is toward straightening, which is opposite to what actually observed (see details D1 and D2 with $A = 2$); on the contrary, in the case $A = 4.5$, both MC and PB approaches perform generally well (see details D1 and D3 with $A = 4.5$), which means that correcting for the effect of secondary flow on the velocity and bed shear stress distributions is important in this case. The PB method performs slightly better than the MC method, since it predicts a less smooth and more localized migration where the channel has an abrupt change of direction; the MC approach, instead, is somewhat biased toward downstream migration (see detail D1 with $A = 4.5$). Notice however that the migration rates over the period considered are rather small (maximum 1.5 meters), therefore it is difficult to actually identify a clearly better performance between the two methods for bank erosion. The observed migration is definitely not captured at the bend shown in detail D2 in Figure 7. Two might be the explanations: (1) effect of the non-linearity of the flow, associated to velocities and bed shear stresses at the bend apex higher than those computed by the linear model (see the flow fields in Figure 4-5 for the detail D2); effect of riffle-pool topography (not considered by the linear model), with fast flow coming from the upstream shallow riffle section shooting against the bank, as observed in the field by Frothingham and Rhoads (2003) for the case of the Embarras River in Illinois, USA.

4. DISCUSSION AND CONCLUSION

When compared to a fully non-linear model for high-curvature planform configuration and on the same idealized bathymetry, the linear model still provides a fairly accurate representation of the hydrodynamic field. However, an earlier transition of the core of high velocity and bed shear stress from consecutive and opposite outer banks is predicted by the non-linear model. This difference becomes significant in the case of tight banks.

In terms of migration prediction, the linear model captures the overall pattern and rates, provided that

the impact of secondary flow on the lateral distribution of streamwise momentum is accounted for. The physically-based method for bank erosion shows a better capability to reproduce the localized nature of migration than the migration-coefficient approach, even if differences are rather small for the case of Trout Creek. The analysis suggests that topographic forcing (in this case the riffle-pool sequences) and non-linearity of the flow might generate localized bank retreat in bends characterized by an abrupt change of channel direction. Trout Creek is currently adjusting to recover a topography which resembles the curvature-induced topography used for the simulations with RVR Meander. The effect of the riffle-pool sequences is therefore expected to decrease in time, and the linear model to provide a good tool for stability and meander-migration considerations, even in presence of low width-to-depth ratio that would question the applicability itself of a 2D depth-averaged shallow-water model.

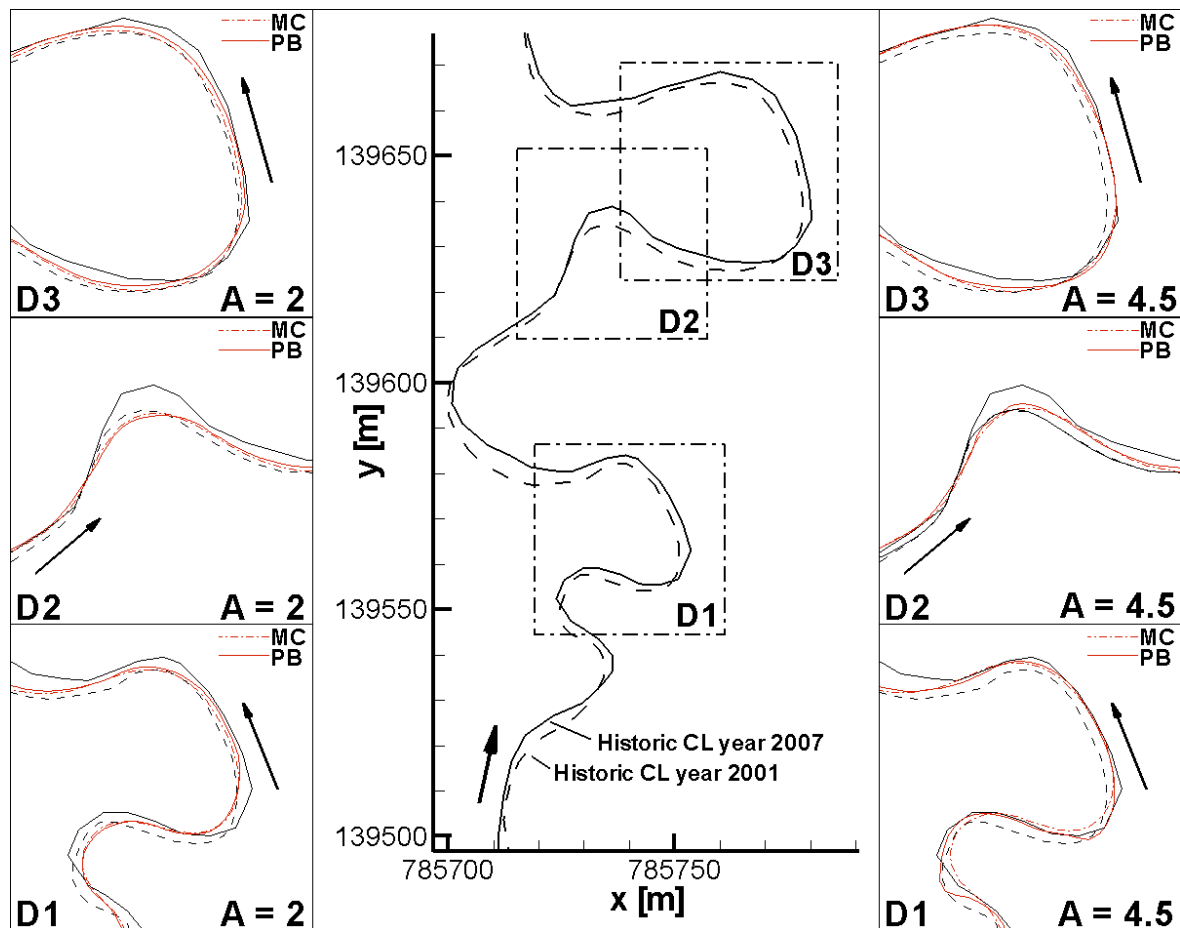


Figure 7 Trout Creek: comparison of observed 2007 centerline and simulated with the linear model (RVR Meander).

5. ACKNOWLEDGMENTS

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